Macro factors and sovereign bond spreads: a quadratic no-arbitrage model

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Discussion:

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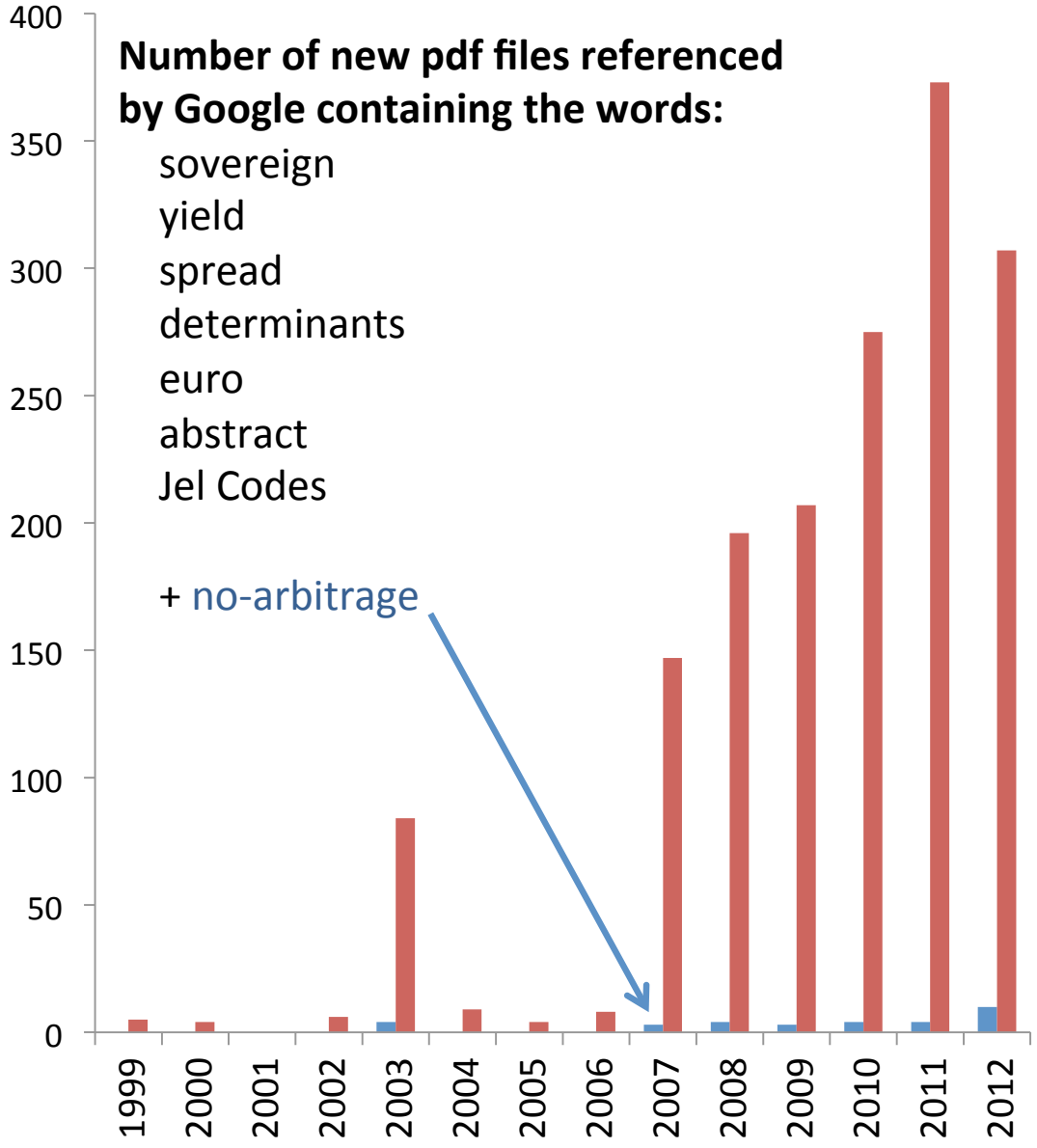
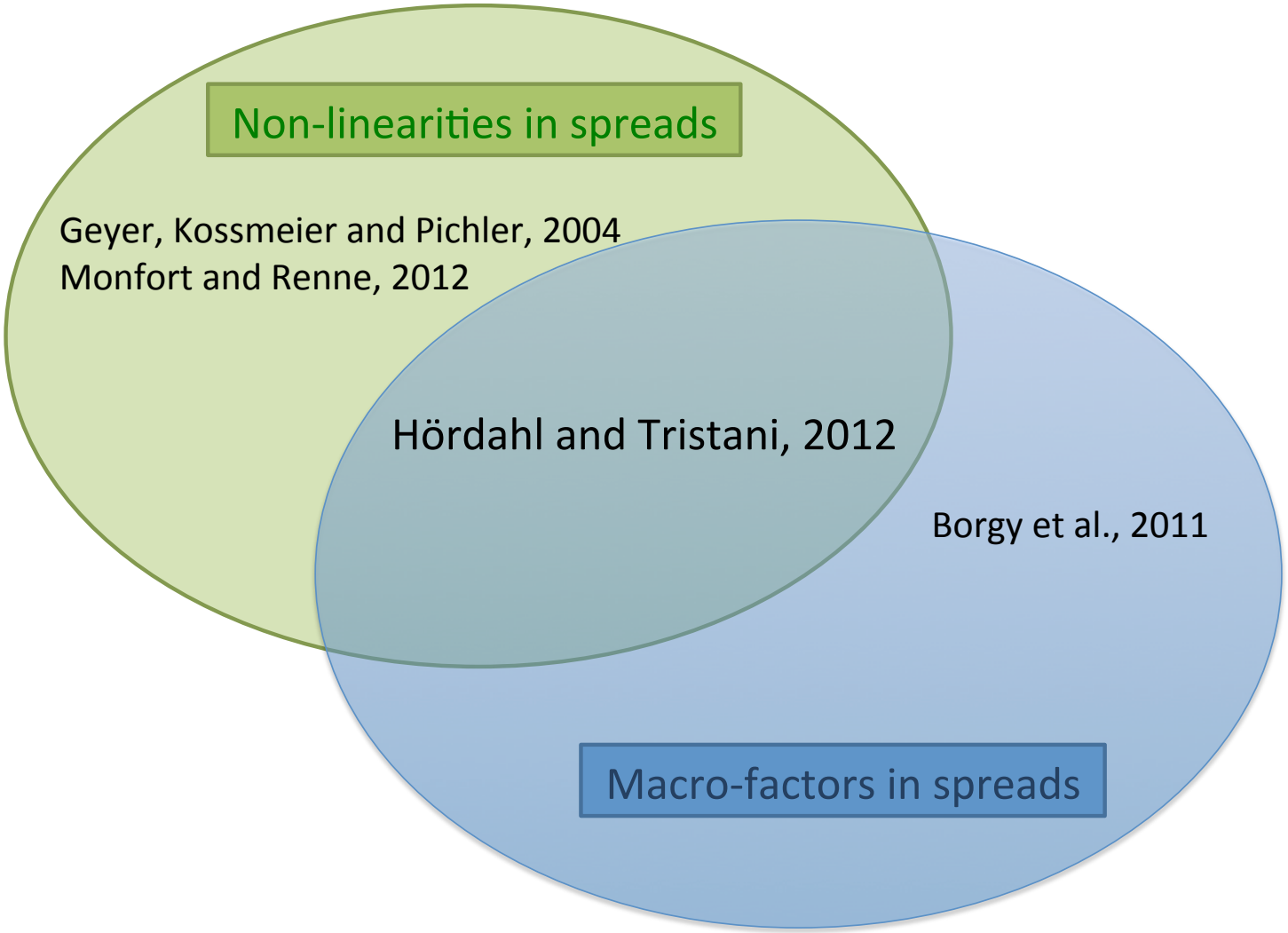
Euro-area no-arbitrage term-structure models

- Many papers studying euro-area spreads using regression techniques.
- Far less papers exploiting the cross-sections of yields (maturities + countries) in a no-arbitrage framework; though this allows to:
  - compute risk premia and derive physical PDs,
  - improve the estimation of the relationship between factors and yields.



Euro-area no-arbitrage term-structure models



The paper in a nutshell

- Estimation of the joint dynamics of 5 term structures of spreads vs Germany (France, Greece, Italy, Portugal and Spain)
- Main results:
  - The model-implied proba. of default (PDs) are related to
    - macroeconomic factors: growth rates and debt/GDP ratios.
    - a common latent factor.
  - Probabilities of default depend non-linearly on Debt/GDP.
  - Risk premia account for the main part of spreads
    - Risk premia are predominantly driven by the common factor.
  - Technical achievements:
    - The model captures salient features of spread volatilities.
    - Good forecasting performances.
    - Interesting time-varying impulse responses.

Model

- The factors: $X_t^i = [C_t, Y_t^i]$
  - Unobserved common factor: C_t
  - Observed country-specific factors $Y_t^i = [g_t^i, d_t^i]$.
- Dynamics of the factors:

$$X_t = \begin{bmatrix} \phi_{C,C} & 0 & 0 \\ 0 & \phi_{g,g} & 0 \\ \phi_{d,c} & \phi_{d,g} & \phi_{d,d} \end{bmatrix} X_{t-1} + \begin{bmatrix} \varepsilon_{C,t} \\ \varepsilon_{g,t} \\ \varepsilon_{d,t} \end{bmatrix}$$

Model

- Default process ($D_{i,t} = 1$: default; $D_{i,t} = 0$: no default):

$$P\left(D_{i,t} = 1 \middle| D_{i,t} = 0, X_t\right) = 1 - \exp(-\Lambda_{i,t})$$

or

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- Default intensity:

$$\begin{aligned} \Lambda_{i,t} &= \lambda_0^i + \lambda_C^i C_t + \lambda_g^i g_t^i + \lambda_d^i d_t^i + \ldots \\ &\quad \ldots \lambda_{dd}^i \times \left(d_t^i\right)^2 \end{aligned}$$

Model

- A stochastic discount factor is specified, exponential affine in X_t, such that under $\mathbb{Q}$:

$$X_t = \begin{bmatrix} \phi^*_{c,c} & 0 & 0 \\ 0 & \phi^*_{g,g} & 0 \\ \phi^*_{d,c} & \phi^*_{d,g} & \phi^*_{d,d} \end{bmatrix} X_{t-1} + \begin{bmatrix} \varepsilon^*_{c,t} \\ \varepsilon^*_{g,t} \\ \varepsilon^*_{d,t} \end{bmatrix}$$

- In that context:

$$\begin{aligned} s_t^{t+n} &= -\frac{1}{n} \log E^{\mathbb{Q}} \left(\exp(-\Lambda_{t+1} - \ldots - \Lambda_{t+n}) \right) \\ &= a_n + b_n X_t + X_t c_n X_t \end{aligned}$$

(Leippold and Wu, 2002, Ang et al., 2008, Gourieroux and Sufana, 2011)

- Estimation technique:
  - Part of X_t is latent (factor C_t).
  - Non-linear filtering required (a) to compute the likelihood function and (b) estimate C_t.

Comments/Questions

About the common factor

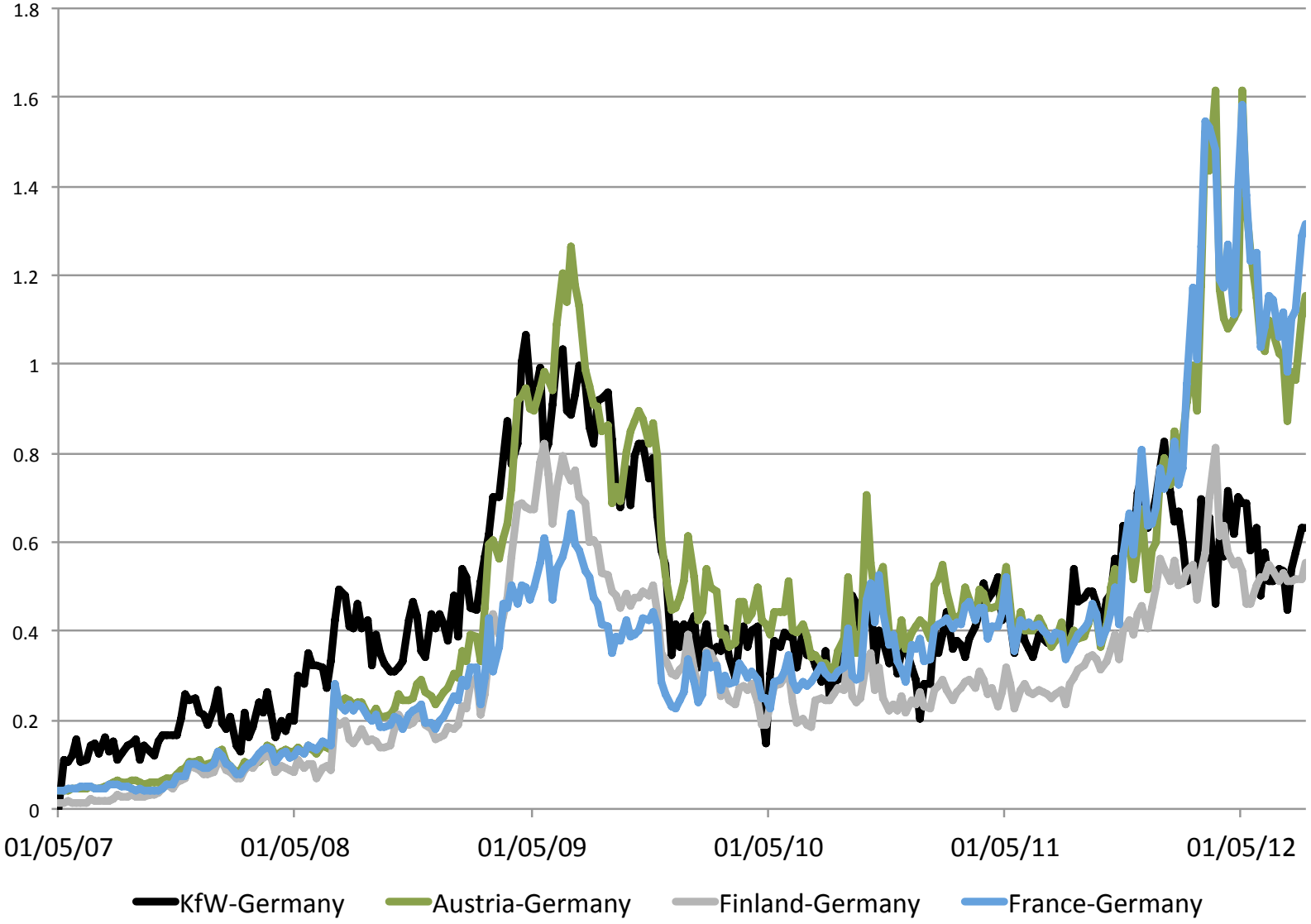
- Why not including the common factor C_t in the quadratic intensities?
- If C_t is the manifestation of self-fulfilling market dynamics, it would make sense to have the intensity depending on it in a non-linear way.
- Other potential explanations for C_t:
  - changes in the prospects for global economic activity,
  - changes in the expected recovery rates,
  - political uncertainty,
  - liquidity-pricing effects.

Comments/Questions

Liquidity effects in spreads

Comments/Questions

Liquidity effects in spreads



Comments/Questions

Common-factor prices of risk

- In the model:

$$\begin{cases} (2 \times N) + 1 & \text{sources of risk} \\ (3 \times N) & \text{prices of risk} \end{cases}$$

- The prices of risk associated with C_t differ across countries.
- The risk associated with C_t is not priced in the same way by the creditors of the different countries.
- Consistent with the view of completely segmented euro-area sovereign-bond markets.

Comments/Questions

Complexities associated with the quadratic framework

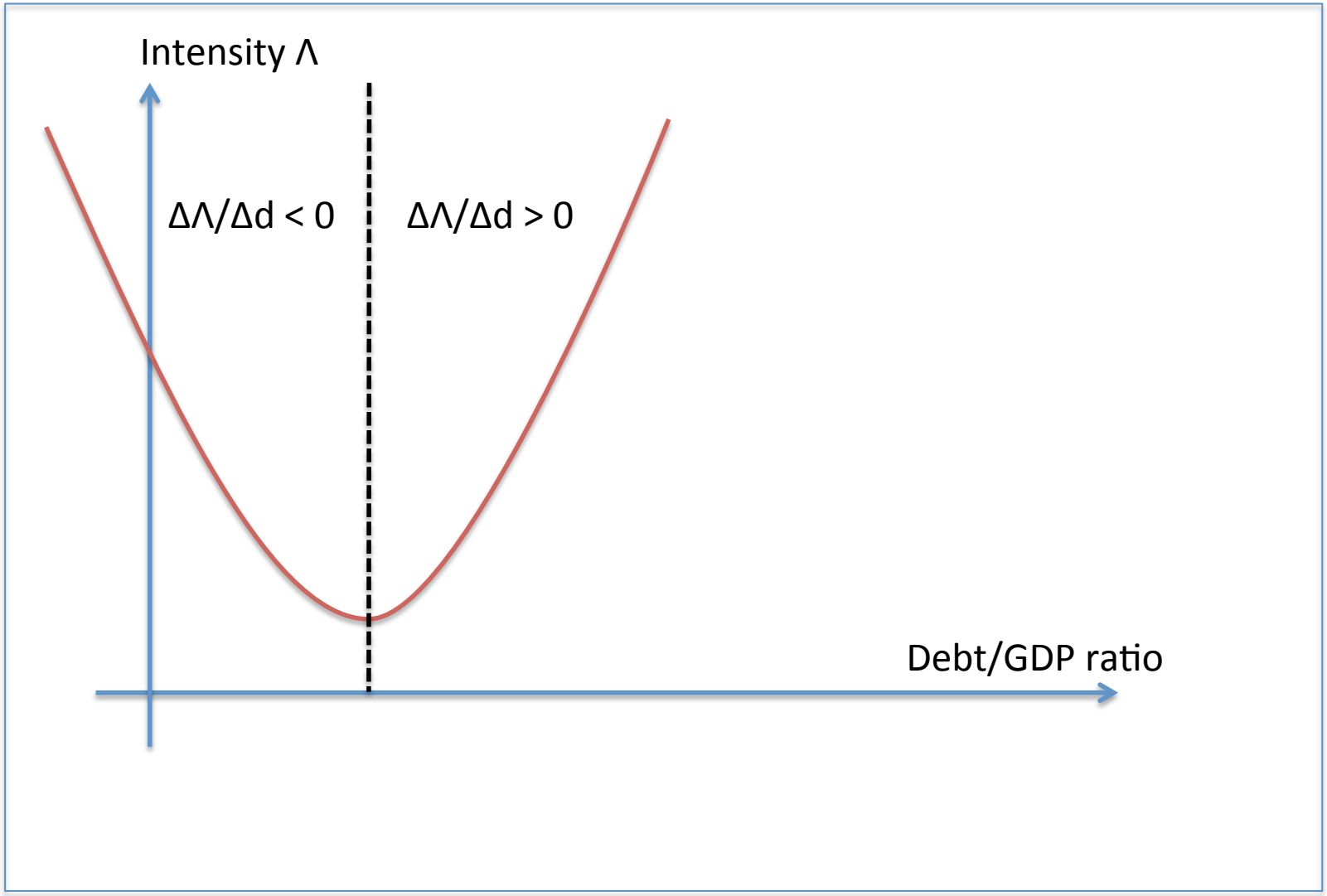
- Quadratic framework:
  - Makes it possible to model non-linearities
  - Could be used to impose positiveness of yields/spreads (e.g. Kim and Singleton, 2011)
- Decomposing yields/spreads is not straightforward:

$$s_t^{t+n} = a_n + \begin{bmatrix} b_c & b_g & b_d \end{bmatrix} X_t + X_t \begin{bmatrix} c_{c,c} & 0 & c_{c,d} \\ 0 & c_{g,g} & c_{g,d} \\ c_{c,d} & c_{g,d} & c_{d,d} \end{bmatrix} X_t$$

  - Interaction terms can be sizable, how to split these onto the different components?
  - why not using the shocks (implicitly identified here) to compute the different components?
- Potential caveat of the quadratic framework: the relationship between the intensity and the factors is not monotonous.

Comments/Questions

Non-monotonous dependencies



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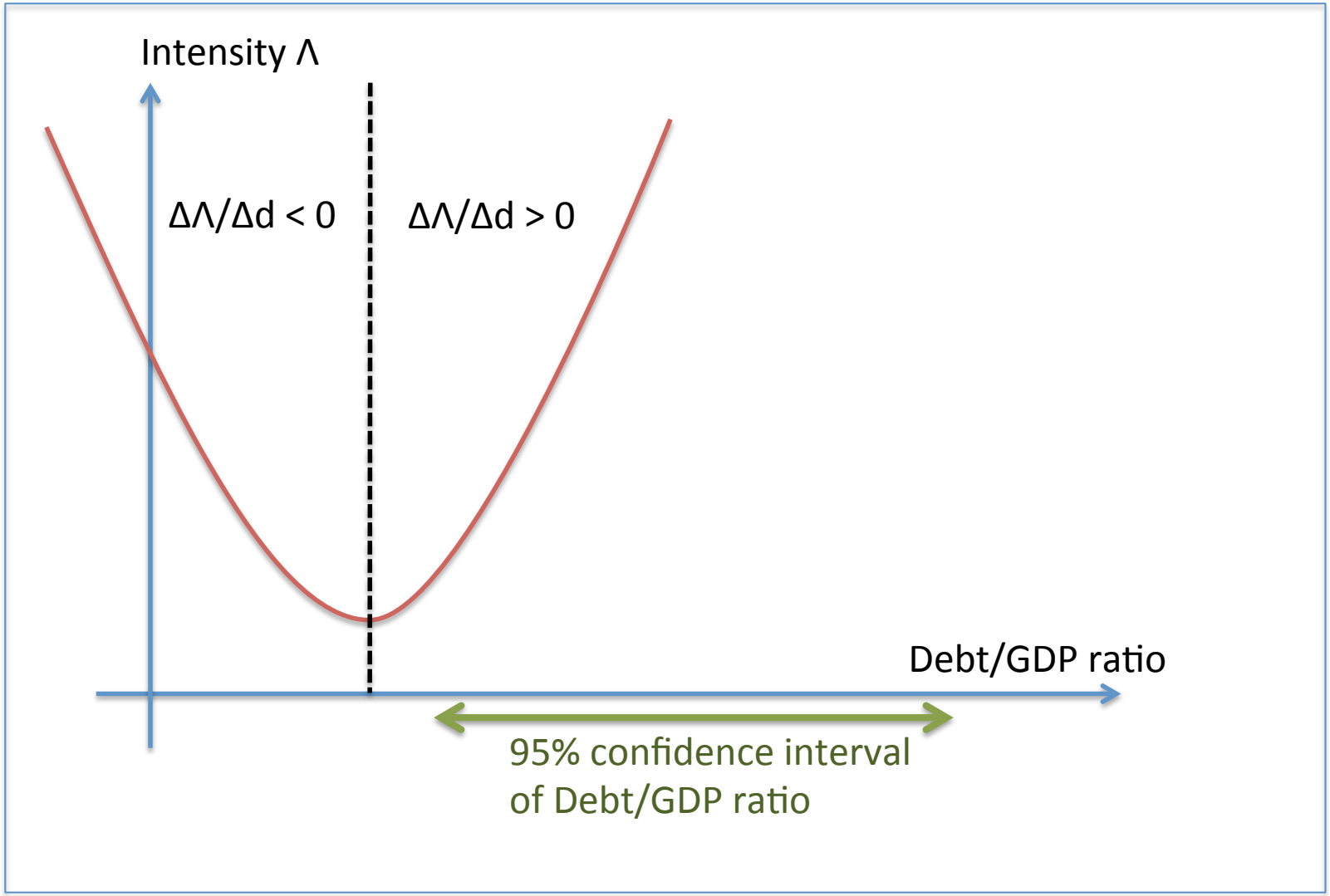

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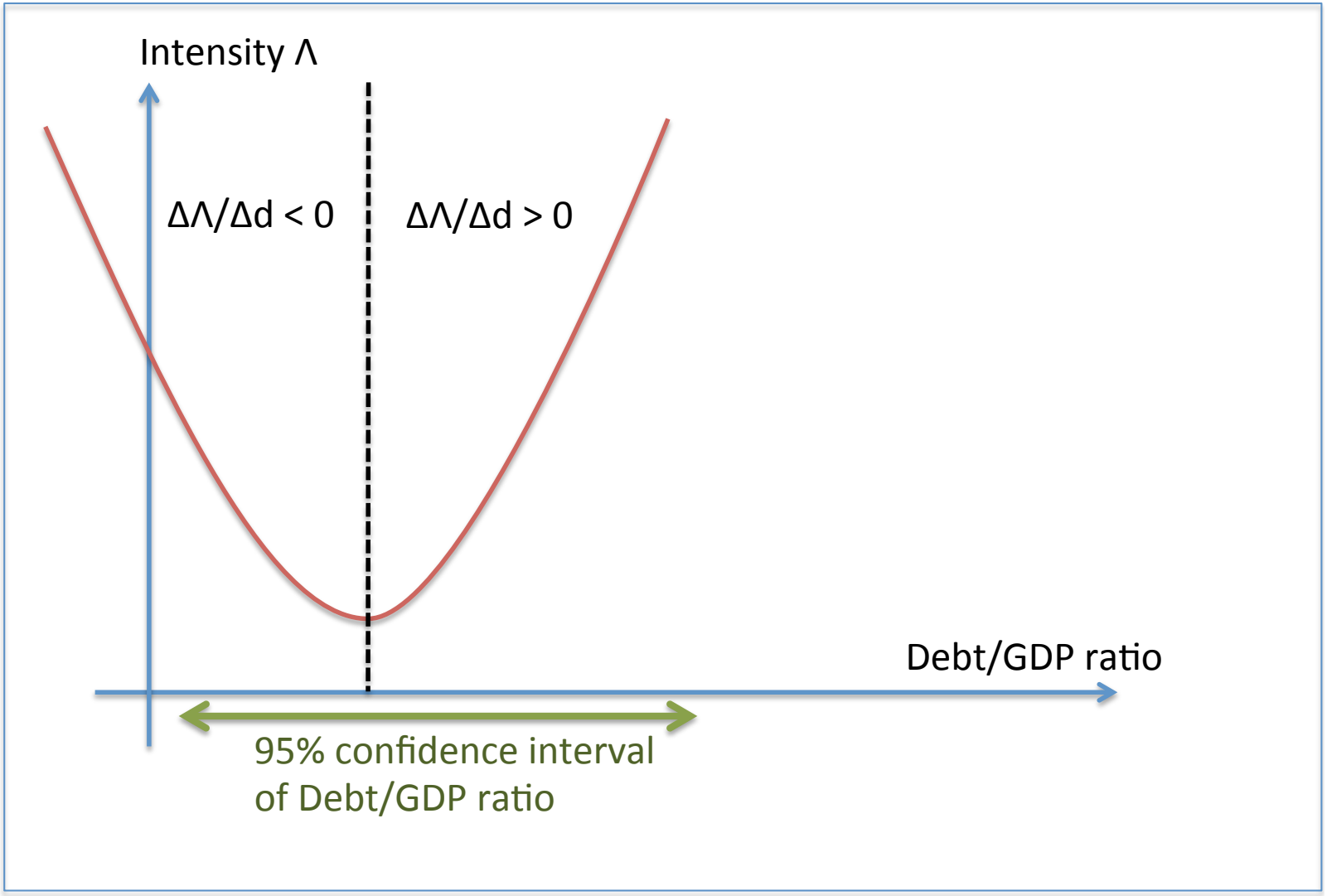

Comments/Questions

Risk premia

- Risk premia (in terms of yields) are...
  - ... negative if bond returns tend to be positive in "bad states of the world" (insurance)
  - ... positive if bond returns tend to be negative in "bad states of the world" (risk exposure)
- Negative risk premia over the first part of the sample, ...
  ... meaning that before 2008, agents would reckon that decreases in sovereign PDs (positive bond returns) correspond to bad states of the world.
- Alternative (econometric) explanation: the unconditional average of the default intensity is too large.
  - Small sample: the unconditional average is imprecisely estimated.
  - Could be constrained to imply lower PDs before 2008 (in the early 2000's, 10-year PD for Spain: 5%, for Greece: 18%).

Comments/Questions

Forecasting ability of the model

$$\text{Risk Premium} = \underbrace{(\text{Observ. spread})}_{\mathbb{Q}-\text{expected } \Lambda_t\text{'s}} - \underbrace{(\text{spread if investors were risk-neutral})}_{\mathbb{P}-\text{expected } \Lambda_t\text{'s}}$$

- Assuming investor beliefs $\sim$ survey-based forecasts (CF), model-implied forecasts and Consensus Forecasts should coincide.
- If not, it notably implies that the model-implied risk-neutral PDs would not coincide with the ones of the professional forecasters.
- Two potential objectives that do not necessarily coincide:
  - to have a model that is good at forecasting,
  - to have a model that is able to reproduce agents' expectations (and hence to compute risk premia).

Comments/Questions

Including Greece in the dataset

- Including Greece in the dataset is bold...
- ... but challenging for this kind of model over the whole estimation period.
- The hypothesis of an (expected) constant recovery rate is debatable in the Greek case.

Comments/Questions

Dealing with missing observations

- The fiscal variables feature missing values.
- Pre-filtering using the Kalman filter and a simple autoregressive law of motion for the fiscal variable.
- Why not dealing with that simultaneously with the main estimation? Filtering techniques can easily handle missing values.

To sum up

- The paper nicely illustrates the potential of the quadratic framework to handle non-linearities in sovereign-bond pricing.
- In spite of a deep technical content, the paper remains clear.
- Different modeling choices and interpretations should be investigated/discussed further.
- A very niece piece of work.