# Comparing Different Regulatory Measures to Control Stock Market Volatility: A General Equilibrium Analysis[*]

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This version: February 3, 2014

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**Abstract**

In this paper, we compare the effects of different regulatory measures used to reduce excess volatility of stock-market returns, which is generated by investors trading on sentiment. The regulatory measures we study are the Tobin tax, shortsale constraints, and leverage constraints. The main contribution of our research is to evaluate these regulatory measures within the same dynamic, stochastic general equilibrium model of a production economy, so that one can compare both the direct and indirect effects of the different measures on the financial and real sectors within the same economic setting. We find that of the three measures we consider, only the leverage constraint is effective in reducing stock-market volatility, and this is accompanied by positive effects on the real sector: an increase in the levels of consumption growth and investment growth, and a decrease in their volatilities. In contrast, both the Tobin tax and shortsale constraints *increase* volatility in financial markets, and have negative effects on the real sector: a decrease in the growth rates of output and investment and an increase in the volatility of consumption-growth.

*Keywords*: Financial regulation, Tobin tax, borrowing constraints, shortsale constraints, stock market volatility, incomplete markets, heterogeneous investors.

*JEL:* G01, G18, G12, E44

# 1 Introduction

One of the main objectives of a regulator is to maintain stable financial markets. Major instruments for attaining such stability include fiscal and monetary policy measures, as well as direct restrictions on trading in financial markets. In this paper, we compare the effects of different regulatory measures in financial markets that a regulator can use to reduce the volatility of stock-market returns and ensure orderly financial markets. The measures we study are the Tobin tax on financial transactions, shortsale constraints, and borrowing constraints, all of which have been proposed by regulators in response to the financial crisis. For example, on 1 August 2012, France introduced a financial transaction tax of 0.20%; on 25 July 2012, Spain's Comisión Nacional del Mercado de Valores (CNMV) imposed a three-month ban on short-selling stocks, while Italy's Consob prohibited shortselling of 29 banks and insurance stocks; and, tighter leverage constraints have been proposed following the subprime crisis—for instance, Reuters reported on 17 October 2008 that European Commissioner Joaquin Almunia said: "Regulation is going to have to be thoroughly anti-cyclical, which is going to reduce leverage levels from what we've seen up to now."[1]

In particular, the kind of questions we address are the following: If a regulator wishes to reduce the volatility of stock markets, which it judges to be excessive, should it introduce a Tobin tax, shortsale constraint, or borrowing constraint? That is, which regulatory measure has the strongest effect on the volatility of stock market returns? What exactly is the channel through which each regulatory measure works? What will be the impact of this regulatory measure on real variables, such as output and investment, and on financial variables, such as the level of the stock market and the market price of risk, the level and term structure of the riskless interest rate, and the trading volume in financial markets? And, at a broader level, would more tightly regulated markets function better, be more stable, and increase productivity and welfare? The main contribution of our research is to evaluate these regulatory measures within the same dynamic, stochastic general equilibrium model of a production economy, so

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[1] For a review of research on the Tobin tax, see Anthony, Bijlsma, Elbourne, Lever, and Zwart (2012) and McCulloch and Pacillo (2011); for a review of the literature on shortsale constraints, see Beber and Pagano (2013); and, for a review of studies on regulatory constraints on leverage, see Crawford, Graham, and Bordeleau (2009).

that one can compare both the direct and indirect effects of these different measures on the financial and real sectors of an economy within the same economic setting.

The dynamic, stochastic general equilibrium model we construct is of a production economy that is populated by multiple investors who have different beliefs; some of these investors may be fully rational while others may trade on sentiment. Trading between investors who differ in their beliefs leads to asset returns that are "excessively volatile," as has been documented empirically.[2] We then introduce a variety of regulatory measures in this model. After the regulations have been imposed by the regulator, financial markets are neither complete nor frictionless, even if they were before, so that computing the equilibrium requires special tools. We evaluate the implications of each regulatory measure by solving this dynamic, stochastic general equilibrium model with incomplete financial markets with frictions. The importance of studying these issues in general equilibrium is highlighted in Loewenstein and Willard (2006) and Coen-Pirani (2010), who show that partial-equilibrium analysis can lead to incorrect inferences.

Our model has two central features. The first is the presence of investors with heterogeneous beliefs, rather than a representative investor. Both policymakers and academics have recognized the importance of studying models with heterogeneous investors with different beliefs. For instance, Stiglitz (2010) criticizes representative-investor models and highlights importance of heterogeneous investors as key challenge. Hansen (2010) lists one of the challenges for macroeconomic models to be "Building in explicit heterogeneity in beliefs, preferences . . . ", and Hansen (2007) in his Ely lecture says: "While introducing heterogeneity among investors will complicate model solution, it has intriguing possibilities." Sargent (2008) in his presidential address to the American Economic Association, discusses extensively the implications of the common beliefs assumption for policy. Similarly, Eichenbaum (2010) writes "Nobody disagrees about the odds of drawing four aces from a deck of cards. But there is substantial scope for belief heterogeneity when historical evidence is at best a weak guide. For many questions, this situation is the one we face."

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[2]See Shiller (1981) and the literature that followed that article, some of it questioning its method, but little of it overturning its conclusion.

One way to model investors with heterogeneous beliefs has been proposed by Scheinkman and Xiong (2003).[3] In their model of a "tree" economy, a stream of dividends is paid. Some aspect of the stochastic process of dividends is not observable by anyone. All investors are risk neutral, are constrained from short selling, and receive information in the form of the current dividend and some public signals. Rational investors know the true correlation between innovations in the signal and innovations in the unobserved variables. Irrational investors (they call them "overconfident" but a better word might have been "sentiment-prone") are people who steadfastly believe that this correlation is a positive number when, in fact, it is zero. This causes them to give too much weight to the signal, which then generates fluctuating expectations and excessive stock price movements.[4]

In our analysis, we start with the model developed in Dumas, Kurshev, and Uppal (2009), who have a setting similar to that in Scheinkman and Xiong (2003)) with investors who hold heterogeneous beliefs, except that all investors are risk averse (and are allowed to sell short). Thus, in the eyes of each investor, the behavior of the other investor(s) seems fickle and is seen as a source of risk, which requires a risk premium and *which originates in the financial market itself*, over and above the source of risk originating from the production system.

In order to measure real effects, we extend the model of Dumas, Kurshev, and Uppal (2009) to allow for production.[5] In order to produce a realistic calibration of the equity premium and to stabilize the rate of interest, we also allow for investors with habit in their utility function, as in Campbell and Cochrane (1999), instead of power utility.

The second central feature of our model, once regulatory constraints are introduced, is market incompleteness and frictions. Typically, general-equilibrium models in financial economics

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[3]Scheinkman and Xiong (2003) develop their model in order to illustrate the insight in Harrison and Kreps (1978) that when there is disagreement between investors and shortsales are prohibited, then asset prices may exceed their fundamental value. Panageas (2005) studies the implications of this model for physical investment.

[4]The first central feature of our model is related to the strand of the literature that deals with heterogeneous beliefs in financial-market equilibrium. Heterogeneity of beliefs between investors needs to be sustained or it quickly becomes irrelevant. Differences in the basic model investors believe in, or in some fixed model parameter was proposed earlier by Harris and Raviv (1993), Kandel and Pearson (1995), and Cecchetti, Lam, and Mark (2000), and used more recently, by David (2008). Under this approach, investors are non-Bayesian. Another modeling possibility is differences in priors, while investors remain Bayesian, as in Biais and Bossaerts (1998), Buraschi and Jiltsov (2006), Detemple and Murthy (1994), Gallmeyer (2000), and Duffie, Garleânu, and Pedersen (2002). The model we develop allows for both differences in priors and also differences in the model investors believe in.

[5]And for incomplete markets. See below.

and in macroeconomics have assumed that financial markets are complete. The main reason for making this strong assumption is that it simplifies substantially the task of solving for the equilibrium. With complete financial markets, instead of solving for the dynamic problem of each investor, one can identify the equilibrium by solving at each date and state only the static problem of allocating consumption across investors. Real decisions are made by an easily defined "representative investor." However, once regulatory constraints are introduced, financial markets are no longer complete or frictionless; therefore, identifying the equilibrium in the dynamic economy is difficult because it requires one to solve a forward-backward system of difference equations. We, however, build on the method developed by Dumas and Lyasoff (2012) to show how, even in the presence of a variety of regulatory constraints, the system of forward-backward equations can be reduced to a system of backward (recursive) equations.[6]

These two central features of the model allow us to meet the twin challenge set by Eichenbaum (2010), who noted that the dynamic, stochastic general equilibrium models developed and used by macroeconomists "did not place much emphasis on financial market frictions." The twin challenges he presented were to model heterogeneity in beliefs and persistent disagreement between investors on the one hand and financial market frictions with risk residing internally in the financial system rather than externally in the production system. The twin challenges are met here with one stroke because the heterogeneity of investor beliefs we capture is a fluctuating, stochastic one so that it constitutes, indeed, an internal source of risk.[7]

The main finding of our paper is that of the three measures we consider, only the leverage constraint is effective in reducing stock-market volatility, and this is accompanied by positive effects on the real sector: an increase in the levels of consumption growth and investment growth, and a decrease in their volatilities. In contrast, both the Tobin tax and shortsale constraints *increase* volatility in financial markets, and have negative effects on the real sector:

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[6]The second central feature of our model is related to the literature on solving models of incomplete financial markets. There exists a vast literature on methods to solve general equilibrium models (see, for example, the special issue of the *Journal of Economic Dynamics and Control* (2010, vol. 1)). Our work shows how, even in the presence of borrowing and leverage constraints, the solution to the dynamic general equilibrium model can be characterized in terms of a recursive system of equations. This method makes it possible to solve a large class of dynamic general equilibrium models much more conveniently, and so in our view, our research makes a substantial contribution also on the methodological front.

[7]It constitutes, in fact, two internal sources of risk, which are correlated with each other: sentiment is stochastic and the volatility of sentiment is stochastic (with serial correlation), so that periods of quiescence in the financial market are followed by periods of agitation.

a decrease in the growth rates of output and investment and an increase in the volatility of consumption-growth.

Our work is related to several strands of the academic literature. The literature that is closest to our proposed research is the work on the remedies to the recent financial crisis. For example, Geanakoplos and Fostel (2008) and Geanakoplos (2009) study the effect of exogenous collateral restrictions on the supply of liquidity, while Krishnamurthy (2003) studies how credit constraints can lead to an amplification of shocks in the economy.[8] Ashcraft, Gârleanu, and Pedersen (2010) compare the effectiveness of different monetary tools. Our analysis is related also to the historic debate on the stabilizing or destabilizing effects of speculation. Alchian (1950) and Friedman (1953) are given credit for articulating the doctrine according to which investors who do not predict as accurately as others are driven out of the market.

Our model is related also to the literature on "behavioral equilibrium theory." Our model is an equilibrium model of investor sentiment, in the sense of Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999). The first two of these papers feature only a single group of investors who are non-Bayesian, while our model has two groups with heterogeneous beliefs. The model of Hong and Stein features two investors with heterogeneous beliefs but they are not intertemporal optimizers, in contrast to the investors in our model.

The rest of the paper is organized as follows. In Section 2 we describe our modeling choices for the real and financial sectors, and the preferences and beliefs of investors. In Section 3, we characterize equilibrium in our economy, and also explain how to implement the three regulatory measures we consider: a proportional tax on transactions in the risky financial asset (Tobin tax), a shortsale constraint on the risky asset, and a leverage constraint. In Section 4, we explain how we solve our model and the values chosen for various parameters in order to calibrate the model. Our results are described in Section 5. We conclude in Section 6. Technical results and details of the solution method are relegated to the appendix.

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[8]See also Chabakauri (2013a,b), who studies the effect of portfolio constraints on financial quantities in an exchange economy where agents are heterogeneous with respect to their preferences but have the same beliefs.

# 2 The General Model

In this section, we describe the features of the model we study. In our model, there is a single consumption good, which is produced by a representative firm. The presence of production activity in the model allows us to study, on the real side of the economy, the effects of the excessive volatility in financial markets and of the regulatory measures, which are intended to reduce this excessive volatility.

Time is assumed to be discrete. We denote time by $t$, with the first date being $t = 0$ and the terminal date being $t = T$. In our model we will allow for $K = 2$ investors, who are indexed by $k$ and who have utility functions over consumption with external habit. We assume that there are two sources of risk represented by a stochastic productivity shock $Z$, and a public signal, which are modeled jointly as a Hidden Markov Chain Model, the current state of the economy being unobserved. One of the investors is learning about the current state, rationally weighting the information it receives (current output and current signal), while the other places too much trust in the value of the signal and hence trades on sentiment. The existence of a "sentiment-prone" trader increases the volatility of the financial markets relative to the volatility of the fundamentals as shown in Dumas, Kurshev, and Uppal (2009), and it leads to inefficient production decisions.

While it is not clear according to which investor's beliefs the firm should be managed, we let the rational investor decide on the investment policy of the firm.[9] Nonetheless, the sentiment-prone investor's beliefs have real effects because they affect the equity price at which the firm can issue stock to finance its physical investment.

There are two financial assets: the first asset (denoted by $B_t$) is assumed to be a one-period bond; the other asset is assumed to be stock ($S_t$) paying out the dividend $D_t$ of the representative firm. As the main feature of our model, we impose, on top of the setup described above, a number of exogenous possible regulatory actions, such as a Tobin tax (proportional transaction tax for trading financial assets), shortsale constraints on the stock, and leverage

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[9]We could also consider two alternative cases, which involve a time varying power of decision: (i) the largest stockholder decides on the investment policy, and (ii) the overoptimistic investor decides on the investment policy. The difference in results would be negligible.

constraints; and we study the effect of these regulatory actions on the financial markets and real side of the economy. In the rest of this section, we give the details of the model.

## 2.1 Preferences of an Investor

We assume that each investor maximizes her lifetime utility of consumption, where the utility depends on the external reference point. We consider a simple version of "catching up with the Joneses" preferences with additive external habit level $C_t$:

$$E_k \sum_{t=0}^{T} \beta_k^t \frac{(c_{k,t} - h_k \times C_t)^{1-\gamma_k}}{1 - \gamma_k}. \tag{1}$$

In the above specification, $E_k$ denotes the conditional expectation at $t = 0$ under investor $k$'s subjective probability measure; $c(k, t) > 0$ is the consumption of investor $k$ at date $t$ in state $\omega(t, s)$;[10] the habit level is based on the aggregate consumption of the previous period: $C_t = \frac{1}{K} \sum_{k=1,2} c_{k,t-1}$; $h_k$ is the habit factor; $\beta_k$ is the subjective rate of time preference; and $\gamma_k > 0$ is equal to the coefficient of relative risk aversion when $h_k = 0$ and is higher than that otherwise, all the more so as the "surplus" $c_{k,t} - h_k \times C_t$ becomes smaller. The index $k$ for the parameters $h_k$, $\beta_k$, and $\gamma_k$ indicates that the investors may differ along all dimensions of their utility functions. However, we consider mostly calibrations in which preferences are identical, so that we can focus on the effects of differences in beliefs.

Investors receive an initial endowment of shares in the risky equity of the firm, and they can trade both assets, riskless and risky ones in financial markets. Ownership of the risky asset gives investors the right to the dividend paid by the representative firm.

## 2.2 Production

We assume that there exists a representative firm producing and paying out a single consumption good. At each period $t$ the firm uses the capital stock $K_t$ to generate production $Y_t = K_t \times Z_t$, where $Z_t$ denotes the stochastic technology specified below in Section 2.3. The capital of the firm depreciates at the periodic rate $\delta$, and after investment $I_t$ its law of motion

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[10]To simplify notation, we do not write explicitly the dependence on the state $\omega(t, s)$.

can be described as $K_{t+1} = (1 - \delta)K_t + I_t$. We assume that the change in the capital level is subject to quadratic adjustment costs. The difference between the production and capital expenses (including the adjustment costs) is paid out to the investors as total dividend $D_t$:

$$D_t = Y_t - I_t - \frac{\xi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t,$$

with each investor receiving an amount proportional to his stock holdings at the beginning of period $t$.

The manager of the firm chooses investment $I_t$ to maximize the value of the firm $P_{k,t}$ for the owner $k$, given by the discounted expected value of dividends $D_t$ under the appropriate equivalent martingale measure of the owner, or using the pricing kernel $M_{k,\tau}$ of the respective investor $k$:

$$P_{k,t}^S(K_t) = \max_{I_t,\ldots,I_{T-1}} \left\{ D_t + E_t\left[\sum_{\tau=t+1}^{T} \frac{M_{k,\tau}}{M_{k,t}} D_\tau\right]\right\}, \tag{2}$$

where $E_t$ is the conditional expectation under the true probability measure. We assume for the main part of the analysis that the value of the firm is maximized with respect to the expectations of the rational investor.

## 2.3 Uncertainty and Learning by the Agents

Time is assumed to be discrete, with $t = \{0, 1, \ldots, T\}$.[11] Investors may have their own probability measures that are equivalent to the true one, and that are formed in the way we describe now.

Uncertainty in the economy is generated by a Hidden Markov Model. We assume that the economy can be in one of two unobservable states $X_t \in \{1, 2\}$. The transition between the two unobservable states follows a *Markov* process with a row-stochastic $2 \times 2$ state transition probability matrix $A = (a_{i,j})$, defined as

$$a_{i,j} = P(X_{t+1} = j \mid X_t = i).$$

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[11]Uncertainty is represented by a $\sigma$-algebra $\mathcal{F}$ on the set of states $\Omega$. The filtration $\mathbb{F}$ denotes the collection of $\sigma$-algebras $\mathcal{F}_t$ such that $\mathcal{F}_t \in \mathcal{F}_s, \forall s > t$ , with the standard assumptions that $\mathcal{F}_0 = \{\emptyset, \Omega\}$ and $\mathcal{F}_T = \mathcal{F}$. In addition to time being discrete, we will also assume that the set of states is finite. The probability measure on this space is represented by $P : \mathcal{F} \to [0, 1]$ with the usual properties that $P(\emptyset) = 0, P(\Omega) = 1$, and for a set of disjoint events $\omega_i \in \mathcal{F}$ we have that $P(\cup_i \omega_i) = \sum_i P(\omega_i)$.

While the state of the economy is unobservable for the investors, they observe productivity realization $Z_t$ and a signal $\Xi_t$. For simplicity, we assume that both productivity and the signal can take on only two values. That is, productivity can either be low or high: $Z_t \in \{Z_l, Z_h\}$ and the signal can either be positive or negative: $\Xi_t \in \{Pos, Neg\}$, that is, we have four possible pairs of observations: $\{(Z_l, Pos), (Z_l, Neg), (Z_h, Pos), (Z_h, Neg)\}$, which we denote without loss of generality by $O_t \in (1, \ldots, 4)$.

The observations $O_t$ are related to the (hidden) states of the Markov process by a $2 \times 4$ row-stochastic observation probability matrix $\Psi = (\psi_{i,o})$ with

$$\psi_{i,o} = P(O_t = o \,|\, X_t = i).$$

Given the probabilistic relationship between the observations and the current state of the economy, the investors can use the observations to form conditional state probabilities $p_{t,i} = P(X_t = i \,|\, \mathbb{O}_t)$, where $\mathbb{O}_t = (O_0, \ldots, O_t)$ denotes the series of past and current observations.

For our economy, we assume that both investors know the transition matrix $A$, but might disagree on the observation probability matrix $\Psi$. Specifically, we assume that investor $k$ believes that the observations are generated by the observation probability matrix $\Psi^k$.

Given a series of observations $\mathbb{O}_t = (O_0, \ldots, O_t)$ and an initial state distribution $\pi(i) = P(X_0 = i)$ (*priors*) investor $k$ updates her beliefs about the current state of the economy according to the following recursive "forward algorithm:"

$$\alpha_{t,i} = \left( \sum_{j=1}^{2} p_{t-1,j} \, A_{j,i} \right) \Psi^k(i, O_t),$$

$$p_{t,i} = \alpha_{t,i} \left( \sum_{j=1}^{2} \alpha_{t,j} \right)^{-1},$$

subject to the initial conditions $\alpha_{0,i} = \pi(i) \, \Psi(i, O_0)$ and $p_{0,i} = \alpha_{0,i} \left( \sum_{j=1}^{2} \alpha_{0,j} \right)^{-1}$. This forward algorithm is a result of the straightforward application of Bayes, as explained, for instance in Baum, Petrie, Soules, and Weiss (1970) and Rabiner (1989). The forward algorithm is the nonlinear analog for discrete-time discrete-state Markov chains of the Kalman filter, which is applicable to linear stochastic processes.

The investors *agree to disagree*, that is, even if they agree at one point in time on the state probabilities $p_{t,i}$, the fact that they update their beliefs differently will lead to disagreement about future state probabilities.

## 3  Equilibrium and Regulatory Measures

In this section, we first describe the optimization problem of each investor, and of the representative firm, and then impose market clearing to obtain a characterization of equilibrium. We also show how to implement in our model three different regulatory measures, namely, a proportional tax on transactions in the risky asset (Tobin tax), a shortsale constraint on the risky asset, and a leverage constraint on the portfolios of the investors. We proceed with a short representation of the solution method in the next section, and the derivations with details of the numerical procedure to solve for equilibrium provided in the appendix.

### 3.1  The Optimization Problem of the Firm

The objective of the firm's manager is to maximize the value of the firm equal to the value of all future dividends in equation (2). We can rewrite this value to be maximized more conveniently in the dynamic programming fashion:

$$P_{k,t}^S(K_t) = \max_{I_t} \left\{ \left[ K_t Z_t - I_t - \frac{\xi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \right] + E_{k,t}\left[ \frac{M_{k,t+1}}{M_{k,t}} P_{k,t+1}^S(K_{t+1}) \right] \right\}, \quad (3)$$

subject to the law of motion of the capital:

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{4}$$

Taking the first order conditions and rewriting the expression in a suitable form, the optimality condition to be satisfied for the firm can be written then as follows:

$$E_{k,t}\left[ \frac{M_{k,t+1}}{M_{k,t}}\left( Z_{t+1} + \left(1 + \xi\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)\right)(1-\delta) - \frac{\xi}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2 \right) \right] = 1 + \xi\left(\frac{I_t}{K_t} - \delta\right),$$

$$(5)$$

subject to the law of motion for capital in Equation (4).

## 3.2 The Optimization Problem of an Investor

The objective of each investor $k$ is to maximize lifetime utility given in (1) by choosing consumption, $c(k, t)$, and the portfolio positions in each of the two financial assets, namely, position $\theta_{k,t}^B$ in the one-period riskless bond with price $B_{k,t}$, and the position $\theta_{k,t}^S$ in the risky asset with price $S_{k,t}$, paying the dividend $D_t$, which is defined each period by the manager of the firm. We rewrite the optimization problem (1) of an investor $k$ in the dynamic programming fashion as follows:

$$
V_{k,t} = \max_{\left\{c_{k,t}; \theta_{k,t}^{B,S}\right\}} \left\{ \frac{(c_{k,t} - h_k \times C_t)^{1-\gamma_k}}{1-\gamma_k} + \beta_k E_{k,t} \left[ V_{k,t+1} \left(C_{t+1}, \theta_{k,t}^S \left(S_{k,t+1} + D_{t+1}\right) + \theta_{k,t}^B \right)\right] \right\},
$$
(6)

subject to the flow budget constraint:

$$
c_{k,t} + \theta_{k,t}^S S_{k,t} + \theta_{k,t}^B B_{k,t} = \theta_{k,t-1}^S \left(S_{k,t} + D_t\right) + \theta_{k,t-1}^B,
$$
(7)

and with the habit reference level $C_t = \sum_{k=1}^{2} c_{k,t-1} = D_{t-1}$.

Note that investors in general equilibrium have to agree on the prices of traded assets; however, in case there are states in which the bond or the stock are not traded, investor-specific prices may differ. When the regulators impose a regulatory measure, extra constraints are added to the investors' optimization. We discuss the treatment of such constraints below in Section 3.3.2.

The optimality conditions to be satisfied at each point in time in each random state of the economy can de derived in a standard way, and they are given by the following two equations for each investor $k$:

$$
\beta_k E_{k,t} \left[ \left( \frac{c_{k,t+1} - h_k \times C_{t+1}}{c_{k,t} - h_k \times C_t} \right)^{-\gamma_k} \left(S_{k,t+1} + D_{t+1}\right) \right] = S_{k,t}
$$
(8)

$$
\beta_k E_{k,t} \left[ \left( \frac{c_{k,t+1} - h_k \times C_{t+1}}{c_{k,t} - h_k \times C_t} \right)^{-\gamma_k} \right] = B_{k,t},
$$
(9)

In addition, the original dynamic budget constraint (7) must be satisfied.

## 3.3 Equilibrium in the Economy

Equilibrium in this economy is defined as a set of consumption policies, $c_{k,t}$, portfolio policies, $\theta_{k,t}^{\{B,S\}}$, and investment policy of the representative firm $I_t$, along with the resulting price processes for the financial assets, $\{B_t, S_t\}$, such that the consumption policy of each investor maximizes her lifetime utility; that this consumption policy is financed by the optimal portfolio policy; the investment in the production is chosen to maximize the value of the firm; and financial markets and the market for the consumption and investment clear at each state of nature at all points in time.

### 3.3.1 Equilibrium Conditions without Regulatory Measures

When there are no regulatory measures imposed on the economy, the equilibrium conditions are given by optimality of investment (5) subject to (4) under the respective investor's beliefs ($k = 1$), optimality of consumption (8) and (9) for each investor subject to the individual budget constraint (7), and two sets of market clearing conditions as specified below. First, for financial markets, supply should equal demand for both bond and stock:

$$\theta_{1,t}^B + \theta_{2,t}^B = 0, \tag{10}$$

$$\theta_{1,t}^S + \theta_{2,t}^S = 1, \tag{11}$$

and, second, aggregate consumption and capital expenditures should be financed by firm's output:

$$K_t Z_t = c_{1,t} + c_{2,t} + I_t + \frac{\xi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t.$$

Of course, by Walras's law financial-market clearing implies goods market clearing so that the last equation is redundant.

### 3.3.2 Equilibrium Conditions with Regulatory Measures

We consider three possible regulatory interventions, each being a specific constraint on the financial policy of the investors: (i) a financial transaction tax on the trades in the risky stock

(Tobin tax), (ii) a shortsale constraint on holdings of the risky asset, and (iii) a leverage constraint limiting the effective leverage of the financial wealth of an investor. Each of the regulatory measures leads to a specific change in the conditions for investor's optimization, while keeping the other equilibrium conditions unchanged.

The introduction of the Tobin tax affects the individual budget constraint by spending part of the available funds for the tax, and we need to rewrite the condition (7) as follows:

$$c_{k,t} + \theta_{k,t}^S S_{k,t} + \theta_{k,t}^B B_{k,t} + \kappa_t S_{k,t} \times \left| \theta_{k,t}^S - \theta_{k,t-1}^S \right| = \theta_{k,t-1}^S \left( S_{k,t} + D_t \right) + \theta_{k,t-1}^B,$$

where $\kappa_t$ is the level of proportional transaction costs for trading risky asset prevalent at time $t$. The Tobin tax typically affects the aggregate resource constraint as well, but we assume that the financial tax is not wasted in the economy, and hence we reimburse the tax paid by each investor back to him, in the form of a lumpsum transfer.

The shortsale constraint restricts the holdings of the risky asset to be above a predefined limit $\rho$:

$$\theta_{k,t}^S \geq \rho, \forall k, t.$$

The leverage constraint limits the amount of borrowing, or equivalently, the investment in the risky asset, to be less than a specified level $\alpha$:

$$\frac{\theta_{k,t}^S \times S_{k,t}}{\theta_{k,t}^B \times B_{k,t} + \theta_{k,t}^S \times S_{k,t}} \leq \alpha, \ \forall k, t, \tag{12}$$

We can also rewrite the leverage constraint (12) as a margin constraint, limiting the bond investment relative to the stock investment that serves as collateral:

$$\theta_{k,t}^B \times B_{k,t} \geq \zeta \times \theta_{k,t}^S \times S_{k,t} = \frac{1-\alpha}{\alpha} \times \theta_{k,t}^S \times S_{k,t}, \ \forall k, t,$$

When we incorporate the regulation-induced constraints into the household optimization problems and in equilibrium, a number of additional equations and inequalities show up in the system to be solved at each state of nature. The derivations of the modified optimality conditions and the numerical procedures to solve for equilibrium are given in the Appendix.

# 4 Calibrating and Solving the Model

For the quantitative analysis we calibrate our model described in Section 2 to match several stylized facts of the U.S. economy. We solve our economy for 30 years, assuming that each period in the model corresponds to one year of calendar time where the last 15 years are used as a burn-in period only. The statistics are based on the 10,000 simulated paths of the economy. We explain the calibration procedure below.

We use the deprecation rate that is typically used in the literature: 0.08 at an annual frequency. We assume that the investors have homogeneous preferences, that is, same risk-aversion, same habit parameter and same rate of time preference. We set the rate of time preference for both investors to be 0.9606 p.a., corresponding to the standard choice in the literature of 0.99 at the quarterly frequency. We choose the remaining parameters, that is, risk-aversion, habit parameter, adjustment costs, and the initial level, volatility, and the growth rate of technology to match several financial and macroeconomic quantities. Specifically, we want to match the level of the risk-free interest rate, the equity risk premium, the volatility of the equity premium, output volatility, and investment volatility. Note that in reality most firms use borrowed funds to finance their assets, and hence their equity risk is higher than the risk of the assets. To estimate the equity risk from the risk of the assets we need to apply a specific leverage adjustment. Because in our model the firm is financed by equity only, we artificially lever up the equity risk, corresponding equity premium, and its volatility by factor 2. All statistics reported below correspond to the levered firm.

In addition to assuming that the two investors have the same preference parameters, we assume also that they have the same initial endowment: 0.5 shares of the firm and zero debt. Thus, the only source of heterogeneity between the two investors is the difference in their beliefs, which we describe below.

In the Hidden Markov Chain we set the transition probabilities to be 0.95, that is, the hidden states are highly persistent. This is in line with the literature, e.g., Guvenen (2009) uses an AR(1) autocorrelation of 0.95 at a quarterly frequency, and consistent with empirical research that finds an high autocorrelation of the Solow residuals. For the initial point in time we assume

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that it is equally likely that the economy is in either state ($\pi = 0.5$). We assume that the realized technology level provides information about the current state of the economy, while the signal is pure noise. Given the four pairs of observables $\{(Z_l, Pos), (Z_l, Neg), (Z_h, Pos), (Z_h, Neg)\}$, this implies that the first two pairs (with a low technology level $Z_l$) indicate that the economy is currently in the first hidden state, while the other two pairs (with a high technology level $Z_h$) indicate that the economy is currently in the second hidden state. Specifically, we assume that the probability of observing low technology, conditional on being in the first hidden state, is 0.95, i.e., the probability is 0.475 for each of the pairs $\{(Z_l, Pos), (Z_l, Neg)\}$. Conditional on being in the second hidden state, we similarly set the probability that one observes high technology to 0.95 or, similarly, 0.475 for each of the pairs $\{(Z_h, Pos), (Z_h, Neg)\}$. In summary, the observation matrix for the four pairs under the objective probability measure is given by:

$$\Psi^{Technology} = \begin{bmatrix} 0.475 & 0.475 & 0.025 & 0.025 \\ 0.025 & 0.025 & 0.475 & 0.475 \end{bmatrix}.$$

While one investor ("rational") knows that the signal is pure noise and accordingly uses matrix $\Psi^{Rational} = \Psi^{Technology}$ to update her beliefs, the other investor ("irrational") believes incorrectly that the signal also provides useful information. The irrational investor uses an observation matrix that is a mixture of the case where only technology provides information about the current state, in which case $\Psi^{Technology}$ would the observation matrix, and the case where only the signal provides information about the current state, in which case the observation matrix would be given by:

$$\Psi^{Signal} = \begin{bmatrix} 0.475 & 0.025 & 0.475 & 0.025 \\ 0.025 & 0.475 & 0.025 & 0.475 \end{bmatrix}.$$

For $\Psi^{Signal}$, i.e., the case where only the signal would provide information about the current state, we assumed that, conditional on being in the first unobservable state, the probability of observing pairs 1 and 3, i.e., the signal $Pos$, is 0.475 each. Similarly, we assumed that conditional on being in the second unoservable state, the probability of observing pairs 2 and 4, i.e., the signal $Neg$, is 0.475 each.

Finally, we assume that the irrational agents weights the information from the technology level by weight $(1-w)$ and the signal by weight $w$, resulting in the following observation matrix

under beliefs of irrational Investor:

$$\Psi^{Irrational} = (1 - w) \times \Psi^{Technology} + w \times \Psi^{Signal}.$$

For the baseline calibration we set the level of sentiment of irrational investor to $w = 0.9$.

The set of parameters we use is summarized in Table 1. The risk aversion of 3 is reasonably low due to our choice of habit formation. Similarly, the habit parameter of 0.1 seems to be a good trade-off—not too high, which could lead to unrealistically high implied risk aversions, but high enough to increase the equity premium and its volatility. Furthermore, the model's volatility of technology growth relative to output growth at 1.15 is fairly close to the empirical value of 1.22 reported by Kung and Schmid (2012).

Table 2 displays standard statistics about financial as well as real variables from the model and their empirical counterparts (taken from Guvenen (2009)). While the risk-free rate of 2.3% in the model—one of our calibration targets—is close to its empirical counterpart (1.94%), the equity premium is understated at 3.3% in the model compared to 6.17% in the historical data.[12] In contrast, the equity premium volatility in the model (21.7%) matches its empirical counterpart (19.4%) fairly well. For the remaining financial statistics—not part of our calibration targets—it is noteworthy that the model overstates interest rate volatility (8.3% vs. 5.44%) while understating the Sharpe ratio at 0.15 compared to 0.32 in the U.S. data.

The volatility of output, which was one of the calibration targets, is matched fairly well (3.99% in our economy versus 3.78% in the data). Similarly, the other macroeconomic calibration target—investment volatility (normalized by that of output)—is matched reasonably well, with a volatility of 2.67 in the model compared to 2.39 in the U.S. data. In addition, the correlation between investment and output in the model is reasonably close to the data: 0.82 vs. 0.96.

In contrast, the calibration results for quantities related to consumption are weaker. For example, the volatility of consumption (normalized by output) is about 0.4 in the U.S. data, but is overstated at 0.92 in the model, and, similarly, the correlation between consumption and output is overstated in the model. However, those results are in line with the model quantities

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[12] The recent financial crisis and the corresponding market level declines should have pushed this number down from the reported level in the data.

in Guvenen (2009) and Danthine and Donaldson (2002) that report normalized consumption volatility at 0.75 to 0.98 and 0.82, respectively.

Finally, we want to discuss the effect of the sentiment-prone behavior of the irrational investor on the model quantities. Figures 1 to 4 show various financial and macroeconomic quantities, plotted against the weight that the irrational investor puts on the signal (her level of *sentiment*). Similar to the results in Dumas, Kurshev, and Uppal (2009), the fact that the irrational investor uses the signal to update her beliefs—even though the signal contains no information—creates *excess volatility*. That is, in Figure 1 the volatility of stock returns and the interest rate increases in the level of sentiment. Specifically, comparing the rational economy in which both investors use the correct observation matrix under (zero sentiment) to our baseline economy (90% sentiment) the excess volatility of stock returns is about 3.5% and the interest rate's excess volatility amounts to 3%. Focusing on the macroeconomic quantities in Figures 2 to 4, our computations show that the behavior of the sentiment-prone investor can lead to a considerable reduction in the growth rate of output, consumption and investment. For example, comparing our baseline calibration to the rational economy we find that output growth is lower by about 0.5% p.a. in the model with the sentiment-prone investor. The magnitude is similar for investment growth; moreover, consumption growth declines by about 1% p.a. in the presence of the sentiment-prone investor. For second moments, our analysis also shows that the presence of the sentiment-prone investor leads to higher consumption volatility and investment volatility, though it does not affect output volatility.

In summary, the presence of overconfident investors increases the volatility of financial and macroeconomic quantities, while at the same time reducing the growth rate of consumption and investment.

# 5  Analysis of Measures to Regulate Stock-Return Volatility

In this section, we undertake a quantitative analysis of the model described in the previous sections, examining closely the changes in financial and macroeconomic variables when we apply a particular regulatory measure. More precisely, we consider a calibrated economy, which

is characterized by excessive volatility because of sentiment trading, and we study how the introduction of (i) Tobin tax, (ii) shortsale constraints, or (iii) leverage constraints influence the volatility in financial markets, and the effect on a number of other financial and macroeconomic quantities.

For financial markets, we study stock-market volatility, the interest-rate level and the volatility of the interest rate. For the real side of the economy, we examine consumption growth and consumption-growth volatility, output growth and its volatility, as well as investment and investment-growth volatility. The results for financial variables are provided in Figures 5 to 9, and for macroeconomic variables in Figures 10 to 15. In each of these figures, there are three plots: the top plot is for the case of Tobin tax, the middle plot is for the case of shortsale constraints, and the bottom plot is for the case of leverage constraints.

## 5.1  Tobin Tax

The Tobin tax, proposed by Nobel Laureate economist James Tobin, was originally proposed as a tax on all spot conversions of one currency into another, and it was intended to put a penalty on short-term financial round-trip excursions into another currency. While many economists (e.g., Keynes (1936), Tobin (1978)) suggested that taxes on financial transactions would reduce financial volatility by reducing or speculative (and often irrational) trading, the theoretical and empirical support for such claims is limited and somewhat mixed.

Our general equilibrium analysis allows us to study the effect of the financial transactions tax on volatility of stock returns, where the excessive volatility is generated by sentiment-prone trading. The results of our analysis are provided in the top panel of Figure 5, which shows that the volatility of stock returns is monotonically *increasing* in the level of the Tobin tax. The volatilities of other financial variables are also increasing monotonically in the level of Tobin tax, thus making the general financial environment more risky. For example, we see from the top panel of Figure 6 that interest rate volatility increases from about 8% with zero Tobin tax to more than 9% with the Tobin tax of 2%. The interest rate drops (Figure 7) slightly (by about 0.2%), while the equity risk premium rises (Figure 8) by 1.5%.

For the real sector, the effect of a Tobin tax is overall negative: the Tobin tax increases consumption-growth volatility from below 3.8% to about 4% (top panel of Figure 11), and reduces output growth (Figure 12) and investment growth (Figure 14). The effect of the Tobin tax on consumption growth (Figure 10), output growth volatility (Figure 13), and investment growth volatility (Figure 15) is almost negligible.

Overall, our analysis shows that the Tobin tax is ineffective at reducing stock-return volatility, and its side-effects on the real sector are negative: it reduces output growth and investment growth, while increasing consumption-growth volatility.

## 5.2 Shortsale Constraints

Similar to the Tobin tax, shortsale constraints are typically introduced by regulators in an attempt to calm down the markets, and to limit large price swings due to short-term speculative position-taking.

From the middle panel of Figure 5, we observe that the volatility of the risky stock return does not go down with the severity of the constraint, and it actually rises from the 25% level, where the constraint is (almost) never binding, to 26.5% in the case where we prohibit risky security shortsales and the constants becomes binding more often. The volatility of the riskfree rate in the middle panel of Figure 6 also increases by about 0.5% to 8.7%. The interest rate itself declines slightly, but the effect is very small. Thus, the *effect of shortsale constraints on financial market stability is sharply negative*, and the shortsale constraint clearly does not achieve the desired goal of reducing volatility in stock markets. Even worse is the influence of the shortsale ban on consumption risk sharing and on the real side of the economy. In the middle panels of Figures 10 and 11 we observe a *decrease* in consumption growth and an *increase* in consumption-growth volatility, that is, both indicators are pushed further away from the solution in the setup where both investors are fully rational. The binding shortsale constraint also hampers output and investment growth, and increases the output-growth volatility (Figures 12 to 14); the magnitude of the effect is rather small, however.

Beber and Pagano (2013) provide an extensive review of the literature on shortsale constraints, as well as the *ex ante* and *ex post* comments by the regulators on the shortsale constraints implemented during the financial crisis. Anecdotal evidence suggest that the "The costs (of the short-selling ban on financials) appear to outweigh the benefits (Christopher Cox, SEC Chairman, telephone interview to Reuters, 31 December 2008)." The empirical analysis carried out by Beber and Pagano also sheds the light on other negative effect of a ban on shortsales: it reduces liquidity, slows price discovery, and fails to support asset prices.

Overall, our model suggests that shortsale constraints are not very effective at reducing volatility in financial markets; moreover, they are associated with negative effects on the real sector, reducing growth rates of output and investment, and increasing volatility of output growth.

## 5.3   Leverage Constraints

The main objective of leverage constraints is to limit the riskiness of the individual portfolios by restricting the portfolio composition in terms of value of risky securities. Leverage constraints are often implemented in the form of a margin requirement. When there is only one risky asset, the leverage constraint takes a simple form of a limit on the value of the position in the risky asset relative to the value of the portfolio, as can be seen from Equation (12). It is important to note that the leverage constraint is state-dependent; that is, it allows one to increase stock position in bad states of nature with low prices of the stock, compares to good states, when the stock prices are high, and hence, it is not as restrictive for the risk sharing between investors as the other types of constraints.

From the bottom panel of Figure 5 we see that a stricter leverage constraint quickly *reduces* stock-return volatility from 25% to less than 22%, that is, almost to the level observed in the case with no sentiment-prone investors, thus fulfilling its main regulatory objective. At the same time the leverage constraint pushes the riskfree rate and its volatility down, as seen from the bottom panels of Figures 6 and 7.

The effects of the leverage constraint in the financial markets are monotone and straightforward to interpret. However, the influence of the leverage constraint on consumption sharing and the real side of the economy is not monotone, and the net effect may be either positive or negative depending on the severity of the constraint. For instance, we see from the bottom panel of Figure 10 that at low levels the leverage constraint has almost no effect on consumption growth, then at moderate levels the consumption-growth rate decreases slightly, but when the constraint is tighter, then consumption growth increases almost all the way to the level of the economy with no sentiment-prone investors. Consumption growth volatility in Figure 11 is non-monotonic in the level of leverage constraint, and it goes down for a moderate constraint of $\alpha = 2$, increasing after that when we squeeze the leverage to $\alpha = 1.5$, and falling again for even a stricter constraint. The output and investment growth in the bottom panels of Figures 12 and 14 follow the same pattern by first decreasing slightly for levels of the leverage constraint of about $\alpha = 2$, and then increasing quickly and approaching the fully-rational level for the constraint $\alpha = 1$. Output growth volatility in Figure 13 behaves similarly to output growth itself, while investment growth volatility in Figure 15 first increases by almost 1% to 11.5% for a leverage constraint of $\alpha = 2$, but then decreases quickly to approximately 9.5% for $\alpha = 1$. Thus, along with the positive effect on financial markets, the leverage constraint also works well for the real side of the economy, but only when the constraint is strict enough to push the economy close to the fully-rational equilibrium.

Overall, our analysis shows that of the three measures we have studied, the leverage constraint is the only one that has a positive effect on both the financial and real sectors. In particular, the leverage constraint is successful in *reducing volatility in financial markets.*

## 5.4   Summary of Results

Lasse Heje Pedersen in his talk from 20 October 2008 talk at the International Monetary Fund and the Federal Reserve Board stressed that in the extreme market conditions banning short selling and introducing Tobin tax would not be good ideas to calm markets down.[13] What is required is a *reduction in risks* taken by the market participants, and neither the Tobin tax

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[13]See "Liquidity Risk and the Structure of Financial Crises" presentation from October, 2008 on Lasse Pedersen's web-site.

nor the shortsale constraint for risky securities explicitly curbs risk taking. On the other hand, the leverage constraint (in the form of the margin requirement) has traditionally been used by exchanges to control the risks taken by traders, and implementing leverage constraints at an economy-wide level reduces volatility in financial markets and improves the investment climate.

Our general equilibrium analysis confirms that of the three measures we have considered—financial-transaction tax, shortsale constraints, and leverage constraints—only the last one has positive effects on the economy by reducing the volatility in financial markets, increasing the consumption growth, and boosting investment. The effect of the leverage constraint on the financial markets is monotone, and a more strict leverage constraint always reduces the stock and interest rate volatility. The consumption sharing and the real (production) side are affected in a non-monotone manner: the positive effect is achieved when the constraint is strict enough to push the economy towards its rational state, that is, when the actions of sentiment-prone investors are strictly curbed or effectively eliminated. It is important to note that of the three measures, only the leverage constraint reduces sentiment-based speculation.

In Table 3 we provide a summary of the effects of the regulatory measures on financial markets, on consumption, and on the production side of the economy. We see that both the Tobin tax and the shortsale constraint have only one positive effect on the financial markets: they both lead to a lowering of the riskfree rate, and hence, potentially reducing the costs for financing. However, on the production side, both measures lead to a decrease in investment and output, and lead to an increase in the volatility of either investment or output, or both. In contrast, the leverage constraint, affects positively both the financial and real sides of the economy, by reducing volatility in financial markets and also in consumption and investment growth rates.

## 6 Conclusion

We have undertaken a general-equilibrium analysis of a production economy with heterogenous investors, who are uncertain about the current state of the economy. Incorrect beliefs about the current state by one class of investors leads to trading on sentiment and gives rise to excessive

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volatility in financial markets. excessive volatility has a negative impact on the risk sharing, and as a consequence it reduces consumption, investment and output growth, while increasing the volatility of investment growth. We study three regulatory measures that can be implemented in financial markets to curb the excessive volatility and hence improve welfare: a Tobin tax on financial transactions, a constraint on shortsales of the risky security, and a leverage constraint.

We find that neither the Tobin tax nor shortsale constraints are effective in reducing volatility in financial markets; moreover, both measures have negative consequences on macroeconomic quantities. For example, the Tobin tax increases the volatility of consumption growth and reduces the growth of both output and investment. Similarly, a shortsale constraint has very little effect on most real quantities, and it also reduces output and and increases the volatility of output growth. A constraint on leverage is the only regulatory measure that is effective in both reducing volatility in financial markets and improving the macroeconomic environment for investment. By limiting the magnitude of risky positions, a sufficiently strict constraint on leverage boosts the growth rates of consumption, output, and investment, while reducing substantially the volatility of investment growth.

**Table 1: Model Parameters**

In this table, we provide a summary of the model parameters, divided into three categories: the parameters of the Hidden Markov Chain, parameters for the preferences of investors and their beliefs, and the parameters of the production process. The model parameters are described in Section 2.

| Description | Variable | Value |
|---|---|---|
| *Hidden Markov Chain* | | |
| Autocorrelation hidden states | $A_{1,1}, A_{2,2}$ | 0.95 |
| Precision of technology | $B_{1,1} + B_{1,2}, B_{2,3} + B_{2,4}$ | 0.95 |
| Probability of the initial state | $\pi_k$ | 0.5 |
| | | |
| *Preferencs and Beliefs* | | |
| Sentiment of irrational Agent | $w$ | 0.9 |
| Subject time preference | $\rho_k$ | 0.9606 |
| Risk aversion | $\gamma_k$ | 3 |
| Habit parameter | $h_k$ | 0.1 |
| | | |
| *Production* | | |
| Depreciation | $\delta$ | 0.08 |
| Volatility of technology | $\sigma_T$ | 4.90% |
| Technology growth | $d_T$ | 0.60% |
| Adjustment costs | $\xi$ | 13 |

**Table 2: Financial and Business Cycle Statistics: Model vs. U.S. data**

In this table, we provide a summary of the moments generated by the calibrated model for financial markets and macroeconomic variables. The calibration of the model is described in Section 4, and the moments for the data are taken from Guvenen (2009).

| Description | Variable | Model | Data |
| --- | --- | --- | --- |
| *Financial variables* | | | |
| Risk-free rate | $r_f$ | 2.30% | 1.94% |
| Interest rate volatility | $\sigma(r_f)$ | 8.30% | 5.44% |
| Equity premium | $E[R^{ep}]$ | 3.30% | 6.17% |
| Equity premium volatility | $\sigma(R^{ep})$ | 21.70% | 19.40% |
| Sharpe ratio | $E[R^{ep}]/\sigma(R^{ep})$ | 0.15% | 0.32% |
| | | | |
| *Macroeconomic variables* | | | |
| Output volatility | $\sigma(Y)$ | 3.99% | 3.78% |
| Normalized investment volatility | $\sigma(I)$ | 2.67% | 2.39% |
| Normalized consumption volatility | $\sigma(C)$ | 0.93% | 0.40% |
| Correlation between investment & output | $Cor(I,Y)$ | 0.82 | 0.96 |
| Correlation between consumption & output | $Cor(C,Y)$ | 0.95 | 0.76 |

**Table 3: Summary of Effects of the Regulatory Measures**

In this table, we provide a summary of the effects of three regulatory measures on financial and macroeconomic variables. The text in **green** indicates an overall positive or a mixed effect on a given segment of the economy, and the text in **red** indicates a negative effect.

|  | Tobin tax | Shortsale constraint | Leverage constraint |
|---|---|---|---|
| Financial markets | risk **up** <br><br> financing **cheaper** | risk **up** <br><br> financing **cheaper** | risk **down** <br><br> financing **cheaper** |
| Consumption | uncertainty **up** <br><br> growth **slower** | uncertainty **up** <br><br> growth **slower** | uncertainty **down** <br><br> growth **much faster** |
| Production | investment & output **down** <br><br> risk **up** | investment & output **down** <br><br> risk **up** | investment & output **up** <br><br> risk **mixed** |

## Figure 1: Effect of Sentiment on Financial Variables

The plots in this figure show implications of sentiment-prone trading by one of the investors in the economy on the selected variable. Sentiment is measured by the weight put on the uninformative signal by the "irrational" investor.

*Panel A: Volatility of Stock Returns*
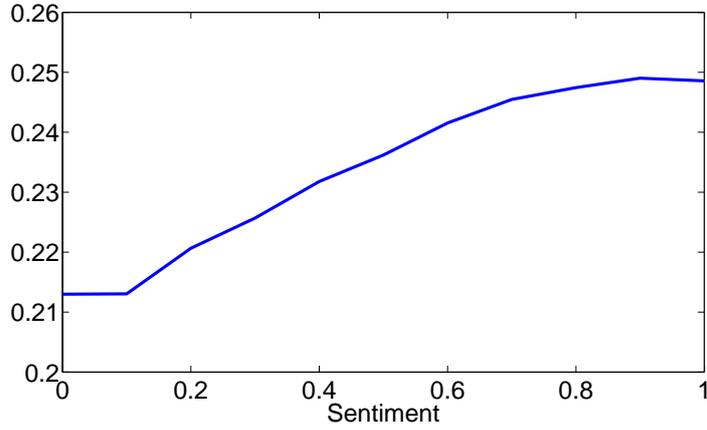


*Panel B: Level of the Interest Rate*
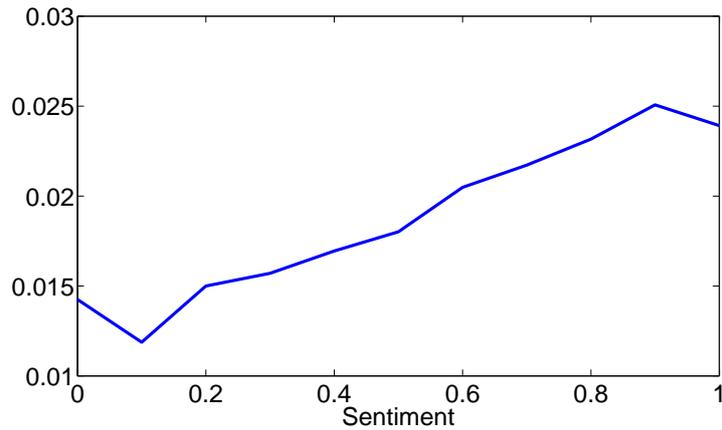


*Panel C: Volatility of the Interest Rate*
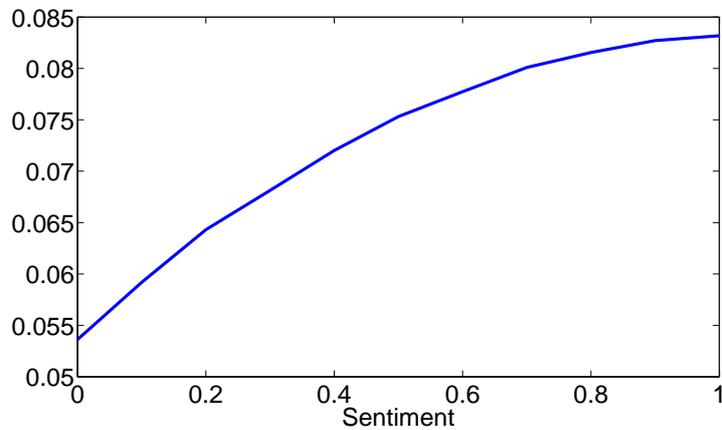
## Figure 2: Effect of Sentiment on Consumption

The plots in this figure show implications of sentiment-prone trading by one of the investors in the economy on the selected variable. Sentiment is measured by the weight put on the uninformative signal by the "irrational" investor.

*Panel A: Aggregate Consumption Growth*
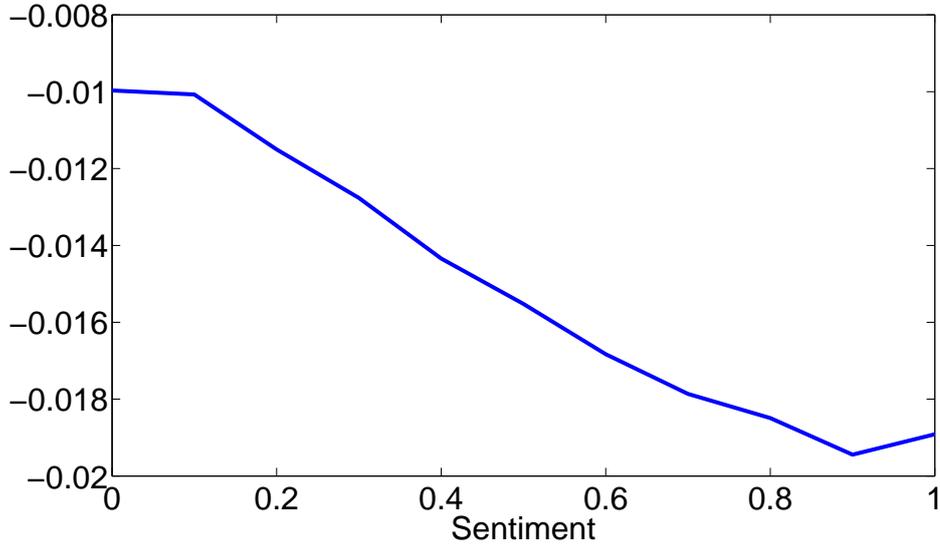


*Panel B: Volatility of Aggregate Consumption Growth*
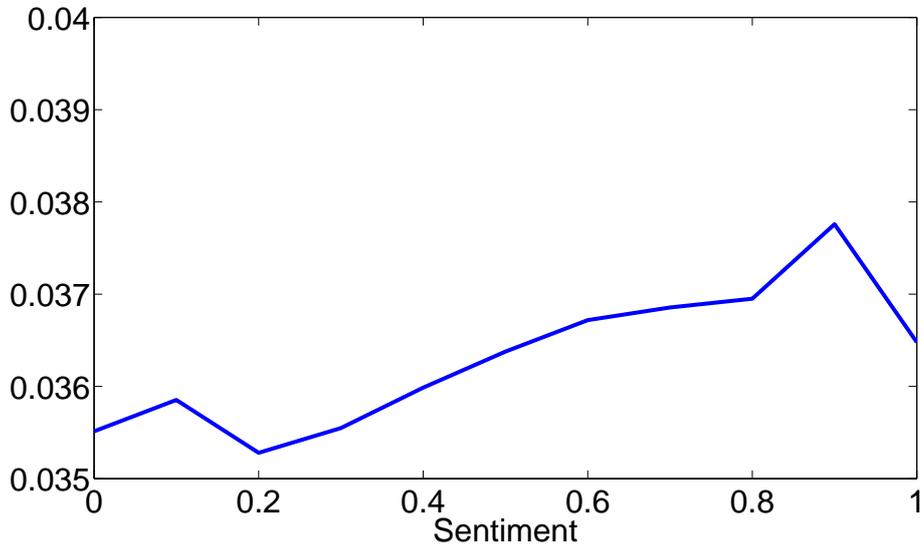
## Figure 3: Effect of Sentiment on Production Output

The plots in this figure show implications of sentiment-prone trading by one of the investors in the economy on the selected variable. Sentiment is measured by the weight put on the uninformative signal by the "irrational" investor.

*Panel A: Output Growth*
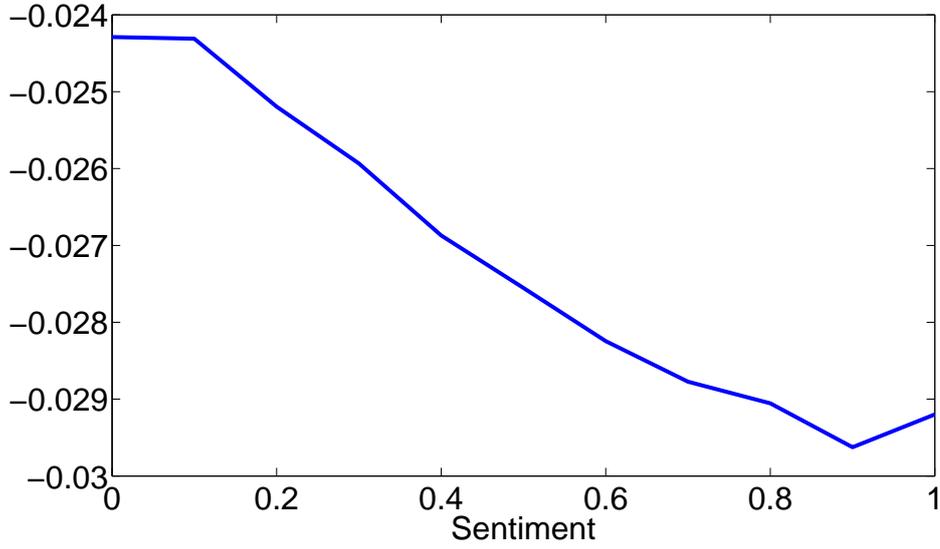


*Panel B: Volatility of Output Growth*
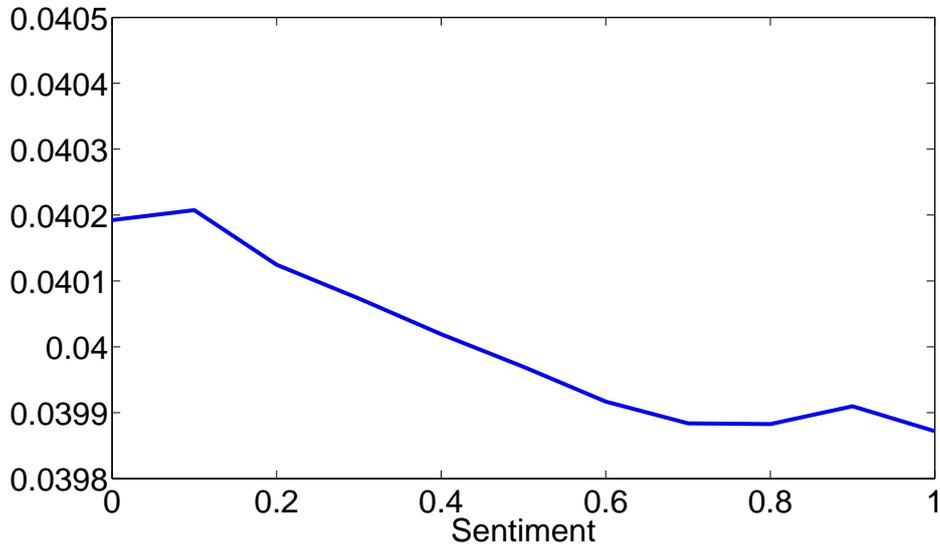
## Figure 4: Effect of Sentiment on Investment

The plots in this figure show implications of sentiment-prone trading by one of the investors in the economy on the selected variable. Sentiment is measured by the weight put on the uninformative signal by the "irrational" investor.

*Panel A: Investment Growth*
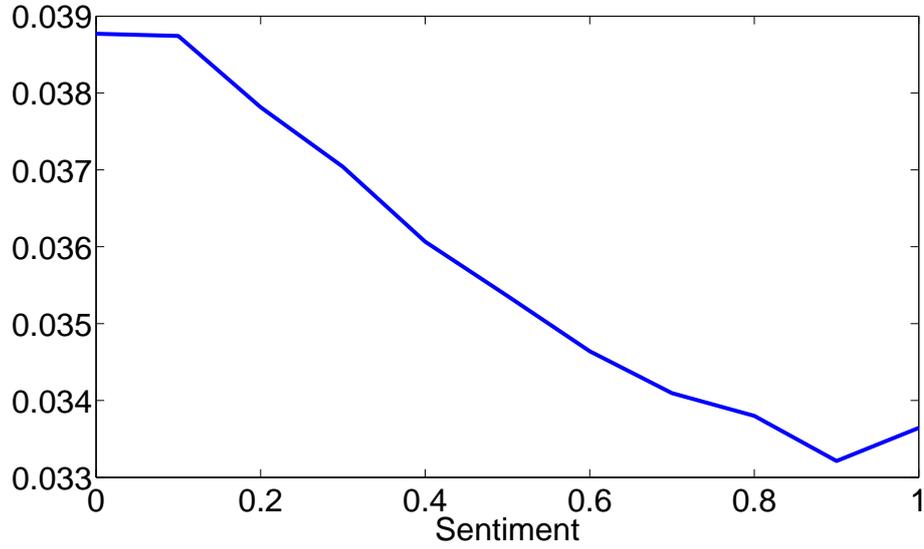


*Panel B: Volatility of Investment Growth Rate*
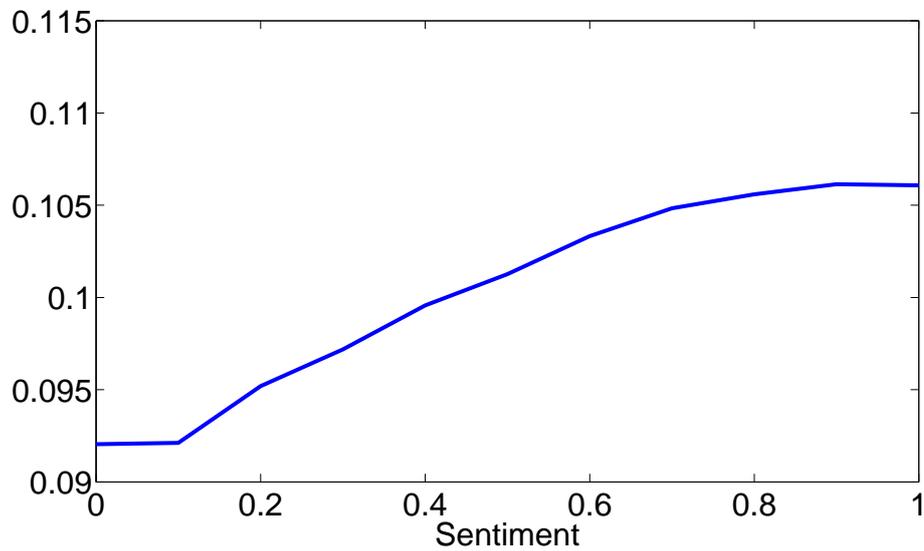
## Figure 5: Volatility of Stock Return

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
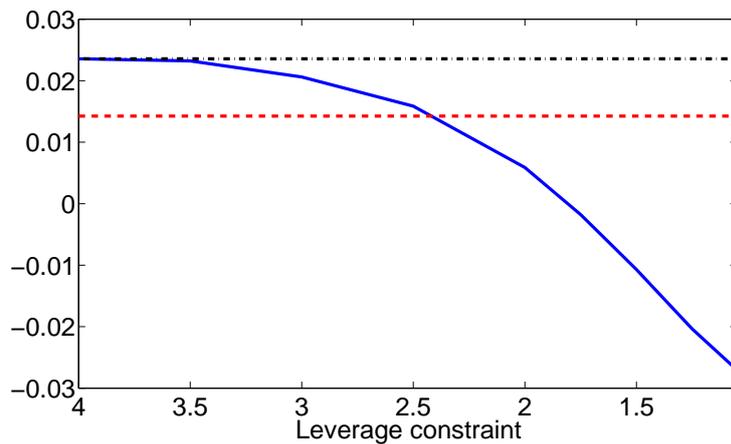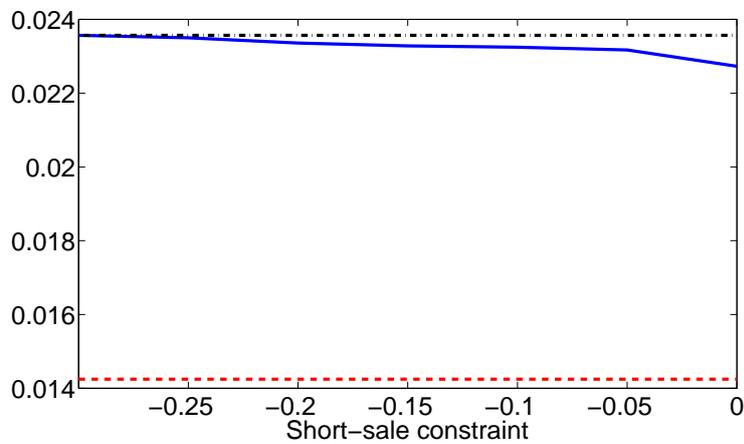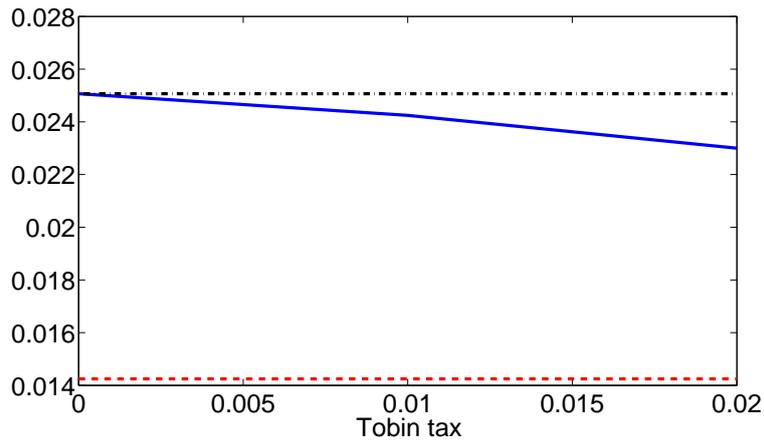
## Figure 6: Volatility of the Riskfree Interest Rate

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
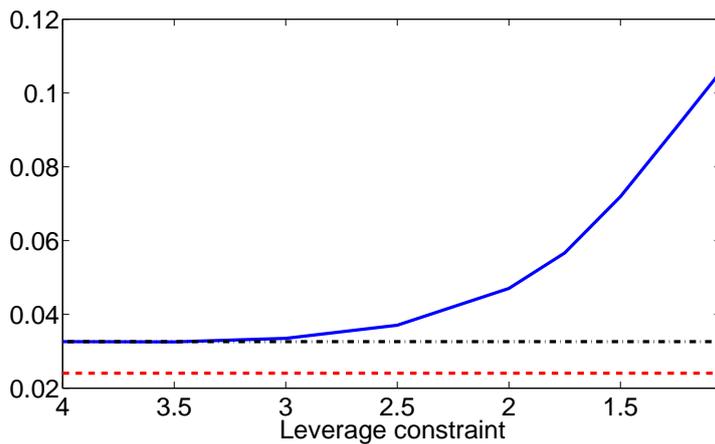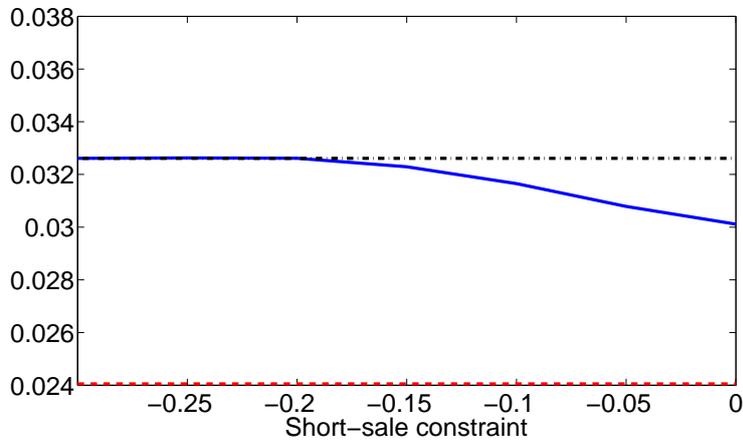
## Figure 7: Riskfree Interest Rate

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
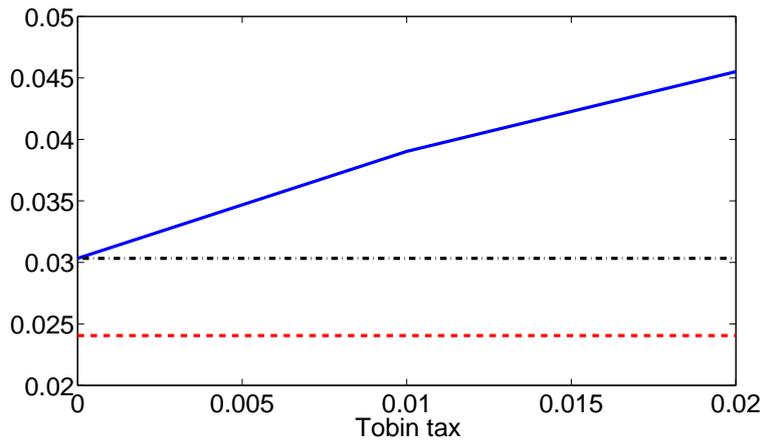
The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
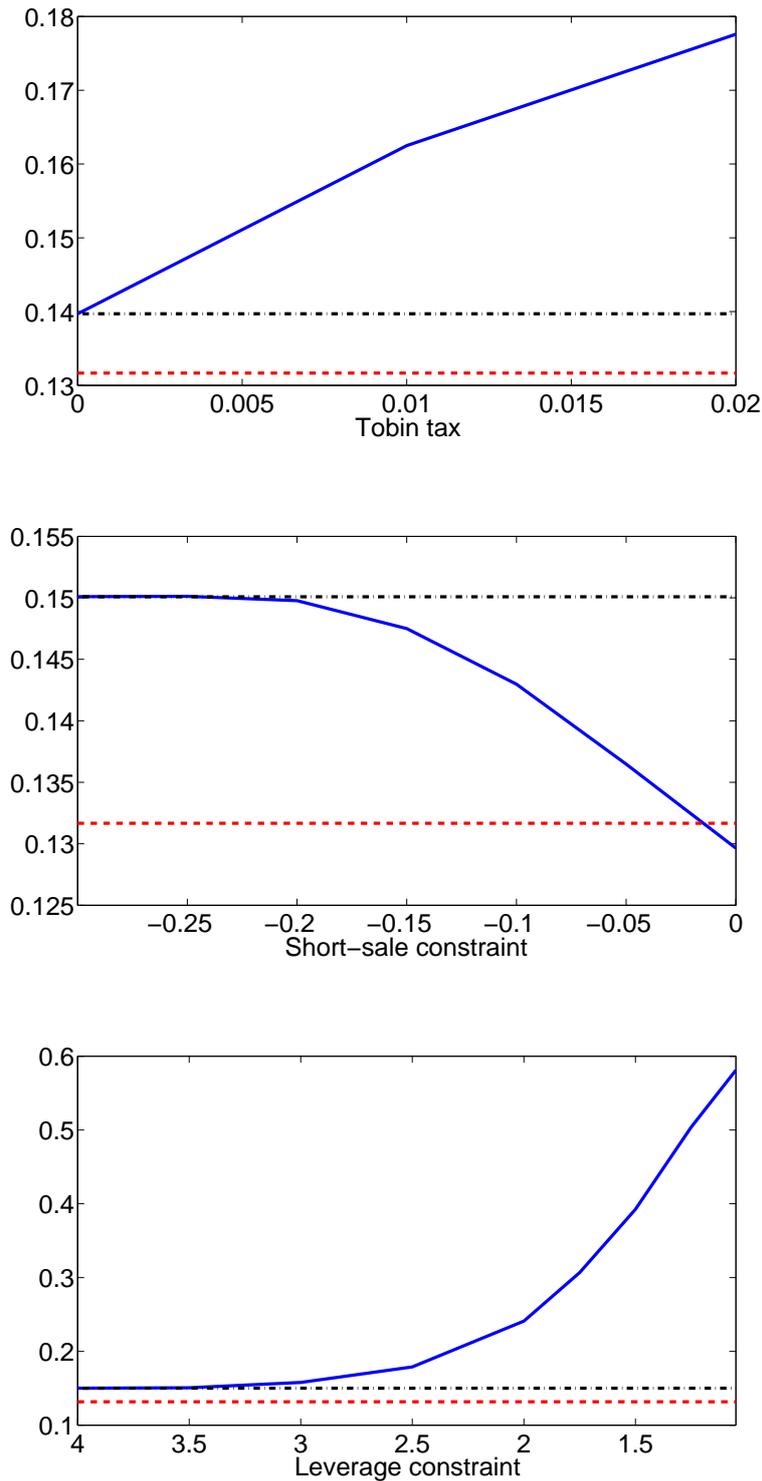
**Figure 9: Sharpe Ratio**

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
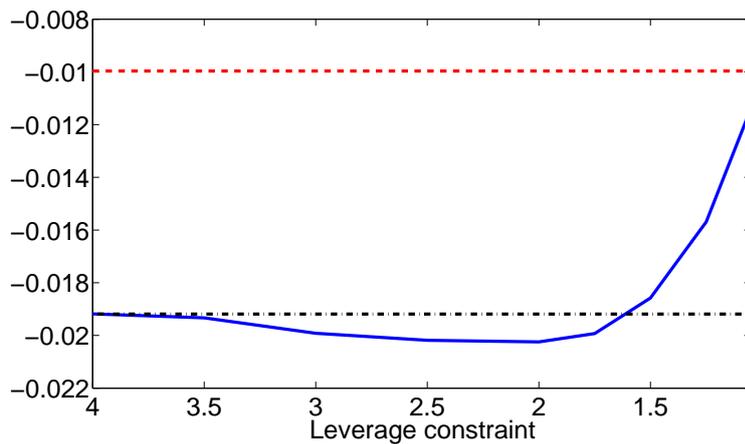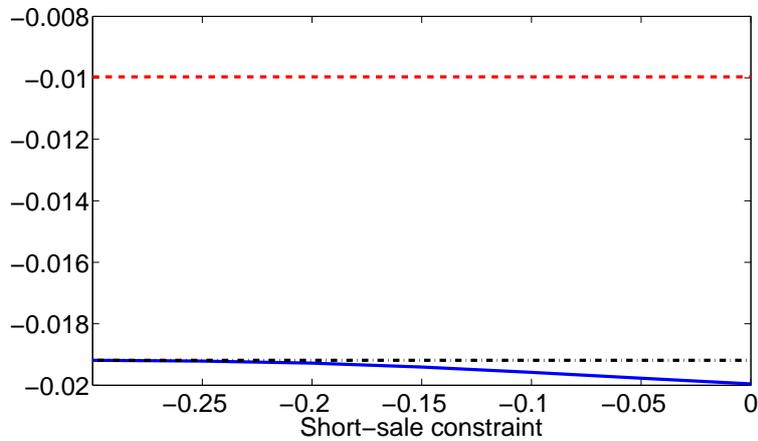
## Figure 10: Consumption Growth

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
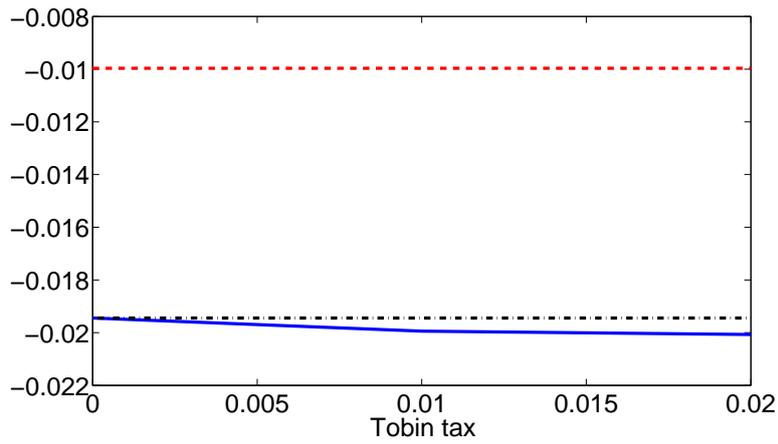
## Figure 11: Volatility of Consumption Growth

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
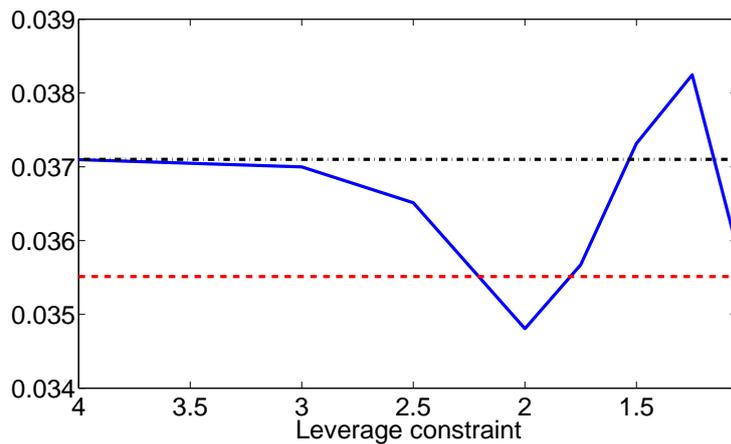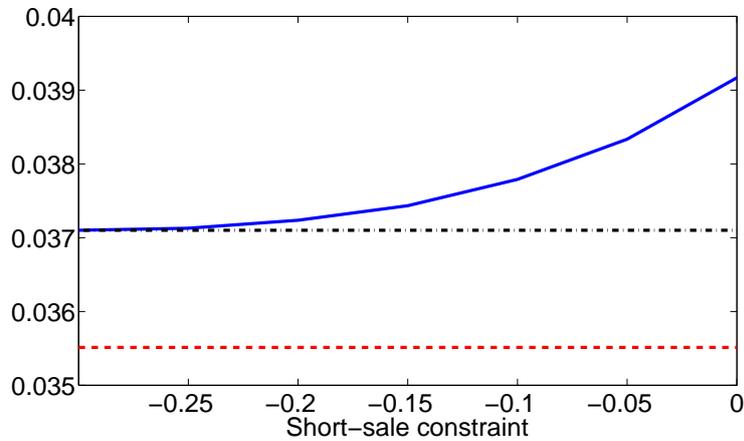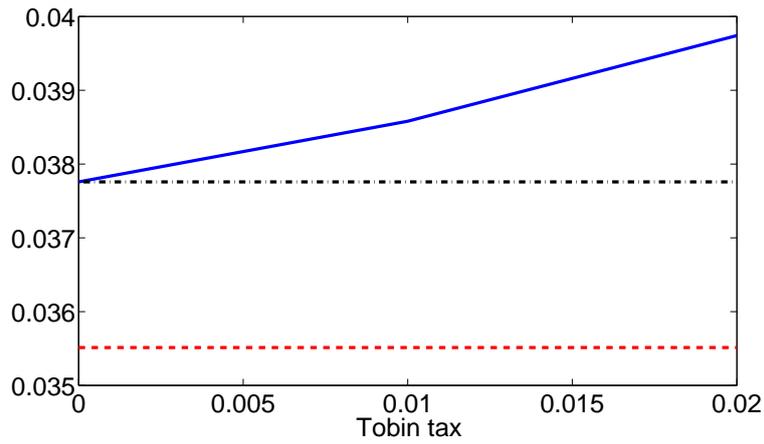


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## Figure 12: Output Growth

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
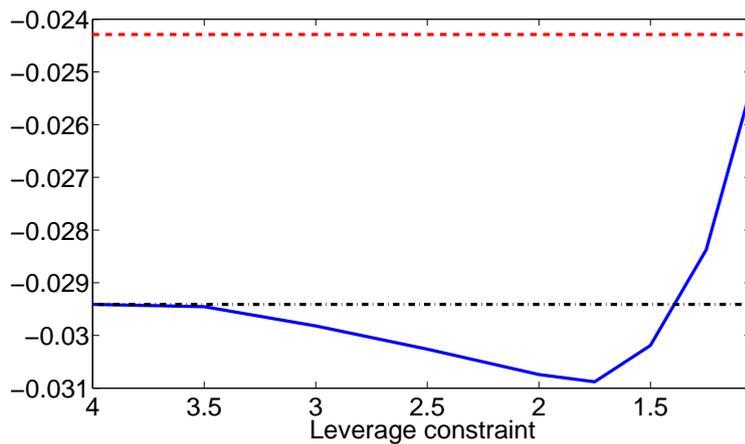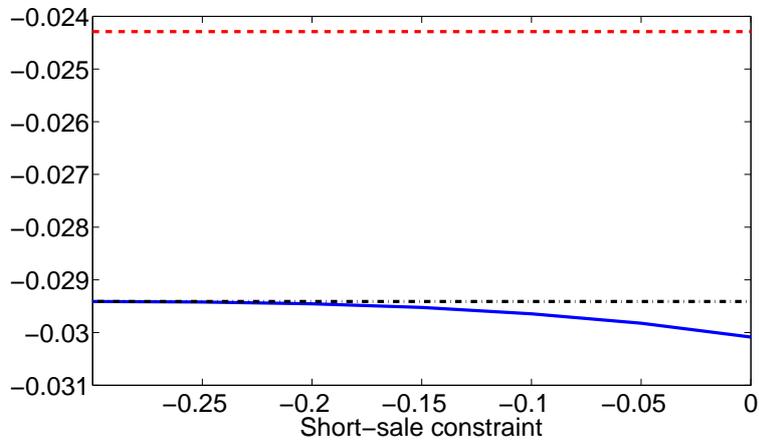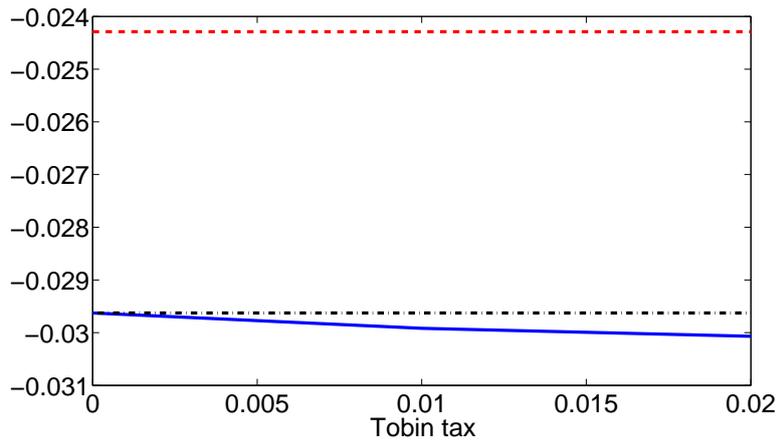
## Figure 13: Volatility of Output Growth

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
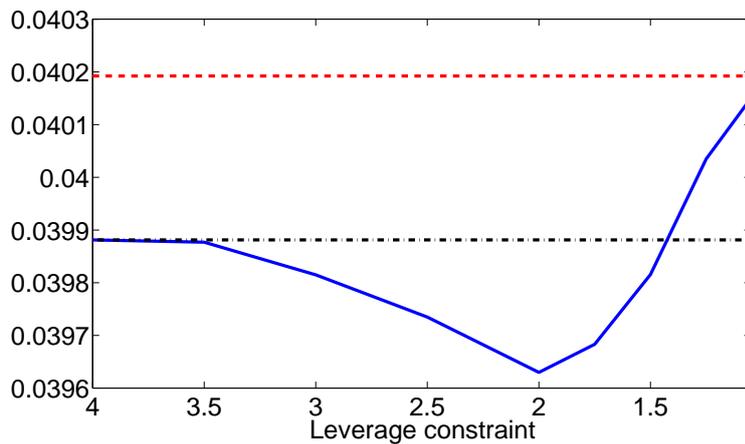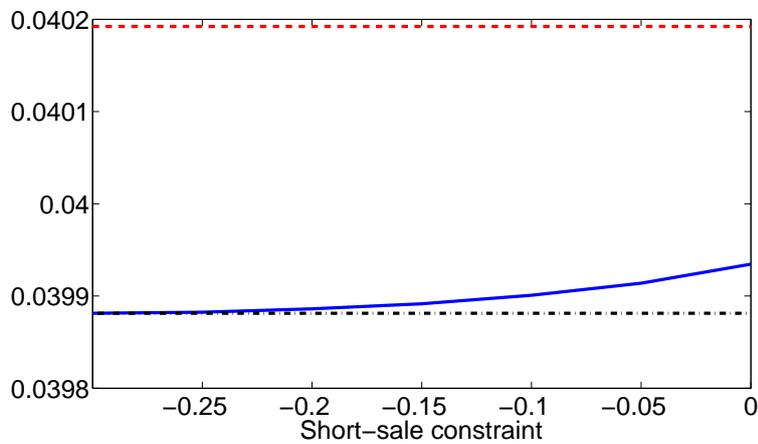
**Figure 14: Investment Growth**

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
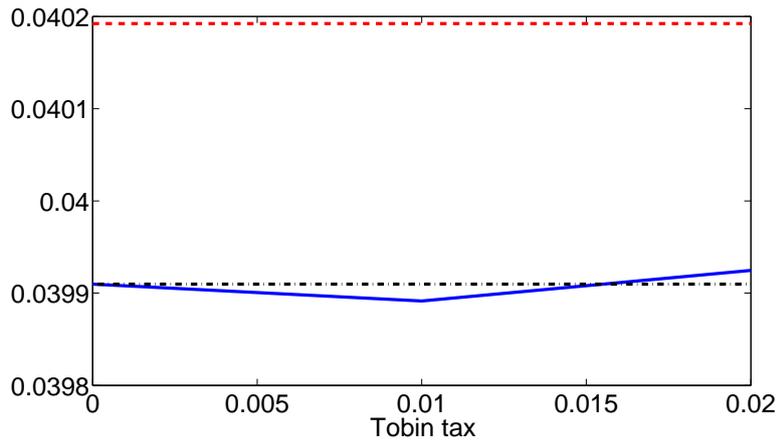
**Figure 15: Volatility of Investment Growth**

The three panels show implications of the introduction of each of the three considered regulatory measures (Tobin tax, short-sale constraint on the risky asset, and leverage constraint) on the quantity of interest. The red dotted line depicts the quantity of interest for the case when *both* investors are learning rationally; the black dash-dotted line depicts the case with excessive volatility due to "sentiment-prone" trading and without any intervention of regulators; and blue line depicts the case with the introduction of a particular regulatory measure in the economy with excessive volatility. The Tobin tax varies from 0% to 2% and is charged on the value of the risky asset traded; the short-sale constraint is the minimum position in a risky asset; and, the leverage constraint is the maximum value of the risky asset in the portfolio of an investor.
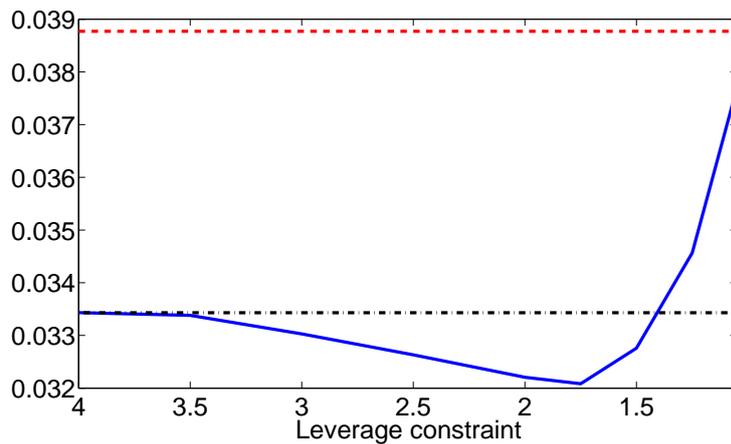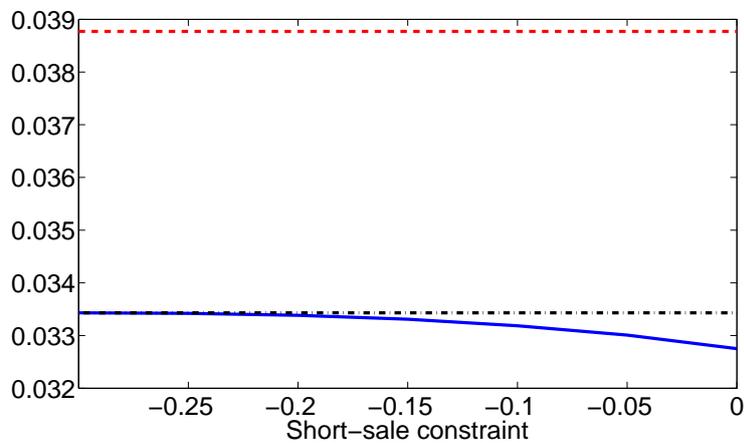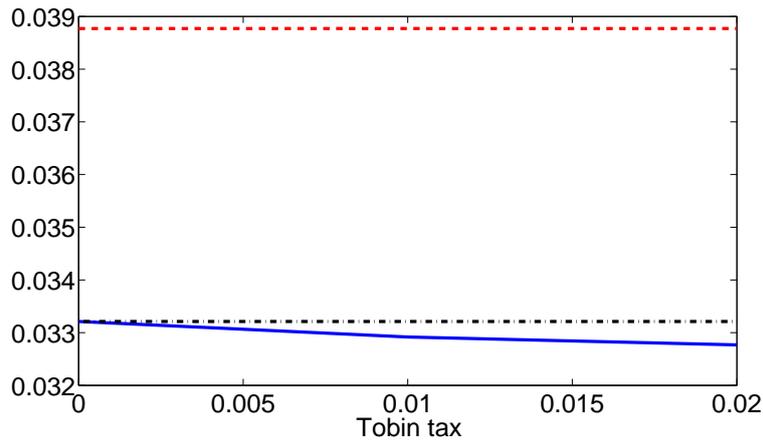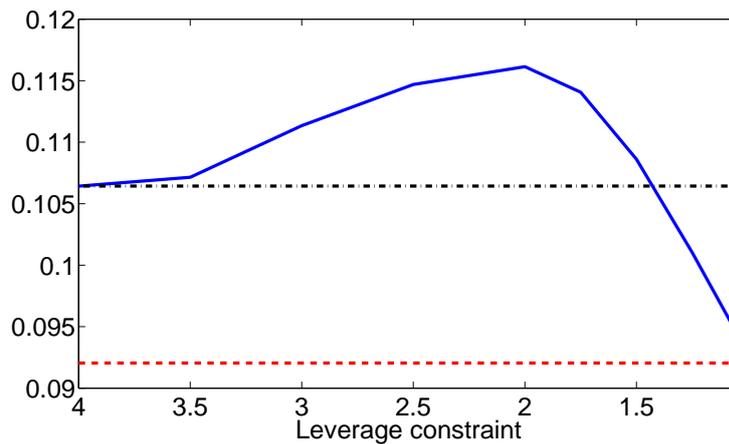


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# A Derivations of Optimality Conditions

## A.1 Optimality Conditions: Representative Firm

Form the Lagrangian from the objective function (3) and the law of motion for physical capital (4), we get:

$$\mathcal{L}(I_t) = \max_{I_t} \left\{ \left[ K_t Z_t - I_t - \frac{\xi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \right] + E_{k,t} \left[ \frac{M_{k,t+1}}{M_{k,t}} P^S_{k,t+1}(K_{t+1}) \right] \right\}$$

$$+ \min_{q_t} q_t \left( (1-\delta)K_t + I_t - K_{t+1} \right).$$

Taking the first-order condition by the investment $I_t$, we get

$$q_t = 1 + \xi \left( \frac{I_t}{K_t} - \delta \right).$$

Note that

$$\frac{\partial \mathcal{L}_t}{\partial K_{t+1}} = E_{k,t} \left[ \frac{M_{k,t+1}}{M_{k,t}} \frac{\partial P^S_{k,t+1}}{\partial K_{t+1}} \right] - q_t = 0,$$

$$\text{and hence, } q_t = E_{k,t} \left[ \frac{M_{k,t+1}}{M_{k,t}} \frac{\partial P^S_{k,t+1}}{\partial K_{t+1}} \right]. \tag{A1}$$

Apply the Envelope Theorem: at the optimal decision point we have

$$\frac{\partial P^S_{k,t}}{\partial K_t} = \frac{\partial \mathcal{L}_t}{\partial K_t} = Z_t - \frac{\partial I_t}{\partial K_t} - \frac{\xi}{2} \left[ 2 \left( \frac{I_t}{K_t} - \delta \right) \left( -\frac{I_t}{K_t^2} + \frac{1}{K_t} \frac{\partial I_t}{\partial K_t} \right) K_t + \left( \frac{I_t}{K_t} - \delta \right)^2 \right]$$

$$+ E_{k,t} \left[ \frac{M_{k,t+1}}{M_{k,t}} \frac{\partial P^S_{k,t+1}}{\partial K_{t+1}} \right] \frac{\partial K_{t+1}}{\partial K_t} + q_t \left( 1 - \delta + \frac{\partial I_t}{\partial K_t} - \frac{\partial K_{t+1}}{\partial K_t} \right)$$

$$= Z_t + \xi \left( \frac{I_t}{K_t} - \delta \right) \frac{I_t}{K_t} - \frac{\xi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 + \left( 1 + \xi \left( \frac{I_t}{K_t} - \delta \right) \right) (1 - \delta). \tag{A2}$$

Combining (A1) and (A2), we get the optimality condition for the firm (5).

## A.2 Optimality Conditions: Individual Investors

Form the Lagrangian from the objective function (6) and flow budget constraint (7):

$$\mathcal{L}(k,t) = \max_{c_{k,t}; \theta_{k,t}^{B,S}} \left\{ \frac{(c_{k,t} - h_k \times C_t)^{1-\gamma_k}}{1-\gamma_k} + \beta_k E_{k,t} \left[ V_{k,t+1} \left( C_{t+1}, \theta_{k,t}^S \left( S_{t+1} + D_{t+1} \right) + \theta_{k,t}^B \right) \right] \right\}$$

$$+ \min_{\lambda_{k,t}} \lambda_{k,t} \left[ \theta_{k,t-1}^S \left( S_{k,t} + D_t \right) + \theta_{k,t-1}^B - c_{k,t} - \theta_{k,t}^S S_{k,t} - \theta_{k,t}^B B_t \right].$$

Taking the first derivative with respect to current consumption gives

$$(c_{k,t} - h_k \times C_t)^{-\gamma_k} = \lambda_{k,t}$$

and taking the derivatives with respect to the portfolio holding gives

$$\beta_k E_{k,t} \left[ \frac{\partial V_{k,t+1}}{\partial W_{t+1}} \left( S_{k,t+1} + D_{t+1} \right) \right] = S_{k,t} \lambda_{k,t},$$

$$\beta_k E_{k,t} \left[ \frac{\partial V_{k,t+1}}{\partial W_{t+1}} \right] = B_{k,t} \lambda_{k,t}.$$

The budget condition remains:

$$\theta_{k,t-1}^S \left( S_t + D_t \right) + \theta_{k,t-1}^B - c_{k,t} - \theta_{k,t}^S S_{k,t} - \theta_{k,t}^B B_{k,t} = 0.$$

Using the Envelope Theorem, replace the marginal indirect utility $\frac{\partial V_{k,t+1}}{\partial W_{t+1}}$ with the marginal utility of consumption to arrive at the optimality conditions (8) and (9) for each investor, subject to the individual budget constraint (7).

## A.3 Optimality Conditions in the Presence of Regulatory Measures

We derive the optimality conditions for the case of financial transactions tax. The cases of shortsale and leverage constraints are analogous and simpler.

To treat the financial transaction tax, we follow Buss and Dumas (2011) and rewrite the optimality conditions in a dual formulation with an investor-specific Lagrange multiplier $R_{k,t}$ that takes into account the cost of trading in case the investors change the risky stock position. The derivations are as follows. First, we rewrite the flow budget constraint of an investor (7)

to take into account the costs of buying and selling risky asset:

$$\underbrace{\left[\theta^S_{k,t} - \theta^S_{k,t-1}\right]^+ (S_{k,t} + \kappa_t)}_{\text{cost of stock purchases}} + \underbrace{\left[\theta^S_{k,t} - \theta^S_{k,t-1}\right]^- (S_{k,t} - \kappa_t)}_{\text{proceeds of stock sales}} + \theta^S_{k,t-1}D_t + \theta^B_{k,t-1} - \theta^B_{k,t}B_t - c_{k,t} = 0; \forall k, t,$$

(A3)

where $\kappa_t$ is the level of proportional transaction costs for the value of stock traded. Define new variables $\widehat{\theta}$ and $\widehat{\widehat{\theta}}$:

$$\theta^S_{k,t} = \widehat{\theta}_{k,t} + \widehat{\widehat{\theta}}_{k,t} - \theta^S_{k,t-1},$$

(A4)

such that

$$\widehat{\widehat{\theta}}_{k,t} \le \theta^S_{k,t-1} \le \widehat{\theta}_{k,t},$$

(A5)

and rewrite the budget equation (A3) accordingly:

$$\left(\widehat{\theta}_{k,t} - \theta^S_{k,t-1}\right)(1 + \kappa_t)S_{k,t} + \left(\widehat{\widehat{\theta}}_{k,t} - \theta^S_{k,t-1}\right)(1 - \kappa_t)S_{t,i} + \theta^S_{k,t-1}D_t + \theta^B_{k,t-1} - \theta^B_{k,t}B_t - c_{k,t} = 0; \forall k, t,$$

(A6)

Form the Lagrangian using the original objective function (6), modified flow budget constraint (A6) included with multiplier $\lambda_{k,t}$, and two inequality conditions arising from (A5) with the multipliers $\mu_{1,k,t}$ and $\mu_{2,k,t}$ for the buying and selling decision, respectively. Note that, because of the transaction costs, our value function is a function of the past portfolio holdings, and not just a function of incoming wealth. Differentiating the resulting Lagrangian by current consumption $c_{k,t}$ and by the new portfolio variables $\widehat{\widehat{\theta}}_{k,t}$ and $\widehat{\theta}_{k,t}$ for the stock, we obtain the following first-order conditions:

$$(c_{k,t} - h_k \times C_t)^{-\gamma_k} = \lambda_{k,t}$$

$$\beta_k E_{k,t}\left[\frac{\partial V_{k,t+1}}{\partial \theta^S_{k,t}}\right] = \lambda_{k,t}(1 + \kappa_t)S_{k,t} - \mu_{1,k,t}$$

$$\beta_k E_{k,t}\left[\frac{\partial V_{k,t+1}}{\partial \theta^S_{k,t}}\right] = \lambda_{k,t}(1 - \kappa_t)S_{k,t} + \mu_{2,k,t}$$

$$\beta_k E_{k,t}\left[\frac{\partial V_{k,t+1}}{\partial \theta^B_{k,t}}\right] = \lambda_{k,t}B_{k,t}$$

$$\left(\widehat{\theta}_{k,t} - \theta_{k,t-1}^S\right)(1 + \kappa_t)S_{k,t} + \left(\widehat{\widehat{\theta}}_{k,t} - \theta_{k,t-1}^S\right)(1 - \kappa_t)S_{k,t}+$$

$$+\theta_{k,t-1}^S D_t + \theta_{k,t-1}^B - \theta_{k,t}^B B_{k,t} - c_{k,t} = 0$$

$$\mu_{1,k,t}(\theta_{k,t-1}^S - \widehat{\widehat{\theta}}_{k,t}) = 0$$

$$\mu_{2,k,t}(\theta_{k,t-1}^S - \widehat{\theta}_{k,t}) = 0$$

$$\mu_{\{1,2\},k,t} \geq 0$$

$$\widehat{\widehat{\theta}}_{k,t} \leq \theta_{k,t-1}^S \leq \widehat{\theta}_{k,t}.$$

Note that the second and the third equations from the first-order conditions imply that

$$\lambda_{k,t}(1 + \kappa_t)S_{k,t} - \mu_{1,k,t} = \lambda_{k,t}(1 - \kappa_t)S_{k,t} + \mu_{2,k,t},$$

and we can replace the two multipliers $\mu_{\{1,2\},k,t}$ with one multiplier $R_{k,t}$ as follows:

$$\lambda_{k,t}R_{k,t}S_{k,t} \stackrel{def}{=} \lambda_{k,t}(1 + \kappa_t)S_{k,t} - \mu_{1,k,t} = \lambda_{k,t}(1 - \kappa_t)S_{k,t} + \mu_{2,k,t}. \tag{A7}$$

From (A7), it follows that

$$\mu_{1,k,t} = \lambda_{k,t}S_{k,t}(1 + \kappa_t - R_{k,t}),$$

$$\mu_{2,k,t} = \lambda_{k,t}S_{k,t}(-1 + \kappa_t + R_{k,t}),$$

and we can rewrite the slackness conditions respectively:

$$(1 + \kappa_t - R_{k,t})(\theta_{k,t-1}^S - \widehat{\widehat{\theta}}_{k,t}) = 0,$$

$$(-1 + \kappa_t + R_{k,t})(\theta_{k,t-1}^S - \widehat{\theta}_{k,t}) = 0.$$

From the redefined slackness conditions we can see that if there is any trade, then $R_{k,t}$ quantifies the paid transaction costs (up to the scaling by the current stock price), and it allows us to reformulate the floating budget equation as follows:

$$\theta_{k,t-1}^S D_t - c_{k,t} + \theta_{k,t-1}^B - \theta_{k,t}^B B_{k,t} - (\theta_{k,t}^S - \theta_{k,t-1}^S)S_{k,t}R_{k,t} = 0.$$

Now apply the Envelope Theorem to get rid of the value function in the FOCs.

$$\frac{\partial V_{k,t+1}}{\partial \theta_{k,t}^S} = \lambda_{k,t+1}R_{k,t+1}(S_{k,t+1} + D_t)$$

$$\frac{\partial V_{k,t+1}}{\partial \theta_{k,t}^B} = \lambda_{k,t+1}B_{k,t+1},$$

and finally get the following system (including (A4)):

$$(c_{k,t} - h_k \times C_t)^{-\gamma_k} = \lambda_{k,t}$$

$$\beta_k E_{k,t} \left[ \lambda_{k,t+1} R_{k,t+1} (S_{k,t+1} + D_t) \right] = \lambda_{k,t} R_{k,t} S_{k,t}$$

$$\beta_k E_{k,t} \left[ \lambda_{k,t+1} B_{k,t+1} \right] = \lambda_{k,t} B_{k,t}$$

$$\theta^S_{k,t-1} D_t - c_{k,t} + \theta^B_{k,t-1} - \theta^B_{k,t} B_{k,t} - (\theta^S_{k,t} - \theta^S_{k,t-1}) S_{k,t} R_{k,t} = 0$$

$$\theta^S_{k,t} = \widehat{\theta}_{k,t} + \widehat{\widehat{\theta}}_{k,t} - \theta^S_{k,t-1}$$

$$(1 + \kappa_t - R_{k,t})(\theta^S_{k,t-1} - \widehat{\widehat{\theta}}_{k,t}) = 0$$

$$(-1 + \kappa_t + R_{k,t})(\theta^S_{k,t-1} - \widehat{\theta}_{k,t}) = 0$$

$$1 - \kappa_t \leq R_{k,t} \leq 1 + \kappa_t$$

$$\widehat{\widehat{\theta}}_{k,t} \leq \theta^S_{k,t-1} \leq \widehat{\theta}_{k,t}.$$

# B   Equilibrium: Numerical Solution Method

The numerical method we use is not based on an approximation of any sort;[14] it is exact (except for local interpolation and, of course, numerical truncation error), which is a distinct advantage considering the non linearities involved.

We wish to obtain the equilibrium in a numerical form by backward-induction. For reasons explained in Dumas and Lyasoff (2012), the system of first-order conditions written in the previous appendixes cannot be solved by backward induction. However, it is possible to solve the equations by backward induction if one changes the sequence in which the equations are solved, performing a re-grouping of equations in such a way that, at a node of time $t$, one solves the equations yielding not consumption at time $t$ and the portfolio chosen at time $t$ but, instead, consumption at the various successor nodes of time $t + 1$ and portfolio at time $t$.

For instance, consider the system of equations derived in Appendix A.3. We shift to time $t+1$ all equations except the first-order conditions with respect to portfolio choice, thus getting

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[14] Approximation methods commonly used are: Taylor series (the "perturbation" method) and linear combinations of polynomials (the "projection" method).

for all states $j$ that can occur at time $t+1$:

$$(c_{k,t+1,j} - h_k \times C_{t+1,j})^{-\gamma_k} = \lambda_{k,t+1,j}$$

$$\beta_k E_{k,t}\left[\lambda_{k,t+1}R_{k,t+1}(S_{k,t+1}+D_t)\right] = \lambda_{k,t}R_{k,t}S_{k,t}$$

$$\beta_k E_{k,t}\left[\lambda_{k,t+1}B_{k,t+1}\right] = \lambda_{k,t}B_{k,t}$$

$$\theta_{k,t}^S D_{t+1,j} - c_{k,t+1,j} + \theta_{k,t}^B - \theta_{k,t+1,j}^B B_{k,t+1,j} - (\theta_{k,t+1,j}^S - \theta_{k,t}^S)S_{k,t+1,j}R_{k,t+1,j} = 0$$

$$\theta_{k,t+1,j}^S = \widehat{\theta}_{k,t+1,j} + \widehat{\widehat{\theta}}_{k,t+1,j} - \theta_{k,t}^S$$

$$(1 + \kappa_{t+1,j} - R_{k,t+1,j})(\theta_{k,t}^S - \widehat{\widehat{\theta}}_{k,t+1,j}) = 0$$

$$(-1 + \kappa_{t+1,j} + R_{k,t+1,j})(\theta_{k,t}^S - \widehat{\theta}_{k,t+1,j}) = 0$$

$$1 - \kappa_{t+1,j} \leq R_{k,t+1,j} \leq 1 + \kappa_{t+1,j}$$

$$\widehat{\widehat{\theta}}_{k,t+1,j} \leq \theta_{k,t}^S \leq \widehat{\theta}_{k,t+1,j}.$$

Solving this equation system provides future consumption simultaneously in all states $j$ of time $t+1$ for each possible gridpoint value of the time-$t$ endogenous state variables $\{\lambda_{k,t}, R_{k,t}\}$ or, more parsimoniously, their ratios between investors. It is actually even more efficient to replace the state variables $\lambda_{k,t}$ with $(c_{k,t} - h_k \times C_t)^{-\gamma_k}$ and to perform the calculation for each possible gridpoint value of the distribution of current consumption between the two investors and each value of the habit variable $C_t$. We can also eliminate the future state prices to get finally:

$$\beta_k E_{k,t}\left[(c_{k,t+1,j} - h_k \times C_{t+1,j})^{-\gamma_k}R_{k,t+1}(S_{k,t+1}+D_t)\right] = (c_{k,t} - h_k \times C_t)^{-\gamma_k}R_{k,t}S_{k,t}; k = 1,2$$

$$\beta_k E_{k,t}\left[(c_{k,t+1,j} - h_k \times C_{t+1,j})^{-\gamma_k}B_{k,t+1}\right] = (c_{k,t} - h_k \times C_t)^{-\gamma_k}B_{k,t}; k = 1,2$$

$$\theta_{k,t}^S D_{t+1,j} - c_{k,t+1,j} + \theta_{k,t}^B - \theta_{k,t+1,j}^B B_{k,t+1,j} - (\theta_{k,t+1,j}^S - \theta_{k,t}^S)S_{k,t+1,j}R_{k,t+1,j} = 0; k = 1,2; j = 1,..4$$

$$\theta_{k,t+1,j}^S = \widehat{\theta}_{k,t+1,j} + \widehat{\widehat{\theta}}_{k,t+1,j} - \theta_{k,t}^S; k = 1,2; j = 1,..4$$

$$(1 + \kappa_{t+1,j} - R_{k,t+1,j})(\theta_{k,t}^S - \widehat{\widehat{\theta}}_{k,t+1,j}) = 0; k = 1,2; j = 1,..4$$

$$(-1 + \kappa_{t+1,j} + R_{k,t+1,j})(\theta_{k,t}^S - \widehat{\theta}_{k,t+1,j}) = 0; k = 1,2; j = 1,..4$$

$$1 - \kappa_{t+1,j} \leq R_{k,t+1,j} \leq 1 + \kappa_{t+1,j}$$

$$\widehat{\widehat{\theta}}_{k,t+1,j} \leq \theta_{k,t}^S \leq \widehat{\theta}_{k,t+1,j}.$$

There are two investors and the exogenous variables follow a Markov chain with four states, as explained in Section 2.3. Then the above system contains 36 equations. Coupled with the firm's first-order condition (5) and the two market-clearing conditions (10) and (11), that's a system of 39 equations for the 39 unknowns: $\left\{ c_{k,t+1,j}, \theta_{k,t}^S, \theta_{k,t}^B, \widehat{\widehat{\theta}}_{k,t+1,j}, \widehat{\theta}_{k,t+1,j}, R_{k,t+1,j}, I_t, S_{k,t} \right\}$. The time-$t$ securities prices can actually be eliminated, thus leaving a 37-equation system.

The complementary slackness conditions cause the system to be ill-defined. As in Buss and Dumas (2011), we replace them with the following:

$$(1 + \kappa_{t+1,j} - R_{k,t+1,j})(\theta_{k,t}^S - \widehat{\widehat{\theta}}_{k,t+1,j}) = \varepsilon,$$

$$(-1 + \kappa_{t+1,j} + R_{k,t+1,j})(\theta_{k,t}^S - \widehat{\theta}_{k,t+1,j}) = \varepsilon,$$

and, as we approach the final solution, we let $\varepsilon$ approach zero. That is called the "Interior-Point Algorithm;" see Armand, Benoist, and Orban (2008).

We have already mentioned three endogenous state variables $\frac{c_{1,t}}{c_{1,t}+c_{2,t}}$, $\frac{R_{1,t}}{R_{2,t}}$ and habit $C_t = \frac{1}{K}\sum_{k=1,2} c_{k,t-1}$, which require a 3D grid. But we treat in the same way, that is, on a grid, a fourth variable, which is, in fact, exogenous and which is imbeded in the conditional expected value operators $E_{k,t}$. Beliefs differ; hence the subscript $k$ on these operators. The way in which beliefs differ has been explained in Section 2.3. Recall that we consider two hidden states and four observable states of a Markov chain. Investors have immutable beliefs about the transition probabilities from the two time-$t$ hidden states to the two time-$t+1$ hidden states and, from there, to the four time-$t+1$ observable states. But each of the two investors has to estimate the probability of being at time $t$ in one of the two hidden states. These estimated probabilities change over time in response to the observations made in the past. By means of simulations of the hidden Markov chain (the workings of which are exogenous to our model), we identify the full, discrete list of pairs of these estimated probabilities that can occur at any time and we place these on a grid, which constitutes the fourth dimension.

The capital stock $K_t$ is an endogenous state variable as well but, under our assumptions, a scale-invariance property holds so that it is only a scale variable, which can be factored out of all the equations.

To recapitulate, the steps of the procedure are as follows:

1. Set up a 4D grid of points applicable at all times for the three endogeneous state variables and the state probability beliefs. The domain of that 4D grid is bounded and the bounds are known a priori.

2. At time $T$ set the terminal (ex-dividend) values $S_{k,T}$, $B_{k,T}$, $\theta^B_{k,T}$ and $\theta^B_{k,T}$ at the value 0.

3. At time $t = T-1, T-2, ...., 0$, solve the system of 37 (or 39) equations described above, for each point of the 4D grid and piecewise interpolate, over the grid of the three endogenous state variables, the functions $S_{k,t}$, $B_{k,t}$, $\theta^B_{k,t}$. and $\theta^B_{k,t}$.[15]

4. (for completeness) At time $t = 0$, solve for the unknowns $c_{k,0}$ and $R_{k,0}$ using the time-0 budget constraints:

$$\theta^S_{k,-1} D_0 - c_{k,0} + \theta^B_{k,-1} - \theta^B_{k,0} \left( \left\{ \frac{c_{1,0}}{c_{1,0}+c_{2,0}}, \frac{R_{1,0}}{R_{2,0}}, C_0 \right\} \right) B_{k,0} \left( \left\{ \frac{c_{1,0}}{c_{1,0}+c_{2,0}}, \frac{R_{1,0}}{R_{2,0}}, C_0 \right\} \right)$$

$$-(\theta^S_{k,0} \left( \left\{ \frac{c_{1,0}}{c_{1,0}+c_{2,0}}, \frac{R_{1,0}}{R_{2,0}}, C_0 \right\} \right) - \theta^S_{k,-1}) S_{k,0} \left( \left\{ \frac{c_{1,0}}{c_{1,0}+c_{2,0}}, \frac{R_{1,0}}{R_{2,0}}, C_0 \right\} \right) R_{k,0} = 0; k = 1, 2,$$

where $\theta^S_{k,-1}$ and $\theta^B_{k,-1}$ are initial endowments of securities and $C_0$ is some initial habit level.

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[15]The state probability variables require no interpolation since they can only take a fixed set of discrete values.

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