Modeling financial sector tail risk, with application to the euro area*

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Abstract

We develop a novel high-dimensional non-Gaussian framework to infer joint and conditional measures of financial sector risk. For this setting we also derive a conditional law of large numbers which permits the computation of joint and conditional risk measures within seconds. Joint risk assessments are based on a dynamic multivariate skewed and fat-tailed copula which accommodates asymmetries, heavy tails, as well as non-linear and time-varying dependence in asset values. We apply the modeling framework to assess the risk from multiple financial firm defaults in the euro area during the financial and sovereign debt crisis, and find unprecedented joint tail risks during 2011-12. Augmenting the model with additional explanatory variables helps to explain the systematic correlation dynamics.

Keywords: systemic risk; dynamic equicorrelation model; generalized hyperbolic distribution; law of large numbers; large portfolio approximation.

JEL classification: G21, C32.

1 Introduction

In this paper we develop a novel high-dimensional non-Gaussian framework that allows us to infer joint and conditional measures of financial sector risk. The framework is designed to give model-based answers to questions such as: What is the probability of observing a joint default of at least a certain fraction of currently active financial firms in a given economic region over the next 12 months? And how does that time-varying joint tail probability change in a situation in which a given financial firm at risk actually defaults? In order to evaluate the corresponding risk measurements quickly (within seconds), we derive and apply a conditional law of large numbers that arises naturally in our setting. As a result, we do not need to rely on simulation methods.

Our modeling framework is based on the multivariate Generalized Hyperbolic skewed-t (GHST) density with dynamic volatility and dependence parameters as put forward in Lucas, Schwaab, and Zhang (2013), but in this paper extended to handle large cross-sectional dimensions both in terms of parameter and risk factor estimation and also the efficient evaluation of joint and conditional risk measures. The non-Gaussian setting allows us to capture empirically the salient stylized facts in the co-movement of financial firms' equity values, such as asymmetries, heavy (joint) tails, and non-linear and time varying dependence. At the same time, the proposed model remains sufficiently flexible to be fitted to a large cross section. Since the model can be treated as a statistical factor model, it can also be used to explore the role of economic variables in driving the time-varying dependence structure. At each point in time, the multivariate risk model is consistent with available information about firms' marginal probabilities of default.

Since the onset of the financial crisis in 2007, financial stability monitoring has become key priorities in many central banks, in addition to their respective monetary policy mandates, see Adrian, Covitz, and Liang (2013). In many cases, central banks or national competent authorities have received micro-prudential and macro-prudential tasks which involve the analysis of risks to a system of financial intermediaries. The cross sectional dimensions involved are usually large. For example, the FDIC currently oversees several thousand banks in the United States, a non-negligible subset of which are large and potentially systemically important. As a second example, the ECB takes over the direct supervision of approximately 130 large European financial sector firms in late 2014. This suggests that prudential oversight is a high-dimensional problem even if attention is limited to only large and systemically important banks. Independent of prudential oversight, financial institutions also set economic capital buffers and risk limits to withstand the materialization of multiple bad outcomes, and the business relationships typically extend to a large number of active financial counterparties.

The construction of useful joint and conditional risk measures, however, is not straightforward. The risk of a default cluster in a portfolio typically involves a high cross-sectional dimension, but extending a copula or multivariate density model beyond, say, five time series is difficult and rarely considered in the literature. Second, the default dependence among financial institutions is non-linear and time-varying. For example, the connectedness across financial firms is stronger during times of turmoil, when fire-sale externalities connect all financial firms through market prices in addition to their direct business links, see for example Brunnermeier and Pedersen (2009). As a result, taking into account higher correlations during times of stress, in addition to merely accommodating rising uncertainty (volatilities) and elevated marginal risks, is an important feature of financial sector joint tail risk assessment.

We overcome the econometric problems implied by our high-dimensional non-Gaussian setting and the time-variation in parameters by proceeding in two steps. First, we separate the univariate from the multivariate analysis, as in Engle (2002). Second, we impose a parsimonious equicorrelation structure onto our dynamic density, similar to the approach taken by Engle and Kelly (2012). The parsimonious structure ensures that all computations remain tractable. The time variation in volatility and correlation parameters is modeled following the Generalized Autoregressive Score (GAS) framework of Creal, Koopman, and Lucas (2011, 2013). In this setting, the scaled score of the local log-likelihood drives the dynamic behavior of the time-varying parameters. As a result, the log-likelihood is available in closed form and can be easily maximized. Moreover, the correlation and volatility dynamics are robust to the non-normality features present in typical financial time series.

In the econometric application, we furthermore take considerable care to distinguish simultaneous increases in volatility from changes in systematic dependence, as forcefully argued in Forbes and Rigobon (2002). In order to clearly model volatility effects, we introduce leverage terms in the marginal GHST volatility models, see also Black (1976), Nelson (1991), Braun, Nelson, and Sunier (1995) and Harvey (2013). Introducing well-documented leverage effects into financial sector equity data rationalizes the pronounced observed negative skewness, and leaves the degrees of freedom parameter free to capture fat tails due to infrequent extreme market movements in the financial data.

This paper uses market risk methods to inform a portfolio credit risk problem. This effectively connects a literature on non-Gaussian volatility and dependence modeling with another separate literature on portfolio credit risk and loan loss asymptotics. Time-varying parameter models for volatility and dependence have been investigated inter alia by Engle (2002), Demarta and McNeil (2005), Creal, Koopman, and Lucas (2011), Zhang, Creal, Koopman, and Lucas (2011), and Engle and Kelly (2012). Portfolio credit risk models and associated tail risk measures, in turn, have been studied in some detail by Vasicek (1987), Lucas, Klaassen, Spreij, and Straetmans (2001), Gordy (2003), Koopman, Lucas and Schwaab (2011, 2012), and Giesecke, Spiliopoulos, Sowers, and Sirignano (2013). The modeling framework is applied to financial sector risk assessment, as in inter alia Hartmann, Straetmans, and De Vries (2005), Acharya, Engle, and Richardson (2012), Malz (2012), Suh (2012), and Black, Correa, Huang, and Zhou (2012). Finally, we adopt an observation-driven modeling framework to facilitate parameter and latent factor estimation in a non-Gaussian setting. Observation-driven score-based time-varying parameter models are an active area of research, see Creal, Schwaab, Koopman, and Lucas (2013), Harvey (2013), and Oh and Patton (2013).

Three studies in particular relate to our construction of financial stability measures. In each case, a collection of firms is seen as a portfolio of obligors whose multivariate dependence structure is inferred from equity returns. Avesani, Pascual, and Li (2006) assess defaults in a Gaussian factor model setting. The determination of joint default probabilities is in part based on the notion of an nth-to-default CDS basket, which can be set up and priced as suggested in Hull and White (2004). Alternatively, Segoviano and Goodhart (2009) propose a non-parametric copula approach. Here, the banking system's multivariate density is recovered by minimizing the distance between the so-called banking system's multivariate density and a parametric prior density subject to tail constraints that reflect individual default probabilities. We regard each of these approaches as polar cases, and attempt to strike a middle ground. The proposed score-driven framework in our current paper retains the ability to describe the salient equity data features in terms of skewness, fat tails, and time-varying correlations (which the Gaussian copula fails to do), and in addition retains the ability to fit a cross-sectional dimension larger than a few firms (which the non-parametric approach fails to do due to computational problems). In addition, parameter non-constancy is addressed explicitly in our new modeling setup. The two above approaches are inherently static, and rely on a rolling window approach to capture time variation in parameters. Finally, we note here the paper developed by Oh and Patton (2013), who model systemic risk in the U.S. non-financial sector using a score-driven approach and the skewed Student's t density of Hansen (1994), rather than the Generalized Hyperbolic Skewed-t of the current paper.

The remainder of the paper is as follows. Section 2 introduces a modified version of the Merton structural modeling framework for simultaneous default of financial sector firms. In particular, it replaces the usual normality assumption by another one that is closer to the processes observed for typical financial data. Two new risk measures are introduced, as well as an efficient way to compute them using an analytic approximation based on a conditional law of large numbers. The econometric GHST dynamic equicorrelation model is introduced in Section 3. Section 4 presents the empirical results on the likelihood of joint defaults of large financial institutions in the euro area. Section 5 studies the explanatory power of additional economic variables for explaining the time varying correlation dynamics. Section 6 concludes. Appendix A presents technical details about the model specification, while Appendix B presents additional estimation results.

2 A framework for financial sector tail risk

2.1 Non-Gaussian asset value processes

The structural approach due to Merton (1974) and Black and Cox (1976) is probably the most well-known benchmark for understanding credit risk dynamics. In this framework, firms' underlying asset values evolve stochastically over time, and default is triggered if a firm's asset value falls below a default threshold. This threshold is in general determined by a firm's current liability structure. In case of multiple firms, the assumptions regarding the correlation (or more generally, dependence) structure between the firms' asset values becomes important for overall risk.

In its simplest form and for the case of two firms $i = 1, 2$, the Merton model is given by

$$\begin{aligned} \mathrm{d}V_{i,t} &= V_{i,t} \cdot (\mu_i \mathrm{d}t + \sigma_i \mathrm{d}W_{i,t}), \qquad (1) \\ y_{i,t} &= \log\left(V_{i,t}/V_{i,t-1}\right) \sim \mathrm{N}\left(\mu_i - \sigma_i^2/2, \sigma_i^2\right), \end{aligned}$$

where $V_{i,t}$ denotes the asset value of firm i at time t, $W_{i,t}$ is a standard Brownian Motion, μ_i and σ_i^2 are fixed drift and variance parameters, respectively, and $\mathrm{d}W_{1,t}W_{2,t} = \rho \mathrm{d}t$, with $\rho \in (-1, 1)$ denoting the asset correlation parameter. The log asset returns $y_{i,t}$ are normally distributed. This implies that joint default probabilities can be read off a bi-variate normal distribution. It also implies that asymptotic joint tail dependence is zero.

The normality assumption in (1) is too restrictive for most financial data. In addition, it rules out asymmetries, joint tail dependence, and non-linear dependence across the support of the distribution. Asset returns are usually skewed and heavy-tailed and have time-varying (co)variances. Moreover, the price process does not have a continuous path as the Brownian Motion, but is identified as a semi-martingale with possible jumps; see, for example, Cont and Tankov (2004). To incorporate these empirical features, the Generalized Hyperbolic (GH) Lévy process has gained more attention as a replacement for the Gaussian assumption. Eberlein (2001) provides a useful survey on asset pricing models under the GH Lévy process assumption. GH distributions are infinitely divisible (Barndorff-Nielsen and Halgreen (1977)) and can generate a Lévy process that is a semimartingale.

We rewrite (1) using a Lévy process framework as in Bibby and Sørensen (2001),

$$\mathrm{d}V_{i,t} = \frac{1}{2}v(V_{i,t})[\log(f(V_{i,t})v(V_{i,t}))]'\mathrm{d}t + \sqrt{v(V_{i,t})}\mathrm{d}W_{i,t}, \quad (2)$$

$$y_{i,t} = \log\left(V_{i,t}/V_{i,t-1}\right) \sim GHST(\tilde{\sigma}^2_{i,t}, \gamma_i, \nu), \quad (3)$$

with $v(V_{i,t})$ and $f(V_{i,t})$ two continuously differentiable strictly positive real valued functions defined on $\mathbb{R}$. Following the arguments in Bibby and Sørensen (2003), we can find suitable functions for a prescribed marginal distribution such as the GHST exist. We focus in this paper on the GH skewed-t (GHST) distribution, which is an asymmetric version of the Student's t distribution. Our analysis can be easily extended to other GH distributions. The GHST distribution is an asymmetric fat-tailed distribution with skewness parameter γ_i and kurtosis parameter ν. The distribution is flexible enough to capture the most interesting data features with a limited set of parameters, while keeping close to the structural default modeling and derivatives pricing literature. In addition, the dynamic version of the GH distribution proposed in Zhang, Creal, Koopman, and Lucas (2011) can accommodate for time-varying volatilities and correlations.

In the modified Merton framework (2), firm i defaults at time t if $y_{i,t}$ falls below the firm specific default threshold $y^*_{i,t}$. Therefore, at time t, the firm's marginal probability of default $p_{i,t}$ is given by

$$p_{i,t} = F(y^*_{i,t}; \tilde{\mu}_i, \tilde{\sigma}_{i,t}, \gamma_i, \nu), \quad (4)$$

where $F(\cdot; \tilde{\mu}, \tilde{\sigma}, \gamma, \nu)$ is the cumulative distribution function (cdf) of a univariate GHST distribution with parameters $\tilde{\mu}, \tilde{\sigma}, \gamma, \nu \in \mathbb{R}$. Similarly, the joint default probability of two borrowers is given by

$$p_{i,j,t} = F(y^*_{i,t}, y^*_{j,t}; \tilde{\mu}, \tilde{\Sigma}, \gamma, \nu), \quad (5)$$

where $F(\cdot; \tilde{\mu}, \tilde{\Sigma}, \gamma, \nu)$ is the bivariate GHST cdf with parameters $\tilde{\mu}, \gamma \in \mathbb{R}^2$, $\nu \in \mathbb{R}$, and $\tilde{\Sigma} \in \mathbb{R}^{2\times 2}$. Generalizing this to m or more defaults is straightforward.

2.2 Non-Gaussian dependence and risk factor model

This section introduces our multivariate copula model for correlated defaults and demonstrates that it is also a multi-factor credit risk model. The multivariate Generalized Hyperbolic Skewed Student's t (GHST) model implies marginal GHST distributions, which in turn are consistent with GH Lévy processes for firms' asset values as discussed in Section 2.1.

In our setting, changes in log asset value $y_{i,t}$ triggers a default of firm i if $y_{i,t}$ falls below a threshold value $y^*_{i,t}$. The $y_{i,t}$s, $i = 1, \ldots, N$, are linked together via a GHST copula,

$$y_{i,t} \quad = \quad (\varsigma_t - \mu_\varsigma)\gamma + \sqrt{\varsigma_t}\tilde{L}_{i,t}\epsilon_t, \qquad i = 1, \ldots, N, \tag{6}$$

where $\epsilon_t \in \mathbb{R}^N$ is a vector of standard normally distributed risk factors, $\tilde{L}_t$ is an $n \times n$ matrix of risk factor sensitivities, and $\gamma \in \mathbb{R}^N$ is a vector controlling the skewness of the copula. The random scalar $\varsigma_t \in \mathbb{R}^+$ is assumed to be an inverse-Gamma distributed risk factor that affects all sovereigns simultaneously, where ς_t and ϵ_t are independent, and $\mu_\varsigma = \text{E}[\varsigma_t]$. As a result, default dependence in model (6) stems from two sources: common exposures to the normally distributed risk factors ϵ_t as captured by the time-varying matrix $\tilde{L}_t$; and a common exposure to the scalar risk factor ς_t. The former captures connectedness through correlations, while the latter captures such effects through the tail-dependence of the copula. To see this, note that if ς_t is non-random, the first term in (6) drops out of the equation and there is zero tail dependence. Conversely, if ς_t is large, all asset values are affected at the same time, making joint defaults of two or more firms more likely.

The threshold values $y^*_{i,t}$ typically follow from the availability of estimates for the firm's marginal default probabilities $p_{i,t}$. The probability of default $p_{i,t}$ of firm i at time t is given by

$$p_{i,t} = \Pr[y_{i,t} < y^*_{i,t}] = F_{i,t}(y^*_{i,t}) \quad \Leftrightarrow \quad y^*_{i,t} = F^{-1}_{i,t}(p_{i,t}), \tag{7}$$

where $F_{i,t}(\cdot)$ is the cumulative GHST distribution function of $y_{i,t}$. Clearly, our main interest is not in the marginal default probability $p_{i,t}$, which we have available as input data, but rather in joint default probabilities such as $\Pr[y_{i,t} < y^*_{i,t}\ ,\ y_{jt} < y^*_{jt}]$ and conditional default probabilities such as $\Pr[y_{i,t} < y^*_{i,t} \mid y_{jt} < y_{jt}]^*$, for $i \neq j$.

In this paper we use Expected Default Frequency (EDF) estimates obtained from Moody's Analytics (formerly Moody's KMV) for $p_{i,t}$ to determine $y^*_{i,t}$. An alternative would be to use CDS implied default probabilities. The drawback of the latter, however, is that they are generally larger than the true default probabilities due to the inclusion of a default risk premium component. The use of EDF estimates, therefore, seems more appropriate for our current objectives.

The copula model (6) is a multi-factor credit risk model. To demonstrate this, we rewrite (6) as

$$\begin{aligned} y_{i,t} &= (\varsigma_t - \mu_\varsigma)\gamma_i + \sqrt{\varsigma_t} z_{i,t}, \qquad i = 1, \ldots, N \\ z_{i,t} &= \eta_{i,t}\kappa_t + \Lambda_t \epsilon_{i,t}, , \end{aligned} \tag{8}$$

where γ_i is the ith row of γ, common risk factors are $\kappa_t \sim \mathrm{N}(0,1)$ and $\varsigma_t \sim \mathrm{IG}(\nu/2, \nu/2)$, and idiosyncratic risks $\epsilon_t \sim \mathrm{N}(0, \mathrm{I}_N)$ are serially and mutually independent. The vector $\eta_t = (\eta_{1,t}, \cdots, \eta_{N,t})'$ contains risk factor loadings, or sensitivity parameters, while the diagonal matrix Λ_t scales the idiosyncratic risks. This is a two-factor model with a common Gaussian factor κ_t and a mixing factor ς_t. Section 2.3 demonstrates that factor model coefficients η_t and Λ_t can be expressed in terms of reduced form parameters ρ_t, γ, and ν, which in turn are estimated based on the GHST-DECO model in Section 3.

2.3 Joint risk measures and conditional law of large numbers

This section defines our unconditional and conditional tail risk measures and demonstrates how they can be computed easily and quickly (within seconds). Section 3 demonstrates the the LLN approximation is reliable even in low-dimensional cases such as $N = 10$. While the present discussion is framed in terms of evaluating financial sector risk, the applicability of the presented techniques to general credit portfolios should be entirely clear.

The time-varying probability of at least a certain number of financial sector firms defaulting over the 12 months is a natural measure of financial system risk. Such a measure is for example constructed and tracked in the European Central Bank's biannual Financial Stability Report, see for example ECB (2010). We here use the same definition of a financial system risk measure. After estimating the conditional covariance matrix through the dynamic-GHST copula model, the time-varying copula is used to calculate the probability of default of a part of the financial system of financial institutions.

A direct method to compute joint default probabilities is based on Monte Carlo simulations of firm asset values. As discussed in Section 2, a firm defaults if its asset value falls below a pre-specified threshold, which in turn has been obtained from Moody's Analytics' EDF estimates. In the multivariate setting, these thresholds jointly determine a systemic distress region. In a simulation setting, joint default probabilities are estimated by counting the fraction of simulations from the multivariate dynamic GHST copula that lies inside the systemic distress region. While simulating the risk measures is conceptually easy, the simulation based systemic risk measurements become inefficient as the dimension of the dataset becomes large. As marginal default probabilities are typically small, we need a large number of simulations to obtain realizations of joint defaults, particularly if 3, 4, or more joint defaults are considered.

We therefore explore the equicorrelation structure to obtain an alternative joint risk measurement approach. This alternative approach is based on an analytic approximation and can easily be applied in a large system setting. To obtain our approximation, consider the system of banks as a portfolio. We define a joint tail risk measure (TRM) as the time-varying probability that a certain fraction of banks defaults over a pre-specified period. To compute this probability, we use a conditional Law of Large Number (CLLN) result that is typically applied in the credit risk context; see for example Lucas, Klaassen, Spreij, and Straetmans (2001). Let $c_{N,t}$ denote the fraction of defaults at time t,

$$c_{N,t} = \frac{1}{N} \sum_{i=1}^{N} 1\{y_{i,t} < y_{i,t}^{*}\}, \tag{9}$$

where $y_{i,t}^{*}$ is the default threshold.

For the factor model (8), which we here restate for ease of reference,

$$y_t = (\varsigma_t - \mu_\varsigma)\gamma + \sqrt{\varsigma_t} z_t, \qquad z_t = \eta_t \kappa_t + \Lambda_t \epsilon_t,$$

with $\kappa_t \sim \mathrm{N}(0,1)$ and $\epsilon_t \sim \mathrm{N}(0, \mathrm{I}_N)$, the parameters η_t and Λ_t should be such that

$$\mathrm{Var}(y_t) = \mu_\varsigma \Lambda_t^2 + \mu_\varsigma \eta_t \eta_t' + \sigma_\varsigma^2 \gamma\gamma' = (1-\rho_t^2)\mathrm{I} + \rho_t^2 \ell\ell' = R_t. \tag{10}$$

Solving equation (10), we get

$$\eta_{i,t} = (\rho_t - \sigma_\varsigma^2 \gamma_i^2)^{1/2}/\mu_\varsigma^{1/2}, \qquad \lambda_{i,t} = (1-\rho_t^2)^{1/2}/\mu_\varsigma^{1/2}. \tag{11}$$

This allows to infer the factor model coefficients from the reduced form estimates later on.

As the indicators $1\{y_{i,t} < y_{i,t}^*\}$s are conditionally independent given κ_t and ς_t, we can apply a conditional law of large numbers to obtain

$$c_{N,t} \approx \frac{1}{N}\sum_{i=1}^N \mathrm{E}(1\{y_{i,t} < y_{i,t}^*\} \mid \kappa_t, \varsigma_t) = \frac{1}{N}\sum_{i=1}^N \mathrm{P}[y_{i,t} < y_{i,t}^* \mid \kappa_t, \varsigma_t] := C_{N,t}, \tag{12}$$

for large N. Note that

$$\mathrm{P}[y_{i,t} < y_{i,t}^* | \kappa_t, \varsigma_t] = \Phi\left(\frac{(y_{i,t}^* + \mu_\varsigma \gamma_i - \varsigma_t \gamma_i)/\sqrt{\varsigma_t} - \eta_{i,t}\kappa_t}{\lambda_{i,t}}\right). \tag{13}$$

where $\Phi(\cdot)$ denotes the cumulative standard normal distribution. Also note that $C_{N,t}$ is a function of the random variables κ_t and ς_t only, and not of $\epsilon_{i,t}$ or $y_{i,t}$.

Given these results, we define a joint tail risk measure (TRM) as

$$p_t = \mathrm{P}(C_{N,t} > \bar{c}_{p,t}) = \mathrm{P}(C_{N,t}(\kappa_t, \varsigma_t) > \bar{c}), \tag{14}$$

i.e., the probability that the fraction of credit portfolio defaults exceeds a given fraction $\bar{c} \in [0,1]$. Whereas $C_{N,t}$ is a complex function of κ_t and ς_t, the function is much more regular when seen as a function of κ_t only for given ς_t. In particular, it is monotonically decreasing in κ_t in that case. We use this to efficiently compute unique threshold levels $\kappa^*_{t,N}(\bar{c},\varsigma)$ for each value of ς by solving $C_{N,t}(\kappa^*_{t,N}(\bar{c},\varsigma),\varsigma) = \bar{c}$ for a fixed point. Given these threshold values, we can efficiently compute the joint default probability as

$$p_t = \mathrm{P}(C_{N,t} > \bar{c}) = \int \mathrm{P}(\kappa_t < \kappa^*_{t,N}(\bar{c},\varsigma_t))p(\varsigma_t)\mathrm{d}\varsigma_t. \tag{15}$$

We use a standard one-dimensional numerical integration routine for this purpose. Integrating out the tail risk factor over the positive real line is a matter of fractions of a second.

A related conditional tail risk measure is the probability that the fraction of defaults in the system exceeds a certain value $\bar{c}^{(-i)}$ given that firm i defaults, i.e., given that $y_{i,t} < y^*_{i,t}$. The fact that firm i defaults holds some information on the common factors κ_t and ς_t. How much information precisely depends on ρ_t and the other parameters in the model. Define $C^{(-i)}_{N-1,t}$ as the limiting expression for the fraction of portfolio defaults abstracting from firm i. Also recall that $p_{i,t}$ is the default probability of firm i at time t as obtained from the Moody's Analytics estimates. We define a firm-specific systemic influence measure ($\mathrm{SIM}_{i,t}$) as the conditional probability

$$\begin{aligned}
& \mathrm{P}(C^{(-i)}_{N-1,t} > \bar{c}^{(-i)} | y_{i,t} < y^*_{i,t}) \\
= \; & p^{-1}_{i,t} \cdot \mathrm{P}(C^{(-i)}_{N-1,t} > \bar{c}^{(-i)} \;,\; y_{i,t} < y^*_{i,t}) \\
= \; & p^{-1}_{i,t} \cdot \int \mathrm{P}(\kappa_t < \kappa^*_{N-1,t}(\bar{c}^{(-i)},\varsigma_t) \;,\; y_{i,t} < y^*_{i,t} | \varsigma_t) p(\varsigma_t)\mathrm{d}\varsigma_t \\
= \; & p^{-1}_{i,t} \cdot \int \Phi_2(\kappa^*_{N-1,t}(\bar{c}^{(-i)},\varsigma_t) \;,\; z^*_{i,t}(y^*_{i,t},\varsigma_t) \;;\; \eta_{i,t}(\eta^2_{i,t}+\lambda^2_{i,t})^{-1/2}) p(\varsigma_t)\mathrm{d}\varsigma_t,
\end{aligned} \tag{16}$$

where $z^*_{i,t}(y^*_{i,t},\varsigma_t) = (y^*_{i,t} - (\varsigma_t - \mu_\varsigma)\gamma_i)/(\varsigma_t \cdot (\eta^2_{i,t}+\lambda^2_{i,t}))^{1/2}$, and $\Phi_2(\cdot,\cdot;\eta_{i,t}(\eta^2_{i,t}+\lambda^2_{i,t})^{-1/2})$ is the cumulative distribution function of the bivariate normal with standard normal marginals and correlation parameter $\eta_{i,t}(\eta^2_{i,t}+\lambda^2_{i,t})^{-1/2}$. To obtain the last equality in (16), note that the non-Gaussian probability becomes Gaussian conditional on ς_t, and that $z^*_{i,t}$ is a standardized argument, with $\mathrm{Var}[z_{i,t}] = \eta^2_{i,t}+\lambda^2_{i,t}$ in the denominator. Finally, the conditional probability (16) is close to the Multivariate extreme spillovers indicator of Hartmann, Straetmans, and de Vries (2005).

Finally, we may average over the firm-specific conditional probabilities to obtain an empirical 'connectedness' measure $\frac{1}{N}\sum_{i=1}^{N}\text{SIM}_{i,t}$. The intuition is that a default should move the tail risk of the remaining financial system more if firms are more connected in terms of business links or fire sale externalities. The average measure captures the time-varying probability that an individual credit event increases the level of financial system tail risk. We infer both financial sector risk measures developed this section from a panel of 73 European financial firms in Section 4.

3 GHST dynamic copula model and parameter estimation

The joint and conditional risk measures defined in Section 2.3 require parameter estimates for γ, ν and ρ. This section explains how these can be obtained by fitting a GHST dynamic copula model to a panel of equity returns.

3.1 The dynamic generalized hyperbolic skewed t model

We approximate the asset returns $y_{i,t}$ by equity returns and assume that equity returns $y_t = (y_{1,t}, \cdots, y_{N,t})'$ follow a multivariate dynamic Generalized Hyperbolic skewed-t (GHST) distribution, as a result of the mixture (6)

$$p(y_t; \tilde{\Sigma}_t, \gamma, \nu) = \frac{\nu^{\frac{\nu}{2}} 2^{1-\frac{\nu+N}{2}}}{\Gamma(\frac{\nu}{2})\pi^{\frac{N}{2}} |\tilde{\Sigma}_t|^{\frac{1}{2}}} \cdot \frac{K_{\frac{\nu+N}{2}}\left(\sqrt{d(y_t)\cdot d(\gamma)}\right) e^{\gamma'\tilde{\Sigma}_t^{-1}(y_t-\tilde{\mu})}}{d(y_t)^{\frac{\nu+N}{4}} \cdot d(\gamma)^{-\frac{\nu+N}{4}}}, \tag{17}$$

$$d(y_t) = \nu + (y_t - \tilde{\mu})'\tilde{\Sigma}_t^{-1}(y_t - \tilde{\mu}), \tag{18}$$

$$d(\gamma) = \gamma'\tilde{\Sigma}_t^{-1}\gamma, \quad \tilde{\mu} = -\frac{\nu}{\nu-2}\gamma. \tag{19}$$

where $K_a(b)$ is the modified Bessel function of the second kind, $\tilde{\Sigma}_t$ is the scale matrix, see Bibby and Sørensen (2003), Σ_t is the covariance matrix, $\gamma \in \mathbb{R}^N$ is a skewness parameter, and $\nu \in \mathbb{R}$ is a kurtosis parameters. It is useful to note here that if y_t has a multivariate

GHST distribution with parameters $\tilde{\mu}$, $\tilde{\Sigma}$, γ, and ν as given in (17), then $Ay_t + b$ also has a GHST distribution with parameters $A\tilde{\mu} + b$, $A\tilde{\Sigma}A'$, γ, and ν, for some matrix A and vector b. In particular, the marginal distributions of a GHST distribution are also GHST.

Though the expression for the density in (17) looks complicated at first sight, our empirical modeling methodology follows the standard recognizable steps. In particular, we parameterize the time-varying covariance matrix $\tilde{\Sigma}_t$ of y_t in the standard way as

$$\tilde{\Sigma}_t = D_t \tilde{R}_t D_t, \tag{20}$$

where D_t is a diagonal matrix holding the volatilities of $y_{i,t}$, and $\tilde{R}_t$ corresponds to the correlation matrix of equity returns y_t. Similar to Engle (2002), we assume that the dynamic covariance matrix Σ_t depends on the unobserved factor f_t via the parameterization of the matrix $\tilde{R}_t = \tilde{R}(f_t)$, and (diagonal) matrix of standard deviations $D_t = D(f_t)$.

Before discussing the parameterization $D(f_t)$ and $\tilde{R}(f_t)$ in Sections 3.2 and 3.3, we introduce the dynamic behavior of the unobserved factor f_t. To accommodate for the fat-tailed nature of the GHST density, we follow Creal, Koopman and Lucas (2013) and Zhang, Creal, Koopman, and Lucas (2011) and assume that the factor f_t follows the Generalized Autoregressive Score (GAS) process. Such score-driven dynamics in a context of fat-tailed observations are known to improve the stability of volatility and correlation estimates. The transition equation for f_{t+1} is given by

$$f_{t+1} = \tilde{\omega} + \sum_{i=0}^{p-1} A_i s_{t-i} + \sum_{j=0}^{q-1} B_j f_{t-j}, \tag{21}$$

$$s_t = \mathcal{S}_t \nabla_t, \quad \nabla_t = \partial \ln p(y_t | \mathcal{F}_{t-1}; f_t, \theta) / \partial f_t, \tag{22}$$

where $\tilde{\omega} = \tilde{\omega}(\theta)$ is a vector of fixed intercepts, $A_i = A_i(\theta)$ and $B_j = B_i(\theta)$ are fixed parameter matrices, and θ denotes the vector of all time invariant parameters in the model.

Result 1. *Let y_t follow a zero mean GHST distribution $p(y_t; \tilde{\Sigma}_t, \gamma, \nu)$, where the time-varying* *covariance matrix is driven by the GAS model* (21)-(22). *Then the dynamic score is*

$$\nabla_t = \Psi_t' H_t' \mathrm{vec}\Big(w_t \cdot (y_t - \tilde{\mu})(y_t - \tilde{\mu})' - w_t^{\gamma} \cdot \gamma\gamma' - \gamma(y_t - \tilde{\mu})' - 0.5\tilde{\Sigma}_t \Big) \tag{23}$$

$$w_t = \frac{\nu + N}{4d(y_t)} - \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(y_t)/d(\gamma)}}, \tag{24}$$

$$w_t^{\gamma} = \frac{\nu + N}{4d(\gamma)} + \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(\gamma)/d(y_t)}}, \tag{25}$$

$$H_t = \tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1}, \qquad \Psi_t = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial f_t}, \tag{26}$$

where we define $k_\nu(\cdot) = \ln K_\nu(\cdot)$ *with first derivative* $k'_\nu(\cdot)$. *The matrices* Ψ_t *and* H_t *are time-varying, parameterization specific, and depend on* f_t, *but not on the data* y_t.

Equation (22) reveals the key feature of the GAS dynamic specification. In essence, the score-driven mechanism takes a Gauss-Newton improvement step for the covariance matrix to better fit the most recent observation. As can be seen in (22), the volatilities and correlations react to differences between the squared observations $y_t y_t'$ and the recent scale matrix estimate $\tilde{\Sigma}_t$. The reaction is asymmetric if $\gamma \neq 0$, in which case there is also a reaction to the level y_t itself (non-squared). The reaction to $y_t y_t'$ is modified by the weight w_t. If the distribution is fat tailed, this weight is decreasing in the Mahalanobis distance $d(y_t)$. This meas that if y_t lies far from 0, volatilities are increased more moderately than in a multivariate GARCH framework. This makes intuitive sense. If y_t is fat-tailed, we expect to see large values of y_t from time to time even without major changes in the covariance matrix of y_t. The key features of (22) are thus easily understood. The remaining (tedious) expressions for H_t and Ψ_t only serve to transform the dynamics of the covariance matrix in (20) into the dynamics of the unobserved factor f_t.

Scaling the score in (22) is often important. Following Zhang, Creal, Koopman, and Lucas (2011), we set the scaling matrix S_t equal to the inverse Fisher information matrix from the Student's t distribution,

$$\mathcal{S}_t = \left\{ \Psi_t'(\tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1})'[gG - \mathrm{vec}(\mathrm{I})\mathrm{vec}(\mathrm{I})'](\tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1})\Psi_t \right\}^{-1}, \tag{27}$$

where $g = (\nu + N)/(\nu + 2 + N)$, and $G = \mathrm{E}[x_t x_t' \otimes x_t x_t']$ for $x_t \sim \mathrm{N}(0, \mathrm{I}_N)$. Zhang, Creal, Koopman, and Lucas (2011) demonstrate that this results in a stable model that outperforms alternative models if the data are fat-tailed and skewed.

Estimation and inference can now be carried out in a straightforward way using Maximum Likelihood inference. As the dimension of the parameter space is extremely large, however, we split the estimation problem in smaller parts by adopting a copula perspective. The copula perspective has the additional advantage that more flexibility can be incorporated in modeling the marginal distributions. For example, when working with the multivariate GHST distributions, all marginal distributions must have the same kurtosis parameter ν. By adopting a copula approach, this can be relaxed. The two stages of the modeling and estimation process can be summarized as follows.

1. We estimate a univariate dynamic GHST model for each equity return series using maximum likelihood and the parameterization in Section 3.2. We then transform the observations into their probability integral transforms $u_{i,t} \in [0, 1]$ using the estimated univariate GHST density. The parameters of the univariate GHST distribution are $\tilde{\mu}_{i,t}$, $\tilde{\sigma}_{i,t}$, γ_i, and ν_i.

2. We estimate the matrix $\tilde{R}_t$ parameterized as described in Section 3.3 using the probability integral transforms $u_{i,t}$ constructed in the first step. The correlation matrix is fitted using maximum likelihood for a multivariate GHST copula density. The parameters are 0, $\tilde{R}_t$, $\gamma \cdot (1, \ldots, 1)'$ for $\gamma \in \mathbb{R}$, and ν.

The number of parameters we need to estimate in each step is now substantially reduced. Together with the DECO specification for the correlation matrix, this makes the model feasible for high dimensions.

3.2 The volatility model D_t with leverage

In the univariate modeling stage, we set $f_t = \log D_{i,t}$, where $D_{i,t}$ is the ith diagonal element of D_t. By modeling the logarithm of the variance, the model behaves in a more stable way and the asymptotic validity of maximum likelihood inference can be formally proven, see Harvey (2013). We also prefer to write the univariate model with the density:

$$
\begin{aligned}
p(y_t; \tilde{\sigma}_t^2, \gamma, \nu) &= \frac{\nu^{\frac{\nu}{2}} 2^{1-\frac{\nu+1}{2}}}{\Gamma(\frac{\nu}{2}) \pi^{\frac{1}{2}} \tilde{\sigma}_t} \cdot \frac{K_{\frac{\nu+1}{2}}\left(\sqrt{d(y_t)(\gamma^2)}\right) e^{\gamma(y_t - \tilde{\mu}_t)/\tilde{\sigma}_t}}{d(y_t)^{\frac{\nu+1}{4}} \cdot (\gamma^2)^{-\frac{\nu+1}{4}}}, && (28) \\
d(y_t) &= \nu + (y_t - \tilde{\mu}_t)^2 / \tilde{\sigma}_t^2, && (29) \\
\tilde{\mu}_t &= -\frac{\nu}{\nu-2} \tilde{\sigma}_t \gamma, \quad \tilde{\sigma}_t = \sigma_t T, && (30) \\
T &= \left(\frac{\nu}{\nu-2} + \frac{2\nu^2\gamma^2}{(\nu-2)^2(\nu-4)} \right)^{-1/2}. && (31)
\end{aligned}
$$

An important empirical feature that we have not addressed so far is the presence of leverage effects in volatility: volatilities may increase more when stock prices fall than when they increase, see for example Black (1976). To allow for the empirically documented leverage effect in the marginal distributions, we therefore add a new feature to the GAS framework, namely a score-driven volatility model with leverage. The modified transition equation for f_t takes the form

$$
\begin{aligned}
f_{t+1} &= \tilde{\omega} + \sum_{i=0}^{p-1} A_i s_{t-i} + \sum_{j=0}^{q-1} B_j f_{t-j} + C(s_t - s_t^{\mu}) 1\{y_t < \mu_t\}, && (32) \\
s_t &= \mathcal{S}_t \nabla_t, \quad \nabla_t = \partial \ln p(y_t | \mathcal{F}_{t-1}; f_t, \theta) / \partial f_t, && (33) \\
s_t^{\mu} &= \mathcal{S}_t \nabla_t^{\mu}, \quad \nabla_t^{\mu} = \partial \ln p(\mu_t | \mathcal{F}_{t-1}; f_t, \theta) / \partial f_t, && (34)
\end{aligned}
$$

where $1\{y_t < \mu_t\}$ is an indicator function for the event $\{y_t < \mu_t\}$. The recentering term s_t^{μ} makes the news impact curve of f_{t+1} as a function of y_t continuous. The composite effect is entirely similar to the leverage effect in the GJR-GARCH model of Glosten, Jagannathan, and Runkle (1993), except for the fact that s_t rather than y_t^2 drives the volatility dynamics. This leverage effect in the score-driven model is a novelty in the current paper. Now we introduce the GAS factor f_t to the time varying volatility σ_t.

Result 2. *If we assume y follows the GH skewed-t density* (A1) *where the scale* $\sigma_t = \sigma_t(f_t)$, *the score driven factor f_t includes the leverage effect. The dynamic score is*

$$\nabla_t = \Psi_t H_t \left(w_t \cdot y_t^2 - \tilde{\sigma}_t^2 - \left(1 - \tfrac{\nu}{\nu-2} w_t\right) \tilde{\sigma}_t \gamma y_t \right), \tag{35}$$

$$w_t = \frac{\nu+1}{2d(y_t)} - \frac{k'_{0.5(\nu+1)}\left(\sqrt{d(y_t)\cdot(\gamma^2)}\right)}{\sqrt{d(y_t)/(\gamma^2)}}, \tag{36}$$

$$\Psi_t = \frac{\partial \sigma_t}{\partial f_t}, \quad H_t = T\tilde{\sigma}^{-3}. \tag{37}$$

The score in (35) already accounts for an asymmetric response of volatility to the level of $y_{i,t}$. This, however, stems from the possibly asymmetric nature of the GHST distribution. If the distribution is symmetric ($\gamma_i = 0$), this asymmetric volatility response disappears. In fact, the asymmetry term in (35) captures a different phenomenon: if a distribution is for example left-skewed, an extreme observation in the left tail gives less rise to an increase in volatility. The intuitive reason is that such observations are more likely, and that therefore observing one of them should not lead one to conclude that volatility must have increased.

After we obtain the volatility series, we can transform the observations $y_{i,t}$ into their probability integral transforms $u_{i,t} \in [0,1]$ using the estimated univariate GHST density. The copula model will take these filtered returns as inputs. As a result, $\tilde{\Sigma}_t$ is identical to $\tilde{R}_t$ and we ignore the volatility term $D_t = \mathrm{I}_N$ in the multivariate analysis.

3.3 The block equicorrelation model

The focus in our current paper is on systemic risk measurement. This means that the individual marginal models as well as all individual correlations are of moderate concern, and that the interest lies in capturing joint clustering effects. Therefore, rather than modeling all 2628 correlations separately, we impose an equicorrelation structure on the correlation matrix $\tilde{R}_t$ similar as in Engle and Kelly (2012). We have

$$\tilde{\Sigma}_t = (1-\rho_t^2)\mathrm{I}_N + \rho_t^2 \ell_N \ell_N', \tag{38}$$

where $\rho_t \in (0,1)$, and ℓ is a $N \times 1$ vector of 1's. The score-driven transition equation for the dynamic correlation now simplifies considerably.

Result 3. *If we assume y follows the GH skewed-t density where the scale matrix* $\tilde{\Sigma}_t = (1-\rho_t^2)\mathrm{I}_N + \rho_t^2 \ell_N \ell_N'$ *and* $\rho_t = (1+\exp(-f_t))^{-1}$, $\tilde{R}_t$ *is a function of the score driven factor* f_t

$$\Psi_t = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial f_t} = (\ell_{N^2} - \mathrm{vec}(\mathrm{I}_N))\,\frac{2\exp(-f_t)}{(1+\exp(-f_t))^3}. \tag{39}$$

With the equations (23)–(37)*, the GAS factor updating system is defined.*

For practical purpose, sometimes it is reasonable to relax the restriction of one single factor. We can introduce a two-block GAS-Equicorrelation model as an extension and a general case. Define $\tilde{\Sigma}$ with a block structure such that

$$\tilde{\Sigma} = \begin{bmatrix} (1-\rho_{1,t}^2)\mathrm{I}_1 & 0 \\ 0 & (1-\rho_{2,t}^2)\mathrm{I}_2 \end{bmatrix} + \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \end{pmatrix} \cdot \begin{pmatrix} \rho_{1,t}\ell_1' & \rho_{2,t}\ell_2' \end{pmatrix}, \tag{40}$$

We are also able to provide the GAS result here.

Result 4. *If we assume y follows the GH skewed-t density where the covariance matrix* $\tilde{\Sigma}_t = \tilde{R}$ *contains* 2×2 *blocks with* $\rho_1 = (1+\exp(-f_{1,t}))^{-1}$ *and* $\rho_2 = (1+\exp(-f_{2,t}))^{-1}$, f_t *is a* 2×1 *vector driven by the dynamic score model. For the system* (23)–(37)*, we have*

$$\tilde{\Psi}_t = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial f_t} = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial \rho_t}\frac{d\rho_t'}{df_t}, \tag{41}$$

$$\frac{d\rho_t'}{df_t} = \begin{pmatrix} \frac{\exp(-f_{1,t})}{(1+\exp(-f_{1,t}))^2} & 0 \\ 0 & \frac{\exp(-f_{2,t})}{(1+\exp(-f_{2,t}))^2} \end{pmatrix}, \tag{42}$$

$$\begin{aligned} \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial \rho_t} &= \left(\mathrm{vec}\begin{pmatrix} \mathrm{I}_1 & 0 \\ 0 & 0 \end{pmatrix}, \mathrm{vec}\begin{pmatrix} 0 & 0 \\ 0 & \mathrm{I}_2 \end{pmatrix} \right) \cdot \begin{pmatrix} -2\rho_{1,t} & 0 \\ 0 & -2\rho_{2,t} \end{pmatrix} \\ &+ \left(\begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \end{pmatrix} \otimes \mathrm{I}_N + \mathrm{I}_N \otimes \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \end{pmatrix} \right) \cdot \left(\begin{pmatrix} \ell_1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \ell_2 \end{pmatrix} \right). \end{aligned} \tag{43}$$

This finishes the derivation of two block scale matrix modeling.

One direction worth exploring is to extend the support of ρ_t from [0,1) to (-1,1) by defining $\rho_1 = (\exp(f_{1,t}) - 1)/(\exp(f_{1,t}) + 1)$ and $\rho_2 = (\exp(f_{2,t}) - 1)/(\exp(f_{2,t}) + 1)$. For the application we have in mind, it's not necessary to study the negative correlation at the moment. Further we extend the block scale matrix to allow m multiple groups.

Assume that N firms fall into m different groups according to their exposure to a common systemic risk factor. Firms have equicorrelation ρ_i^2 within each group and $\rho_i \cdot \rho_j$ between groups i and j. So we have $N = n_1 + n_2 + \cdots + n_m$ random variables that follow a GH distribution with a correlation matrix that has a block equicorrelation structure, where n_i denotes the number of firms in group i. Similarly, we can restrict the skewness parameter to be the same within each block. The scale matrix at time t is given by

$$\tilde{\Sigma}_t = \begin{bmatrix} (1-\rho_{1,t}^2)\mathrm{I}_1 & \cdots & \cdots & 0 \\ 0 & (1-\rho_{2,t}^2)\mathrm{I}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (1-\rho_{m,t}^2)\mathrm{I}_m \end{bmatrix} + \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \cdot \begin{pmatrix} \rho_{1,t}\ell_1' & \rho_{2,t}\ell_2' & \cdots & \rho_{m,t}\ell_m' \end{pmatrix}, \tag{44}$$

where I_i is a $n_i \times n_i$ identity matrix, $\ell_i \in \mathbb{R}^{n_i \times 1}$ is a column vector of ones and $|\rho_{i,t}| < 1$. The block structured matrix allows us to obtain analytical solutions for the determinant of R_t. As a result of the Matrix Determinant Lemma (see Harville (2008)), the determinant of the matrix R_t is

$$\begin{aligned} \det(\tilde{\Sigma}_t) &= \det(\Xi_t + u_t u_t') = (1 + u_t'\Xi_t^{-1}u_t)\det(\Xi_t) \\ &= \left[1 + \frac{n_1\rho_{1,t}^2}{1-\rho_{1,t}^2} + \cdots + \frac{n_m\rho_{m,t}^2}{1-\rho_{m,t}^2}\right](1-\rho_{1,t}^2)^{n_1}\cdots(1-\rho_{m,t}^2)^{n_m}, \end{aligned} \tag{45}$$

with Ξ_t the diagonal matrix in the first term on the righthand side of (A22) and u_t the vector in the second term, such that $R_t = \Xi_t + u_t u_t'$. The determinant of matrix L_t is easy to find as the square root of this value. The analytic expressions facilitate the computation of the likelihood and GAS steps in high dimensions. The time-varying correlation coefficients $\rho_{1,t}, \cdots, \rho_{m,t}$ are driven by the GAS factors from a GHST distribution. We can derive the GAS model with these restrictions.

Result 5. *y_t follows a GHST distribution and the time-varying matrix $\tilde{\Sigma}_t$ has a block equicor-*

relation structure which contains $m \times m$ blocks as (A22)*. We introduce f_t a $m \times 1$ dynamic score driven vector such that $\rho_i = (1 + \exp(-f_{i,t}))^{-1}$ for $i = 1, 2, \cdots, m$. For the system* (23)–(37)*, we have*

$$\tilde{\Psi}_t = \frac{\partial \text{vec}(\tilde{\Sigma}_t)'}{\partial f_t} = \frac{\partial \text{vec}(\tilde{\Sigma}_t)'}{\partial \rho_t} \frac{d\rho_t'}{df_t}, \tag{46}$$

$$\frac{d\rho_t'}{df_t} = \begin{bmatrix} \frac{\exp(-f_{1,t})}{(1+\exp(-f_{1,t}))^2} & 0 & \cdots & 0 \\ 0 & \frac{\exp(-f_{2,t})}{(1+\exp(-f_{2,t}))^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\exp(-f_{m,t})}{(1+\exp(-f_{m,t}))^2} \end{bmatrix}, \tag{47}$$

$$\begin{aligned} \frac{\partial \text{vec}(\tilde{\Sigma}_t)'}{\partial \rho_t} = & \left[\text{vec} \begin{pmatrix} \text{I}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}, \cdots, \text{vec} \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \text{I}_m \end{pmatrix} \right] \cdot \begin{pmatrix} -2\rho_{1,t} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -2\rho_{2,t} \end{pmatrix} \\ & + \left[\begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \otimes \text{I}_N + \text{I}_N \otimes \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \right] \cdot \left[\begin{pmatrix} \ell_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \ell_{n_m} \end{pmatrix} \right]. \end{aligned} \tag{48}$$

Together with (45)*, we could analytically compute the score and update the time-varying coefficients.*

We refer to the appendix for more details. The block-equicorrelation model proposed here differs from Engle and Kelly (2012). We assume the equicorrelation in the off-diagonal blocks to be products of the correlation in the diagonal blocks. We impose this restriction to maintain the factor structure that allows analytical computations of risk measures. But it is surely possible to extend the block structure to a similar setting as Engle and Kelly (2012). For the exploration of the financial sector tail risk, it is a realistic assumption given the tight links among financial institutions.

4 Empirical results

In this section, we compute systemic risk measures in the Euro Area. We include 73 major financial groups from 11 EA countries: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands, and Portugal. Our data are demeand equity log returns taken from Bloomberg. The sample covers a period of 172 months from January 1999 to April 2013, but with missing observations of several firms in the beginning. The sample thus includes the period of the Global Financial crisis and the European Debt crisis. The EDF data used to compute the distress thresholds are provided by Moody's KMV. Dealing with missing values is straightforward in a GAS framework. Both the likelihood and the score steps in the dynamic GHST model adapt automatically if data are not observed at particular times and if there are no sample selection issues.

We perform two analyses. In our first analysis, we select ten major European banks. This small cross sectional dimension allows us to compare the dynamic GHST model with the block GAS Equicorrelation models by estimating both models in parallel for the same data set. The results are presented in Section 4.1. In our second analysis, we impose the GAS Equicorrelation structure in the dynamic GHST model for the complete cross section of 73 financial institutions. The conditional Law of Large Numbers approximation is implemented to compute the Banking Stability Measure and the Systemic Risk Measure. Section 4.2 presents the results.

4.1 Major banks in the Euro area

In the first analysis, we select a geographically diversified sub-sample of the largest banks in 10 different Euro Area countries: Bank of Ireland, Banco Comercial Portugues, Santander, UniCredito, National Bank of Greece, BNP Paribas, Deutsche Bank, Dexia, Eerste Group Bank, and ING. This subsample contains no missing observations. From the descriptive statistics in Table 1 we see that all equity returns are skewed and fat-tailed. Dexia and ING Group stand out with a pronounced skewness of -1.746 and -1.284, and a kurtosis of 11.302 and 9.964, respectively. We also find some large kurtosis coefficients, such as 12.688

for the Bank of Ireland. We model the equity returns from all 10 banks with our skewed and heavy-tailed dynamic GHST model.

We consider four models in this section: the GAS-GHST copula model with full correlation matrix and homogenous skewness, the GAS equicorrelation model with heterogenous skewness, the GAS equicorrelation model with homogenous skewness, and the two-Block GAS-Equicorrelation model. All these models separate the volatility estimation and correlation estimation steps. The dynamics in the covariance matrix are all introduced by the scaled autoregressive scores, but the correlation matrix structure differs. As we choose the copula approach to separate the estimation of volatility and correlation, the correlation models will share the same volatility model outlined in Section 3.2.

The first step estimation produces volatility estimates in univariate GAS-GHST models. The volatility estimates of the univariate series are shown in Figure 1. The underlying parameter estimates are presented in Table 2 in the Appendix. The estimated volatility series are plotted in separate panels in Figure 1. From the graph, we see three highly volatile periods corresponding to either financial crises or global economic recessions. The most recent period with clearly high volatility begins in Sept. 2008, when the failure of Lehman Brothers brought down stock prices of all banks. But the magnitude of this increase differs from one institution to the other. The most volatile time series is the Bank of Ireland's equity returns. In the midst of the Global Financial Crisis, the Irish Banking Crisis hits this largest Irish bank even harder. The Bank of Ireland was recapitalized by the Irish Government in February 2009 and further bailed-out by the ECB and IMF in 2010. The idiosyncratic shock to the Bank of Ireland, on top of the common shock from the Lehman Brother's bankruptcy, drives up its volatility even higher.

We filter the equity returns with the estimated volatilities and take them into the multivariate GHST copula model in the second step. We assume skewness parameters in the copula are uniform across different banks, to reduce the computation burden. The dynamic copula is driven by the GAS factors, sharing common scalar parameters A and B in the dynamics. The GAS correlation model contains forty-five pairwise correlation coefficient. Thus there are forty-five dynamic factors, but the correlation targeting pins down the factor

Table 1: Sample Descriptive Statistics.

The descriptive statistics for the monthly equity returns between January 1999 and April 2013. The sample mean values are all very close to zero. Statistical tests suggest that all excess kurtosis coefficients are significantly different from 0 at the 5% significance level. Except for the National Bank of Greece, all skewness coefficients are significantly different from zero at the 10% significance level.

	Mean	Std.Dev.	Skewness	Kurtosis	Minimum	Maximum
Bank of Ireland	0.000	0.212	-0.380	12.688	-1.119	1.079
Banco Comercial Portugues	0.000	0.102	-0.417	4.569	-0.340	0.346
Santander	0.000	0.092	-0.512	4.639	-0.298	0.300
UniCredito	0.000	0.105	-0.360	5.861	-0.428	0.375
National Bank of Greece	0.000	0.154	-0.075	4.993	-0.534	0.582
BNP Paribas	0.000	0.093	-0.507	5.291	-0.334	0.332
Deutsche Bank	0.000	0.113	-0.346	5.621	-0.463	0.457
Dexia	0.000	0.164	-1.746	11.302	-0.956	0.464
Eerste Group Bank	0.000	0.119	-0.602	8.467	-0.536	0.556
ING	0.000	0.134	-1.284	9.964	-0.727	0.459

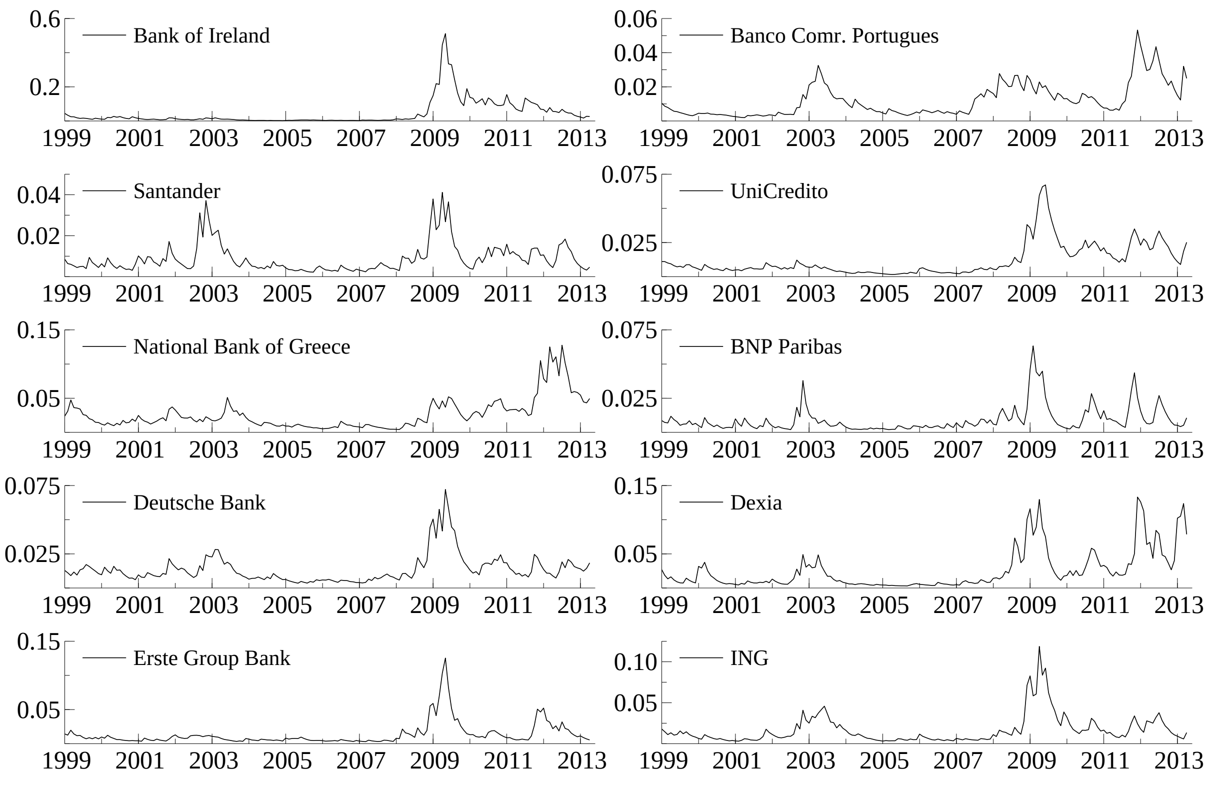

Figure 1: Volatility estimations for the banks' equities

The volatility estimates from the Dynamic GH Skewed-t model for banks' stock returns. The sample period is between January 1999 and April 2013.

mean at the sample average value. The time-varying factors are assumed to follow the GAS model. This model fit the filtered return series with a multivariate GHST distribution with joint degree of freedom and skewness parameter. We further restrict the correlation pattern to follow the GAS Equicorrelation model , and the GAS block Equicorrelation model. The models parameter estimates are reported in Table 2.

The dynamics of these correlations are different over time, but they share some commonality. For instance, all correlations go up during the financial crisis, especially after the failure of Lehman Brothers in September 2008. Figure 2 depicts the average correlation estimates of ten banks with the others. The correlations show a significant rise during the financial crisis 2008 and onwards. Figure 2 plots the average correlation patterns for each countries, but comparing the GAS correlation model with GAS copula model. The overall dynamics of correlation series are similar, but the correlation in copula model takes a more smooth path. As a comparison, we estimate the dynamic GHST model with the GAS-Equicorrelation model, and the two-Block GAS-Equicorrelation model on the same sample. The banks are separated into two groups. The first group contains the Bank of Ireland, Banco Comercial Portugues, Santander, UniCredito and the National Bank of Greece. The second group includes five banks in Euro Area core countries: BNP Paribas, Deutsche Bank, Dexia, Eerste Group Bank, ING. The correlation estimates are plotted in the bottom panels in Figure 2.

If we compare the Equicorrelation model correlation and the average correlation from the GAS model, the dynamic equicorrelation appears to be an average of the pairwise correlations. The flexible GAS-GHST model allows for more heterogenous dynamics on the pair-wise correlation coefficients. But we also see that the equicorrelation model picks up the most salient comovements in the data, such as the drop of correlation in 2001 and the increase after 2008 due to the financial crisis. In the model estimates from the two-block GAS-Equicorrelation matrix, we see that the three correlation estimates exhibit similar time-varying patterns as the equicorrelation dynamics. But we start to see differences in particular periods, for instance around the year 2008. The correlation of banks in the first group is lower than the banks in the second group during the crisis period. We provide the parameter

Table 2: Multivariate Model Estimates for Filtered Data: 10 Euro Area banks.

The parameters estimated in our multivariate GAS-GHST models for ten banks' equity returns. We use univariate GAS-GHST models for the marginal volatility. With the filtered returns, we estimate two dynamic correlation models: the GAS copula model with full correlation structure, and the GAS Equicorrelation model. Most parameters are significant at the 5% level.

	Dynamic Correlation							
	A	B	ω_1	ω_2	γ	Log-lik	AIC	BIC
GAS copula Model	0.016	0.836	0.966		-0.096	529.889	-1049.781	-1034.040
	(0.005)	(0.075)	(0.010)		(0.024)			
GAS Model	0.027	0.776	0.993			-1859.681	3725.362	3734.790
	(0.009)	(0.088)	(0.008)					
GAS EquiCorr (1)	0.121	0.932	0.673		-0.230	-1902.922	3813.845	3826.430
	(0.062)	(0.098)	(0.090)		(0.043)			
GAS EquiCorr (2)	0.060	0.988	0.791	0.890		-2458.319	4924.638	4937.228
	(0.037)	(0.013)	(0.228)	(0.252)				

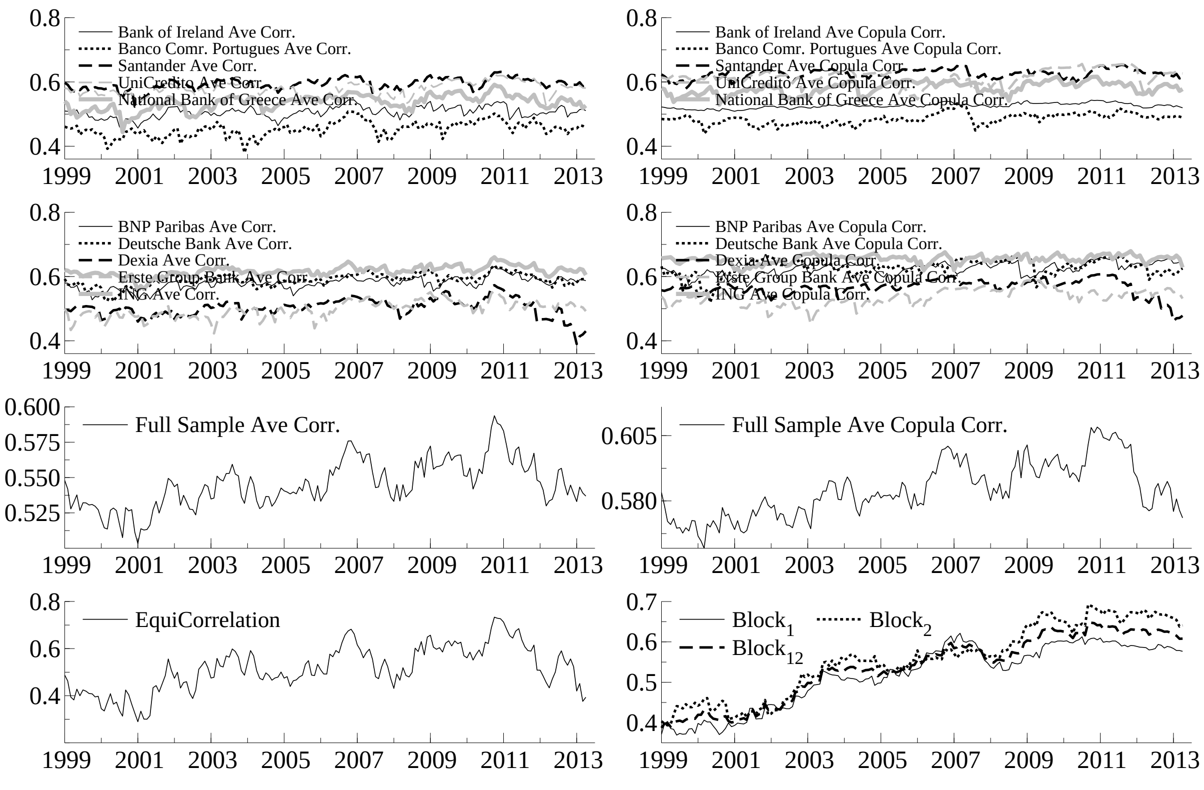

Figure 2: Average estimated correlation from four models.

The average correlation estimates from the GAS models using filtered equity returns of banks.

estimates and log-likelihood values from the dynamic correlation models in Table 2.

With the estimated GHST distributions, either with the full model, the copula model, or with the equicorrelations and block equicorrelations, we can compute the Banking Stability Measure (BSM) and Systemic Risk Measure (SRM) given the default thresholds from inverting the GHST CDF at the observed EDF levels. With the estimated multivariate GHST distributions, we can use simulations to compute the risk indicators. We use 10,000,000 simulations at each time t and count the number of banks under stress. As we obtain the simulations directly, we can compute the conditional and unconditional default probabilities. Alternatively if we use the GAS-Equicorrelation model, we can analytically calculate these measures under the LLN approximation suggested in Section 2.3. The analytical calculation is fast and less cumbersome than the simulation method.

From Figure 3, we see that the dynamic patterns of the risk indicators are very similar irrespective of the computation method used. The Banking Stability measures simulated/calculated from different correlation models are close to each other. The LLN approximated risk measure somewhat understates the risk in normal times and overestimates the risk in crisis times after the year 2008. This is because the number of banks is as small as 10 in our current setting, which makes the LLN approximation less accurate. Figure 4 plots the Systemic Risk Measure proposed in Section 2.3. The simulated (SIM) measure is computed with the straightforward simulation method and the correlation matrix is driven by the estimated GAS model in Result 1. The LLN approximated Systemic Risk Measure is calculated analytically based on the dynamic Equicorrelation estimates. We see the difference in the SRM between these two methods. The approximated SRM with the conditional Law of Large Numbers is always lower than the simulated SRM, but the pattern over time is similar. If we look at the average of the approximated indicator in the last panel, we see a break around the year 2002 in the mean for the analytical SRM. This may be attributed to the introduction of the Euro as a common currency, which tightened the interconnectedness of the European banks.

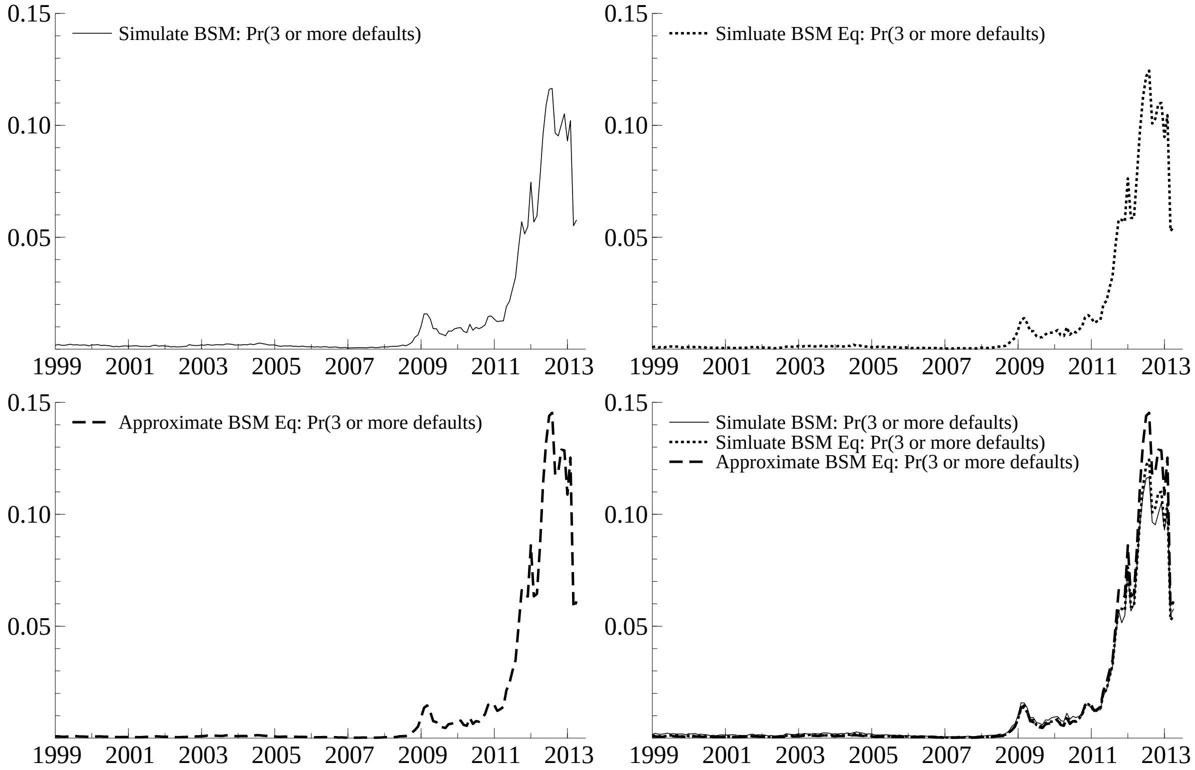

Figure 3: The Banking Stability Measures: a comparison

The Banking Stability Measure constructed from the Dynamic GHST models. A comparison study is provided here with two different correlation assumptions. The top left and bottom left panel contains the BSM with Dynamic Equicorrelation, but the top one is calculated with the analytical computation and the other one is simulated. The top right plot shows the simulated BSM with the full model correlation result. These measures are defined as the probability of three or more firm defaults.

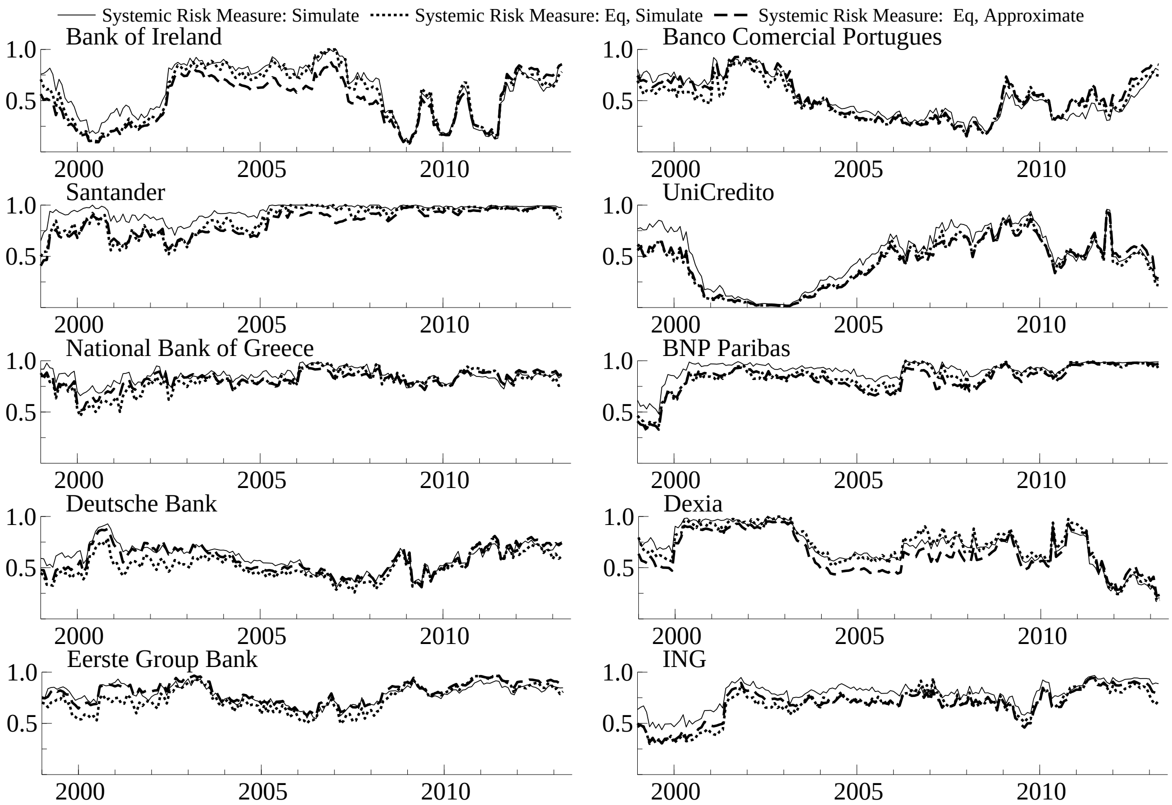

Figure 4: The Systemic Risk Measures: a comparison

The Systemic Risk Measure constructed from the Dynamic GHST models. We show the result for each bank on simulated SRM with correlation estimates from a Full GAS model, as well as the LLN approximated SRM from a Dynamic Equicorrelation model (Eq). SRM is defined as the probability of two or more firms defaulting given firm i failing.

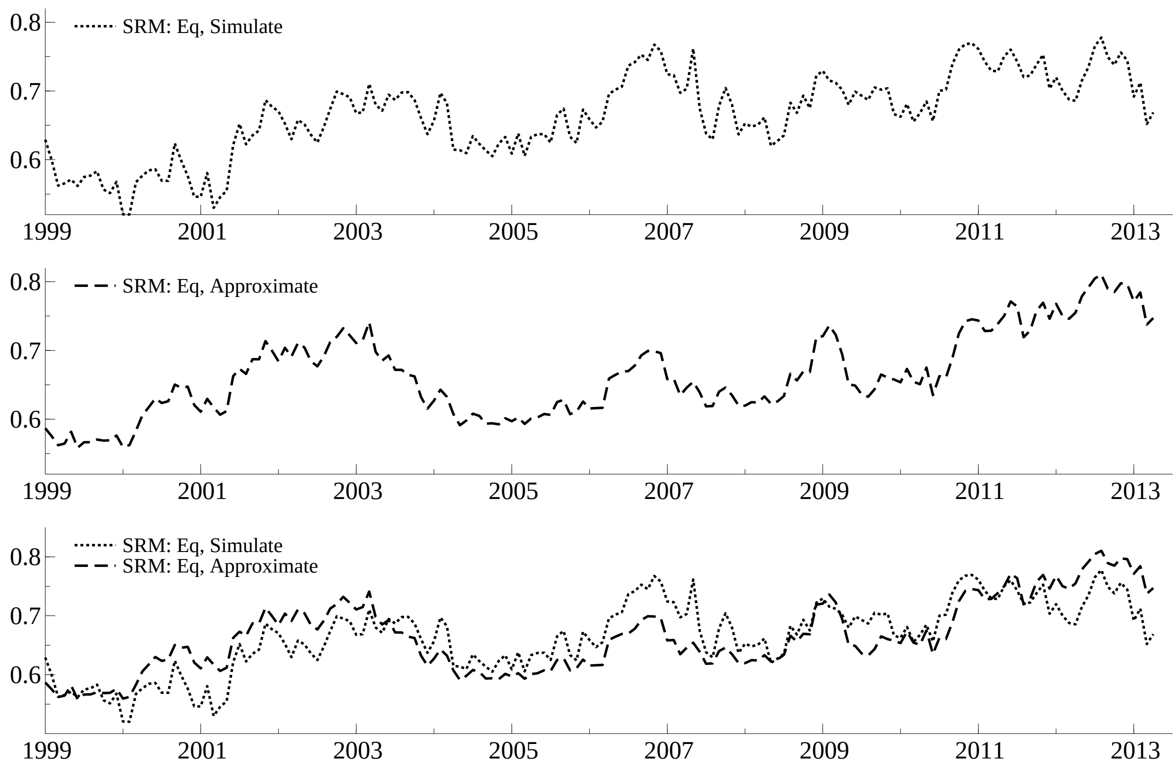

Figure 5: The Systemic Risk Measures

The Systemic Risk Measure constructed from the Dynamic GHST models. We show the result of simulated SRM as well as the LLN approximated SRM from a Dynamic Equicorrelation model (Eq). The final SRM is defined as the average of probability of two or more firms defaulting given one firm failing.

4.2 European large financial institutions

In this section we apply our new joint tail risk measures to financial sector firms located in the euro area. We include 73 major financial groups (banks, insurers, and investment companies) from 11 euro area countries: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands, and Portugal.

Regarding the data, we use demeaned log equity returns at a monthly frequency. These are obtained from Bloomberg. Monthly EDF data is used to compute the distress thresholds and are from Moody's Analytics. The sample covers a period of 172 months from January 1999 to April 2013. The sample thus includes the Global Financial Crisis and the euro area sovereign debt crisis. Not all data from each institution is observed at all times: our shortest time series has only 41 observations. Handling missing data entries is straightforward in our score-driven framework: both the likelihood contribution and the score steps in the dynamic GHST model are zero when data are not observed at a particular time. This means that each firm starts to contribute to the equicorrelation estimate once it enters the sample.

The dynamic correlation is plotted in Figure 6. The estimate hovers slightly below 0.50, with a clear episode of reduced correlations in the early 2000s, and increased correlations during the sovereign debt crisis. As a comparison, we also plot a rolling windows estimate of the correlation based on a window of 12 months. The GAS equicorrelation is slightly more persistent, but the correlation series are otherwise similar.

We compute the financial tail risk measures analytically, here defined as the probability of 10% or more of currently active financial institutions defaulting over the next year. The tail risk measure is plotted in the upper panel of Figure 7. The measure moves relatively little before 2008, and starts to move upwards after the bankruptcy of Lehman Brothers in September 2008. The tail probability then ascends steeply until mid-2012.

The lower panel in Figure 7 presents the average (across institutions) conditional risk measure. It increases up to 60% in September 2008 after the default of Lehman Brothers and further increases during the sovereign debt crisis. We interpret this average conditional measure as an empirical connectedness measure, as one would expect joint tail risk to shift out

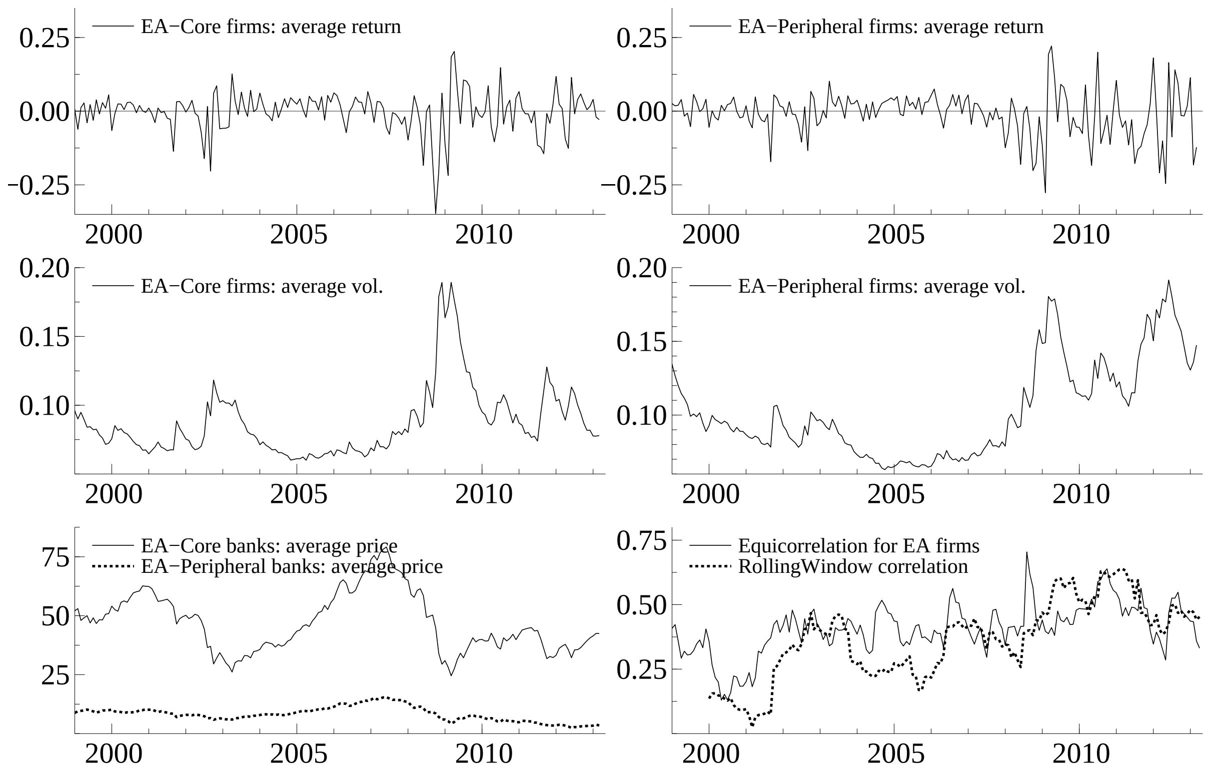

Figure 6: The equity prices, returns, volatility and equicorrelation.

The figure shows the dynamic equicorrelation estimates in the GAS model. As a comparison, we show the rolling window correlation estimates using a window of 12 months.

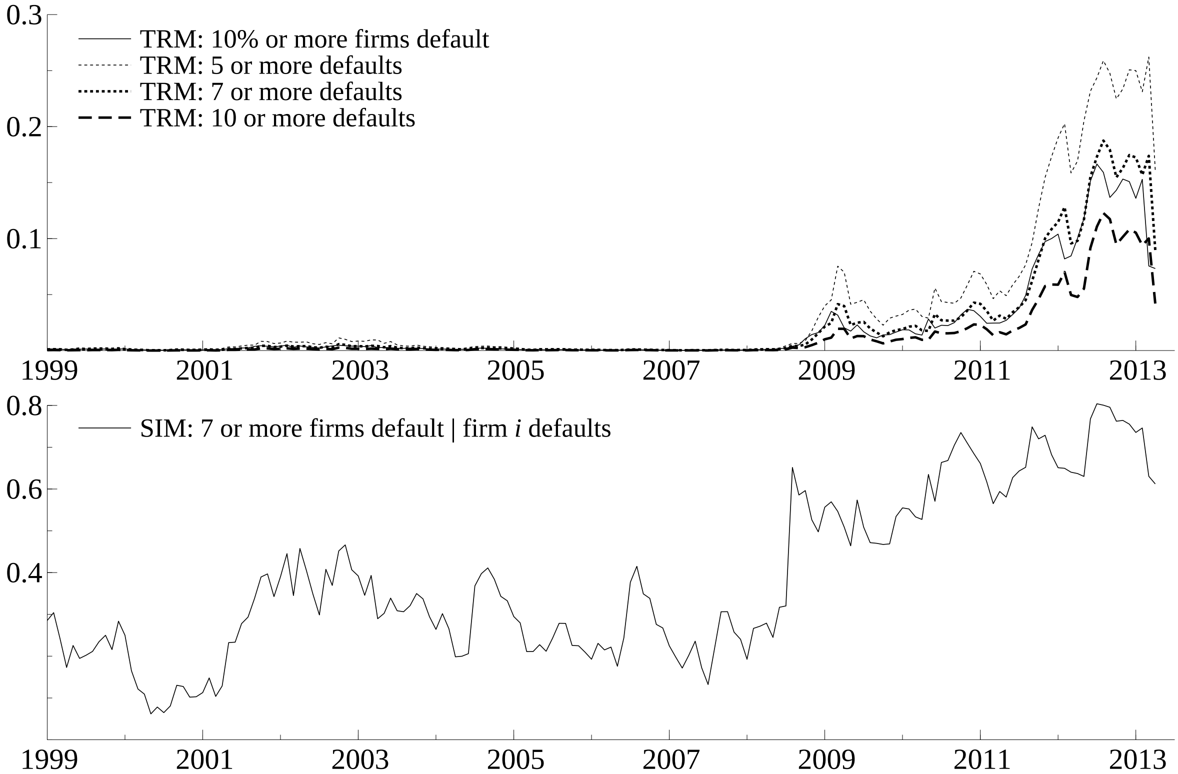

Figure 7: The joint tail risk measures and average conditional risk measure

We plot the joint tail risk measure (top panel) and average conditional risk measure (bottom panel). Both are based on the conditional law of large numbers approximation for the GAS-GHST equicorrelation model. The joint tail risk measure is defined as the probability of more than 10% of financial firms defaulting at time t. The average conditional measure is the average of the default probability of more than seven other firms defaulting in a setting where another firm defaults with certainty.

more in response to a default the more connected they are perceived to be. The conditional risk measure is consistent with the notion that connectedness increases in bad times.

5 Which factors drive bank equity correlations?

This section relates the latent systematic financial system correlations to observed risk factors.

A small literature studies which common factors drive stock returns, idiosyncratic volatilities, as well as stock return correlations, see for example Hou et al. (2011) and Bekaert et al. (2010). To our knowledge, less attention has been devoted to systematic dependence dynamics which underly financial sector stocks, in particular during times of crisis. Disentangling the determinants of financial sector correlation dynamics sheds light on whether it is contagion effects through business links or shared exposure to common economic factors that drive equity correlations. If shared exposure to common risk factors are sufficient to explain shared correlation dynamics, then contagion may work through an impact on marginal risks rather than through increased dependence.

We select a small set of economic covariates to capture correlations dynamics. In this section, the correlations ρ_t depend on both observed (X_t) and one unobserved (f_t) factor. Define $X = (X_1, \cdots, X_t)$ as a $N \times \tau$ matrix of economic variables and β a $1 \times N$ vector of parameters. We set

$$\rho_t = f_t + \beta X_t, \tag{49}$$

where f_t has the familiar score-driven dynamics. The monthly economic variables in X_t are (i) a European stock market volatility index (VSTOXX), (ii) the Euribor-EONIA money market spread, and the (iii) return on the broad S&P European stock market index. Very similar economic covariates are used in Suh (2012). The VSTOXX is a popular measure of the implied volatility of European index options. It is also considered to be a good measure of short term volatility expectations and risk aversion, and thus an appropriate indicator of market turbulence. The Euribor-EONIA spread is a measure of liquidity and credit risk in the overall banking sector. Since these rates refer to unsecured money market transactions,

Table 3: Estimation Results for the GHST Equicorrelation Model
The parameter estimates in the GAS-GHST models with extra economic variables and the GAS-GHST Equicorrelation model. These models are estimated with the filtered returns data. The sample covers the period between January 1999 and April 2013.

	GAS-Eqcorrelation	GAS-Factor(t-1)	GAS-Factor(t)
A	0.406	0.517	0.451
	(0.103)	(0.121)	(0.115)
B	0.837	0.815	0.827
	(0.084)	(0.083)	(0.096)
ω	0.548	0.576	0.548
	(0.053)	(0.099)	(0.098)
ν	20.506	20.066	20.229
	(2.670)	(2.654)	(1.593)
γ	-0.176	-0.181	-0.180
	(0.038)	(0.040)	(0.040)
Euribor-EONIA		0.340	0.042
		(0.129)	(0.128)
S&P index		-0.704	-0.323
		(0.344)	(0.407)
VSTOXX		-0.498	-0.040
		(0.328)	(0.312)
Log-lik	2913.072	2922.797	2913.524

credit risk perceptions are priced into this spread. Finally, the S&P European stock market index tracks the state of equity markets and corporate cash flow conditions. Correlations typically increase as stock market indexes fall.

Table 3 presents the parameter estimates for the augmented dynamic GHST model. We find that the three observed factors help explain the systemic correlation dynamics. The stock market index significantly explains the correlation movement in the next month. The negative sign coincides with the past observations about downside risk and rising correlations during crises. The positive coefficient for the Euribor-EONIA spread suggests that time-varying correlations tend to increase in times of reduced funding liquidity and increased credit risk concerns inside the financial sector. The coefficient for the VSTOXX is not significant. This suggests that the relationship of the systemic correlation and the market volatility is not strong at a monthly frequency.

If we consider the coincident values of the economic variables, all parameter estimates

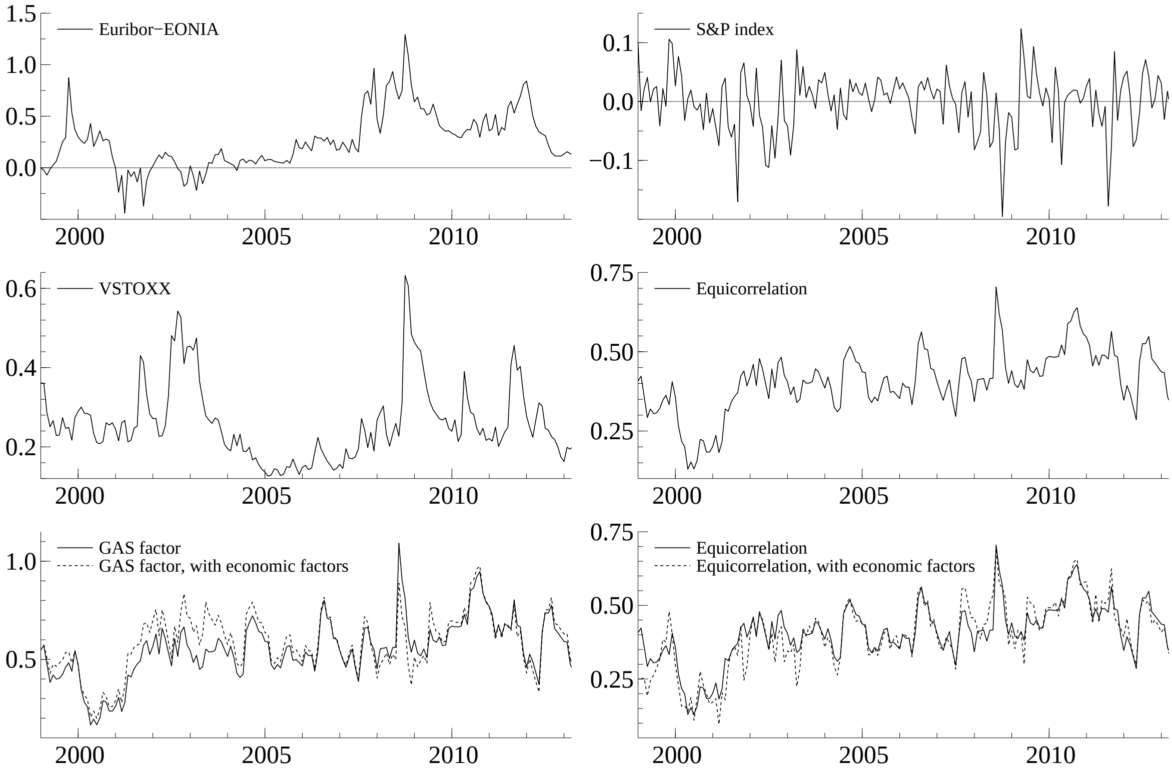

Figure 8: Observed factors and time-varying correlation estimates

The top four panels plot the economic factors used in the augmented model and the correlation estimates from the Equicorrelation model. The bottom-left panel is the plot of factors with and without economic factors. The bottom-right panel compares the correlation from the score-driven model with the correlation in the augmented model with extra economic variables.

cease to be statistically significant. This may be due to a somewhat lagging evolution of the EDF risk measures. While not as pronounced as in issuer ratings, these measures may also incorporate some tradeoff between accuracy and stability over time. Adding the observed economic variables to the score-driven dynamics does not change the dynamic correlation estimates to a large extent, see Figure 8.

6 Conclusion

In this paper, we develop the dynamic GHST model with GAS-Equicorrelation or block GAS-Equicorrelation structure. These models are applicable to large dimensional problems. We also propose two risk measures with a large panel of multiple European financial institutions. The Banking Stability measure we developed indicates the joint default risk in the system. The Systemic Risk Measure takes the average of conditional default probabilities to test the interconnectedness of the financial system. The full dynamic multivariate model with the GH skew-t distribution is used to simulate the possible distress scenarios for the banks. Based on the Monte Carlo simulation, we can analyze the joint and conditional credit risk in individual financial institutions. Another risk measuring model originates from the conditional Law of Large Numbers approximation method. With the application of a Dynamic Equicorrelation model in a large system of financial firms, the approximated risk indicator provides a good measure of credit risks for an unbalanced large panel.

We further showed the explanatory power of some commonly used economic variables (VSTOXX index, Euribor-EONIA spread and European stock market index) to explain systemic correlation dynamics. By introducing these new variables in our dynamic system, the correlation becomes less persistent compared to the pure GAS dynamic model. The residual GAS factor decreases due to the explanatory power of the extra economic variables. It appears that we still miss one or a few more factors to explain the variation in correlation dynamics. Moreover, we might miss a few important firm specific variables, such as the leverage ratio. The current model also enable us to measure the systemic risk contribution of each bank by looking at the conditional probability in the multivariate GH skewed-t

distribution. We leave this for future research.

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A The GHST dynamic model derivations

A.1 The volatility model with leverage

The univariate GHST distribution density

$$
\begin{aligned}
p(y_t;\tilde{\sigma}_t^2,\gamma,\nu) &= \frac{\nu^{\frac{\nu}{2}}2^{1-\frac{\nu+1}{2}}}{\Gamma(\frac{\nu}{2})\pi^{\frac{1}{2}}\tilde{\sigma}_t}\cdot\frac{K_{\frac{\nu+1}{2}}\left(\sqrt{d(y_t)(\gamma^2)}\right)e^{\gamma(y_t-\tilde{\mu}_t)/\tilde{\sigma}_t}}{d(y_t)^{\frac{\nu+1}{4}}\cdot(\gamma^2)^{-\frac{\nu+1}{4}}}, && \text{(A1)}\\
d(y_t) &= \nu+(y_t-\tilde{\mu}_t)^2/\tilde{\sigma}_t^2, && \text{(A2)}\\
\tilde{\mu}_t &= -\frac{\nu}{\nu-2}\tilde{\sigma}_t\gamma, \quad \tilde{\sigma}_t=\sigma_t T, && \text{(A3)}\\
T &= \left(\frac{\nu}{\nu-2}+\frac{2\nu^2\gamma^2}{(\nu-2)^2(\nu-4)}\right)^{-1/2}. && \text{(A4)}
\end{aligned}
$$

Next we introduce the GAS factor f_t to the time varying volatility σ_t.

Proposition 1. *If we assume y follows the GH skewed-t density* (A1) *where the scale $\sigma_t=\sigma_t(f_t)$, the score driven factor f_t with the leverage effect*

$$
\begin{aligned}
f_{t+1} &= \tilde{\omega}+\sum_{i=0}^{p-1}A_i s_{t-i}+\sum_{j=0}^{q-1}B_j f_{t-j}+C(s_t-s_t^{\mu})1\{y_t<\mu_t\}, && \text{(A5)}\\
s_t &= \mathcal{S}_t\nabla_t, \quad \nabla_t=\partial\ln p(y_t|\mathcal{F}_{t-1};f_t,\theta)/\partial f_t, && \text{(A6)}\\
s_t^{\mu} &= \mathcal{S}_t\nabla_t^{\mu}, \quad \nabla_t^{\mu}=\partial\ln p(\mu_t|\mathcal{F}_{t-1};f_t,\theta)/\partial f_t, && \text{(A7)}
\end{aligned}
$$

Then the dynamic score is

$$
\begin{aligned}
\nabla_t &= \Psi_t H_t\left(w_t\cdot y_t^2-\tilde{\sigma}_t^2-\left(1-\frac{\nu}{\nu-2}w_t\right)\tilde{\sigma}_t\gamma y_t\right), && \text{(A8)}\\
w_t &= \frac{\nu+1}{2d(y_t)}-\frac{k'_{0.5(\nu+1)}\left(\sqrt{d(y_t)\cdot(\gamma^2)}\right)}{\sqrt{d(y_t)/(\gamma^2)}}, && \text{(A9)}\\
\Psi_t &= \frac{\partial\sigma_t}{\partial f_t}, \quad H_t=T\tilde{\sigma}^{-3}. && \text{(A10)}
\end{aligned}
$$

Proof. Let $k_\lambda(\cdot)=\ln K_\lambda(\cdot)$ with first derivative $k'_\lambda(\cdot)$. we could directly introduce the GAS factor into σ_t

$$\begin{aligned}\nabla_t &= \frac{\partial \sigma_t}{\partial f_t}\frac{\partial \tilde{\sigma}_t}{\partial \sigma_t}\frac{\partial \ln p(y_t; \tilde{\sigma}_t^2, \gamma, \nu)}{\partial \tilde{\sigma}_t} \\ &= \Psi_t'\left(-\frac{1}{2}w_t\frac{\partial d(y_t)}{\partial \tilde{\sigma}_t} - \frac{1}{\tilde{\sigma}_t} - \frac{\gamma y_t}{\tilde{\sigma}_t^2}\right) \\ &= \Psi_t' H_t\left(w_t \cdot y_t^2 - \tilde{\sigma}_t^2 - \left(1 - \frac{\nu}{\nu-2}w_t\right)\tilde{\sigma}_t\gamma y_t\right),\end{aligned}$$

with

$$w_t = \frac{\nu+1}{2d(y_t)} - \frac{k'_{0.5(\nu+1)}\left(\sqrt{d(y_t)\cdot(\gamma^2)}\right)}{\sqrt{d(y_t)/(\gamma^2)}}, \quad H_t = T\tilde{\sigma}_t^{-3}.$$

We also include the leverage term in the model, by defining $\nabla_t^{\mu} = \nabla_t(y_t)$ with y_t equals to a predetermined threshold μ_t. □

A.2 The dynamic copula model

To derive the dynamic GHST model with GAS driven scale matrix, we start with the GHST distribution

$$p(y_t; \tilde{\Sigma}_t, \gamma, \nu) = \frac{\nu^{\frac{\nu}{2}} 2^{1-\frac{\nu+N}{2}}}{\Gamma(\frac{\nu}{2})\pi^{\frac{N}{2}}|\tilde{\Sigma}_t|^{\frac{1}{2}}} \cdot \frac{K_{\frac{\nu+N}{2}}\left(\sqrt{d(y_t)\cdot d(\gamma)}\right)e^{\gamma'\tilde{\Sigma}_t^{-1}(y_t-\tilde{\mu})}}{d(y_t)^{\frac{\nu+N}{4}}\cdot d(\gamma)^{-\frac{\nu+N}{4}}}, \quad \text{(A11)}$$

$$d(y_t) = \nu + (y_t - \tilde{\mu})'\tilde{\Sigma}_t^{-1}(y_t - \tilde{\mu}), \quad d(\gamma) = \gamma'\tilde{\Sigma}_t^{-1}\gamma, \quad \text{(A12)}$$

$$\tilde{\mu} = -\frac{\nu}{\nu-2}\gamma, \quad \tilde{\Sigma}_t = \tilde{L}_t\tilde{L}_t'. \quad \text{(A13)}$$

And through the derivation of score as the key driving force, we have the following result:

Proposition 2. *If we assume y follows the GH skewed-t density* (A11) *where the time varying scale matrix $\tilde{\Sigma}_t = \tilde{\Sigma}_t(f_t)$, the score driven factor f_t*

$$f_{t+1} = \omega + \sum_{i=0}^{p-1} A_i s_{t-i} + \sum_{j=0}^{q-1} B_j f_{t-j}, \quad \text{(A14)}$$

$$s_t = \mathcal{S}_t\nabla_t, \quad \nabla_t = \partial \ln p(y_t|\mathcal{F}_{t-1}; f_t, \theta)/\partial f_t, \quad \text{(A15)}$$

Thus the dynamic score (1,1) model

$$
\begin{aligned}
f_{t+1} &= \omega + A\mathcal{S}_t \nabla_t + B f_t, && \text{(A16)}\\
\nabla_t &= \Psi_t' H_t' \text{vec}\left(w_t \cdot (y_t - \tilde{\mu})(y_t - \tilde{\mu})' - w_t^{\gamma} \cdot \gamma\gamma' - \gamma(y_t - \tilde{\mu})' - 0.5\tilde{\Sigma}_t\right) && \text{(A17)}\\
w_t &= \frac{\nu + N}{4d(y_t)} - \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(y_t)/d(\gamma)}}, && \text{(A18)}\\
w_t^{\gamma} &= \frac{\nu + N}{4d(\gamma)} + \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(\gamma)/d(y_t)}}, && \text{(A19)}\\
H_t &= \tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1}, \quad \Psi_t = \frac{\partial \text{vec}(\tilde{\Sigma}_t)'}{\partial f_t}. && \text{(A20)}
\end{aligned}
$$

Proof. Let $k_\lambda(\cdot) = \ln K_\lambda(\cdot)$ with first derivative $k'_\lambda(\cdot)$. With the help of the matrix calculus equations regarding an invertible symmetric matrix X,

$$
\frac{\partial(\alpha' X^{-1}\beta)'}{\partial \text{vec}(X)} = -(X^{-1} \otimes X^{-1})\text{vec}(\alpha\beta'), \quad \frac{\partial(\log|X|)'}{\partial \text{vec}(X)} = (X^{-1} \otimes X^{-1})\text{vec}(X); \qquad \text{(A21)}
$$

we obtain

$$
\begin{aligned}
\nabla_t &= \frac{\partial \text{vec}(\tilde{\Sigma}_t)'}{\partial f_t} \frac{\partial \ln p(y_t; \tilde{\Sigma}_t, \gamma, \nu)}{\partial \text{vec}(\tilde{\Sigma}_t)}\\
&= \Psi_t' \left(-w_t \frac{\partial d(y_t)}{\partial \text{vec}(\tilde{\Sigma}_t)} + w_t^{\gamma} \frac{\partial d(\gamma)}{\partial \text{vec}(\tilde{\Sigma}_t)} + \frac{\partial(\gamma' \tilde{\Sigma}_t^{-1}(y_t - \tilde{\mu}))}{\partial \text{vec}(\tilde{\Sigma}_t)} - \frac{1}{2} \frac{\partial(\log(|\tilde{\Sigma}_t|))}{\partial \text{vec}(\tilde{\Sigma}_t)} \right)\\
&= \Psi_t' H_t \text{vec}\left(w_t(y_t - \tilde{\mu})(y_t - \tilde{\mu})' - w_t^{\gamma}\gamma\gamma' - \gamma(y_t - \tilde{\mu})' - 0.5\tilde{\Sigma}_t\right),
\end{aligned}
$$

with $H_t = \tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1}$, and

$$
w_t = \frac{\nu + N}{4d(y_t)} - \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(y_t)/d(\gamma)}}, \quad w_t^{\gamma} = \frac{\nu + N}{4d(\gamma)} + \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(\gamma)/d(y_t)}}.
$$

□

A.3 The Block GAS-Equicorrelation model

We defines a multi-factor structure underlying the dynamic correlation model. In the literature, we call correlation matrices with such a structure a block dynamic equicorrelation matrix. Assume that N firms fall into m different groups according to their exposure to a common systemic risk factor. Firms have equicorrelation ρ_i^2 within each group and $\rho_i \cdot \rho_j$ between groups i and j. So we have $N = n_1 + n_2 + \cdots + n_m$ random variables that follow a GH distribution with a correlation matrix that has a block equicorrelation structure, where n_i denotes the number of firms in group i. The correlation matrix at time t is given by

$$\tilde{\Sigma}_t = \begin{bmatrix} (1-\rho_{1,t}^2)\mathrm{I}_{n_1} & \cdots & \cdots & 0 \\ 0 & (1-\rho_{2,t}^2)\mathrm{I}_{n_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (1-\rho_{m,t}^2)\mathrm{I}_{n_m} \end{bmatrix} + \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \cdot \begin{pmatrix} \rho_{1,t}\ell_1' & \rho_{2,t}\ell_2' & \cdots & \rho_{m,t}\ell_m' \end{pmatrix}, \tag{A22}$$

It is straightforward to see that

$$\frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial f_{i,t}} = -2\rho_{i,t} \cdot \mathrm{vec} \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \mathrm{I}_i & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix} + \left[\begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \otimes \mathrm{I}_N + \mathrm{I}_N \otimes \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \right] \cdot \begin{pmatrix} 0 \\ \vdots \\ \ell_{n_i} \\ 0 \end{pmatrix} \tag{A23}$$

The result in the main text is a general expression that contains the derivative with respect to f_t. The main GAS dynamic system remains the same.

B Additional results

Table B1: Sample Skewness and Kurtosis Statistics.

Descriptive statistics for the CRSP stock returns between January 1999 and April 2013. All observations are monthly log returns. All names are large European financial firms including banks, insurance companies and investment firms.

Name	Size	Std,Dev,	Skewness	Kurtosis	Minimum	Maximum
ACKERMANS & VAN HAAREN	172	0.071	-0.199	3.981	-0.254	0.212
AEGON	172	0.133	-1.134	7.214	-0.649	0.311
AGEAS (EX-FORTIS)	172	0.181	-3.917	35.995	-1.574	0.599
ALLIANZ	172	0.111	-0.692	5.941	-0.458	0.398
ALLIED IRISH BANKS	172	0.198	-0.956	10.137	-1.026	0.863
ALPHA BANK	172	0.177	1.531	14.531	-0.555	1.196
ASSICURAZIONI GENERALI	172	0.080	-0.544	4.914	-0.301	0.244
ATRIUM EUROPEAN RLST.	120	0.118	-0.427	12.784	-0.504	0.620
AXA	172	0.122	-0.977	6.792	-0.516	0.356
AZIMUT HOLDING	104	0.103	-0.253	3.056	-0.293	0.288
BANK OF IRELAND	172	0.212	-0.380	12.688	-1.119	1.079
BANKINTER 'R'	172	0.106	0.144	4.438	-0.327	0.380
BANCA CARIGE	172	0.075	-1.255	6.299	-0.297	0.215
BANCA MONTE DEI PASCHI	165	0.098	-0.516	4.586	-0.320	0.296
BANCA POPOLARE DI MILANO	172	0.109	-0.275	4.428	-0.374	0.335
BANCA PPO.DI SONDRIO	172	0.058	-0.062	4.244	-0.177	0.192
BANCA PPO.EMILIA ROMAGNA	172	0.082	-0.798	5.891	-0.308	0.218
BBV.ARGENTARIA	172	0.092	-0.190	4.387	-0.309	0.289
BANCO COMR.PORTUGUES 'R'	172	0.102	-0.417	4.569	-0.340	0.346
BANCO DE VALENCIA	172	0.232	-6.384	66.738	-2.387	0.722
BANCO ESPIRITO SANTO	172	0.095	-0.563	5.626	-0.360	0.317
BANCO POPOLARE	172	0.108	-0.635	5.705	-0.460	0.311
BANCO POPULAR ESPANOL	172	0.093	-0.516	6.186	-0.383	0.317
BANCO DE SABADELL	143	0.089	0.247	4.878	-0.290	0.358
BANCO SANTANDER	172	0.092	-0.512	4.639	-0.298	0.300
BNP PARIBAS	172	0.093	-0.507	5.291	-0.334	0.332
BOLSAS Y MERCADOS ESPANOLES	80	0.089	-0.149	3.928	-0.274	0.271
CATTOLICA ASSICURAZIONI	148	0.079	-0.818	6.824	-0.368	0.260
CNP ASSURANCES	172	0.080	-0.470	3.424	-0.259	0.166
COFINIMMO	172	0.039	-1.146	7.043	-0.205	0.092
COMMERZBANK	172	0.146	-0.699	6.249	-0.667	0.466
CIE.NALE.A PTF.	172	0.053	-0.070	3.818	-0.146	0.176
CORIO	172	0.060	-0.842	4.451	-0.216	0.144
CREDIT AGRICOLE	135	0.116	-0.316	3.684	-0.336	0.279
BCA.PICCOLO CDT.VALTELL	172	0.082	-0.954	7.701	-0.406	0.220
CAIXABANK	65	0.100	-0.132	3.711	-0.258	0.258
DELTA LLOYD GROUP	41	0.098	-0.153	3.015	-0.268	0.230
DEUTSCHE BANK	172	0.113	-0.346	5.621	-0.463	0.457
DEUTSCHE BOERSE	145	0.094	-0.338	3.991	-0.297	0.301
DEUTSCHE POSTBANK	105	0.120	-1.599	11.122	-0.548	0.397
DEXIA	172	0.164	-1.746	11.302	-0.956	0.464
EUROBANK ERGASIAS S A	172	0.184	0.613	9.228	-0.608	1.015
ERSTE GROUP BANK	172	0.119	-0.602	8.467	-0.536	0.556
EURAZEO	172	0.095	-0.480	4.722	-0.367	0.355
FONCIERE DES REGIONS	172	0.078	-0.799	8.078	-0.423	0.288
GECINA	172	0.095	-0.425	8.838	-0.408	0.374
GBL NEW	172	0.061	-0.510	4.800	-0.261	0.185
SOCIETE GENERALE	172	0.113	-0.446	4.394	-0.401	0.344
HANNOVER RUCK.	172	0.086	-0.903	7.191	-0.449	0.237
ICADE	172	0.087	-0.129	5.257	-0.277	0.348
IMMOFINANZ	172	0.156	-3.143	26.014	-1.029	0.587
ING GROEP	172	0.134	-1.284	9.964	-0.727	0.459
INTESA SANPAOLO	172	0.106	-0.696	4.369	-0.345	0.285
KBC GROUP	172	0.136	-0.691	8.185	-0.572	0.459
KLEPIERRE	172	0.073	-0.957	6.449	-0.352	0.192
MAPFRE	172	0.088	-0.390	5.165	-0.350	0.252
MARFIN INV.GP.HDG.	172	0.202	0.900	6.647	-0.437	1.018
MEDIOBANCA BC.FIN	172	0.091	0.176	3.630	-0.228	0.309
MUENCHENER RUCK.	172	0.095	-0.558	9.969	-0.453	0.464
NATIONAL BANK OF GREECE	172	0.154	-0.075	4.993	-0.534	0.582
NATIXIS	172	0.116	0.130	7.225	-0.493	0.533
BANK OF PIRAEUS	172	0.184	0.367	9.526	-0.767	1.022
POHJOLA PANKKI A	172	0.082	-1.368	13.139	-0.541	0.244
RAIFFEISEN BANK INTL	95	0.146	-0.829	5.917	-0.606	0.372
SAMPO 'A'	172	0.075	-0.335	3.663	-0.235	0.202
SCOR SE	172	0.126	-2.915	23.032	-0.965	0.366
SOFINA	172	0.060	-0.769	4.828	-0.229	0.181
UNIONE DI BANCHE ITALIAN	117	0.094	-0.489	3.864	-0.303	0.236
UNIBAIL-RODAMCO	172	0.064	-0.639	3.498	-0.221	0.153
UNICREDIT	172	0.105	-0.360	5.861	-0.428	0.375
VIENNA INSURANCE GROUP A	172	0.082	-0.602	14.001	-0.492	0.362
WENDEL	172	0.118	-0.857	4.692	-0.382	0.338
WERELDHAVE	172	0.059	-0.320	2.826	-0.172	0.140

Table B2: The Estimation Results: 73 Euro Area financial firms.

The parameter estimates in the GAS-GHST Equicorrelation model. The volatility model is fitted to individual time series. The correlation model is estimated with the filtered returns data. The sample covers the period between January 1999 and April 2013.

Name	A	B	ω	ν	γ	C
ACKERMANS & VAN HAAREN	0.120	0.782	-2.408	103.590	-1.456	-0.069
	(0.016)	(0.023)	(0.054)	(23.565)	(0.053)	(0.013)
AEGON	0.083	0.926	-2.798	10.028	-0.456	0.017
	(0.025)	(0.027)	(0.284)	(3.091)	(0.223)	(0.030)
AGEAS (EX-FORTIS)	0.055	0.900	-3.049	8.484	-0.211	0.080
	(0.022)	(0.030)	(0.171)	(3.109)	(0.175)	(0.033)
ALLIANZ (XET)	0.016	0.892	-3.262	6.078	-0.388	0.094
	(0.037)	(0.045)	(0.290)	(1.008)	(0.030)	(0.017)
ALLIED IRISH BANKS	0.045	0.952	-3.229	7.562	-0.242	0.064
	(0.018)	(0.015)	(0.261)	(2.511)	(0.146)	(0.022)
ALPHA BANK	0.085	0.935	-2.711	3.808	-0.009	0.051
	(0.030)	(0.029)	(0.268)	(0.799)	(0.042)	(0.029)
ASSICURAZIONI GENERALI	0.075	0.878	-2.840	7.241	0.092	0.014
	(0.023)	(0.050)	(0.119)	(2.026)	(0.113)	(0.023)
ATRIUM EUROPEAN RLST.	0.216	0.714	-3.822	2.655	-0.280	0.205
	(0.045)	(0.053)	(0.123)	(0.203)	(0.073)	(0.047)
AXA	0.039	0.759	-2.842	6.593	-0.076	0.149
	(0.032)	(0.066)	(0.107)	(1.556)	(0.105)	(0.041)
AZIMUT HOLDING	0.004	0.876	-2.530	93.706	-1.737	0.028
	(0.005)	(0.009)	(0.028)	(103.963)	(0.037)	(0.007)
BANK OF IRELAND	0.058	0.960	-3.056	8.763	-0.282	0.040
	(0.019)	(0.014)	(0.337)	(3.147)	(0.170)	(0.021)
BANKINTER 'R'	0.030	0.878	-2.884	5.387	-0.072	0.083
	(0.025)	(0.048)	(0.152)	(1.303)	(0.083)	(0.033)
BANCA CARIGE	0.296	0.900	-1.787	8.366	-0.562	-0.165
	(0.035)	(0.020)	(0.449)	(1.821)	(0.143)	(0.041)
BANCA MONTE DEI PASCHI	0.000	0.932	-3.230	6.634	-0.307	0.075
	(0.000)	(0.027)	(0.176)	(1.952)	(0.148)	(0.020)
BANCA POPOLARE DI MILANO	0.017	0.887	-2.848	6.579	0.200	0.078
	(0.025)	(0.048)	(0.134)	(1.597)	(0.105)	(0.030)
BANCA PPO.DI SONDRIO	0.050	0.960	-3.281	5.131	0.020	0.026
	(0.017)	(0.013)	(0.263)	(1.307)	(0.069)	(0.022)
BANCA PPO.EMILIA ROMAGNA	0.226	0.983	-1.654	2.208	-0.021	-0.053
	(0.036)	(0.015)	(2.307)	(0.227)	(0.022)	(0.032)
BBV.ARGENTARIA	0.056	0.836	-2.967	8.224	-0.340	0.082
	(0.031)	(0.088)	(0.153)	(4.712)	(0.304)	(0.066)
BANCO COMR.PORTUGUES 'R'	0.111	0.906	-2.814	5.729	0.061	0.043
	(0.025)	(0.028)	(0.162)	(1.442)	(0.078)	(0.029)
BANCO DE VALENCIA	0.073	0.968	-3.426	2.075	-0.015	0.044
	(0.037)	(0.033)	(0.320)	(0.141)	(0.027)	(0.031)
BANCO ESPIRITO SANTO	0.160	0.955	-3.559	3.018	-0.016	0.055
	(0.029)	(0.019)	(0.357)	(0.463)	(0.025)	(0.030)
BANCO POPOLARE	0.007	0.942	-3.288	31.299	-2.211	0.048
	(0.000)	(0.002)	(0.018)	(1.257)	(0.028)	(0.003)
BANCO POPULAR ESPANOL	0.107	0.875	-2.393	47.650	-1.830	-0.035
	(0.007)	(0.008)	(0.068)	(19.524)	(0.045)	(0.006)
BANCO DE SABADELL	0.000	0.883	-2.656	200.000	-15.485	0.009
	(0.025)	(0.063)	(0.227)	(0.000)	(0.150)	(0.001)
BANCO SANTANDER	0.057	0.815	-3.085	7.077	-0.364	0.113
	(0.026)	(0.062)	(0.141)	(2.149)	(0.179)	(0.043)
BNP PARIBAS	0.046	0.810	-3.128	8.170	-0.374	0.142
	(0.049)	(0.052)	(0.230)	(2.603)	(0.224)	(0.067)
BOLSAS Y MERCADOS ESPANOLES	0.006	0.640	-2.613	94.987	-3.018	0.033
	(0.250)	(0.089)	(0.700)	(105.571)	(2.115)	(0.252)
CATTOLICA ASSICURAZIONI	0.000	0.933	-3.353	5.590	-0.249	0.057
	(0.000)	(0.024)	(0.166)	(1.426)	(0.126)	(0.015)
CNP ASSURANCES	0.171	0.718	-2.554	33.482	-0.304	-0.029
	(0.041)	(0.112)	(0.173)	(30.091)	(0.323)	(0.063)
COFINIMMO	0.000	0.884	-3.850	5.020	-0.249	0.045
	(0.000)	(0.046)	(0.133)	(1.229)	(0.117)	(0.018)
COMMERZBANK (XET)	0.014	0.927	-3.051	7.818	-0.291	0.085
	(0.028)	(0.017)	(0.169)	(2.279)	(0.157)	(0.023)
CIE.NALE.A PTF.	0.158	0.943	-3.507	3.507	0.059	0.004
	(0.033)	(0.025)	(0.330)	(0.635)	(0.045)	(0.034)
CORIO	0.020	0.983	-3.797	5.596	-0.477	0.018
	(0.017)	(0.004)	(0.670)	(1.259)	(0.183)	(0.016)
CREDIT AGRICOLE	0.007	0.942	-3.288	31.299	-2.211	0.048
	(0.000)	(0.002)	(0.018)	(1.257)	(0.028)	(0.003)
BCA.PICCOLO CDT.VALTELL	0.088	0.953	-2.848	4.739	0.046	-0.007
	(0.024)	(0.030)	(0.260)	(1.218)	(0.073)	(0.022)
CAIXABANK	0.000	0.794	-3.578	2.641	-0.103	0.237
	(0.000)	(0.041)	(0.186)	(0.252)	(0.045)	(0.041)
DELTA LLOYD GROUP	0.000	0.900	-1.577	200.000	-1.271	-0.065
	(0.022)	(0.023)	(0.327)	(0.000)	(0.250)	(0.007)
DEUTSCHE BANK (XET)	0.000	0.861	-3.084	16.933	-2.825	0.089
	(0.000)	(0.017)	(0.100)	(4.327)	(0.619)	(0.018)
DEUTSCHE BOERSE (XET)	0.101	0.695	-2.411	29.749	-1.690	-0.049
	(0.017)	(0.063)	(0.041)	(6.550)	(0.047)	(0.012)
DEUTSCHE POSTBANK (XET)	0.000	0.873	-3.188	14.953	-0.251	0.147
	(0.000)	(0.018)	(0.103)	(8.188)	(0.256)	(0.026)

Table B2: The Estimation Results: 73 Euro Area financial firms. (continued)

The parameter estimates in the GAS-GHST Equicorrelation model. The volatility model is fitted to individual time series. The correlation model is estimated with the filtered returns data. The sample covers the period between January 1999 and April 2013.

Name	A	B	ω	ν	γ	C
DEXIA	0.004	0.848	-3.247	19.976	-1.335	0.104
	(0.001)	(0.004)	(0.026)	(0.740)	(0.021)	(0.005)
EUROBANK ERGASIAS S A	0.092	0.959	-2.790	3.473	0.043	0.051
	(0.028)	(0.025)	(0.383)	(0.587)	(0.041)	(0.028)
ERSTE GROUP BANK	0.000	0.874	-3.193	17.949	-0.432	0.140
	(0.000)	(0.020)	(0.126)	(8.350)	(0.057)	(0.018)
EURAZEO	0.013	0.873	-2.889	5.761	-0.058	0.081
	(0.022)	(0.051)	(0.151)	(1.357)	(0.084)	(0.026)
FONCIERE DES REGIONS	0.141	0.900	-2.792	7.871	-0.108	-0.029
	(0.030)	(0.036)	(0.211)	(2.311)	(0.128)	(0.028)
GECINA	0.053	0.925	-3.507	4.256	-0.087	0.066
	(0.031)	(0.033)	(0.286)	(1.147)	(0.059)	(0.029)
GBL NEW	0.000	1.000	30.232	200.000	0.362	-0.061
	(0.000)	(0.001)	(60.599)	(0.000)	(0.913)	(0.043)
SOCIETE GENERALE	0.070	0.867	-3.043	11.492	-0.119	0.142
	(0.026)	(0.033)	(0.171)	(3.947)	(0.188)	(0.045)
HANNOVER RUCK. (XET)	0.000	0.972	-5.091	16.708	-1.351	0.059
	(0.000)	(0.001)	(0.053)	(1.038)	(0.040)	(0.003)
ICADE	0.223	0.893	-3.408	2.095	0.004	0.083
	(0.064)	(0.039)	(0.379)	(0.252)	(0.011)	(0.057)
IMMOFINANZ	0.111	0.940	-4.593	2.103	-0.038	0.105
	(0.027)	(0.013)	(0.266)	(0.095)	(0.034)	(0.038)
ING GROEP	0.051	0.909	-2.898	12.729	-0.958	0.044
	(0.009)	(0.007)	(0.049)	(0.907)	(0.027)	(0.008)
INTESA SANPAOLO	0.060	0.942	-2.570	5.481	-0.005	0.019
	(0.020)	(0.024)	(0.221)	(1.190)	(0.077)	(0.021)
KBC GROUP	0.071	0.932	-3.332	5.533	-0.063	0.070
	(0.022)	(0.024)	(0.235)	(1.392)	(0.086)	(0.025)
KLEPIERRE	0.025	0.923	-3.326	5.154	-0.150	0.047
	(0.022)	(0.040)	(0.196)	(1.228)	(0.089)	(0.030)
MAPFRE	0.000	0.822	-3.078	5.028	-0.179	0.149
	(0.001)	(0.071)	(0.182)	(1.146)	(0.102)	(0.032)
MARFIN INV.GP.HDG.	0.090	0.898	-2.395	4.163	0.102	0.073
	(0.026)	(0.046)	(0.176)	(0.832)	(0.065)	(0.037)
MEDIOBANCA BC.FIN	0.058	0.938	-2.572	6.809	0.109	0.006
	(0.020)	(0.036)	(0.166)	(1.912)	(0.105)	(0.019)
MUENCHENER RUCK. (XET)	0.060	0.909	-2.961	7.991	-0.081	0.055
	(0.025)	(0.033)	(0.182)	(2.322)	(0.119)	(0.024)
NATIONAL BK.OF GREECE	0.060	0.940	-2.718	4.269	0.111	0.073
	(0.025)	(0.021)	(0.246)	(0.808)	(0.060)	(0.029)
NATIXIS	0.081	0.948	-2.832	10.538	-0.627	0.020
	(0.024)	(0.020)	(0.502)	(4.540)	(0.294)	(0.034)
BANK OF PIRAEUS	0.124	0.918	-2.327	4.627	0.073	0.033
	(0.032)	(0.031)	(0.210)	(1.052)	(0.063)	(0.031)
POHJOLA PANKKI A	0.000	0.958	-3.155	4.654	0.037	0.039
	(0.000)	(0.028)	(0.309)	(0.951)	(0.061)	(0.013)
RAIFFEISEN BANK INTL.	0.054	0.797	-2.683	17.207	-0.559	0.144
	(0.040)	(0.044)	(0.207)	(11.602)	(0.421)	(0.072)
SAMPO 'A'	0.054	0.942	-3.109	5.868	-0.335	0.046
	(0.017)	(0.024)	(0.288)	(1.308)	(0.049)	(0.022)
SCOR SE	0.000	0.916	-3.404	4.089	-0.143	0.104
	(0.000)	(0.028)	(0.239)	(0.811)	(0.074)	(0.022)
SOFINA	0.035	0.852	-3.150	5.284	-0.169	0.035
	(0.028)	(0.088)	(0.136)	(1.211)	(0.096)	(0.034)
UNIONE DI BANCHE ITALIAN	0.000	0.858	-2.902	199.999	-2.049	0.058
	(0.000)	(0.040)	(0.098)	(0.034)	(0.653)	(0.013)
UNIBAIL-RODAMCO	0.000	0.948	-3.330	10.823	-0.446	0.032
	(0.000)	(0.032)	(0.239)	(4.486)	(0.275)	(0.014)
UNICREDIT	0.078	0.935	-2.904	5.786	0.230	0.051
	(0.022)	(0.029)	(0.212)	(1.314)	(0.103)	(0.025)
VIENNA INSURANCE GROUP A	0.121	0.921	-3.905	2.801	0.060	0.097
	(0.036)	(0.027)	(0.293)	(0.365)	(0.032)	(0.038)
WENDEL	0.067	0.868	-2.749	12.174	-0.580	0.046
	(0.035)	(0.041)	(0.208)	(5.492)	(0.292)	(0.037)
WERELDHAVE	0.022	0.976	-4.478	4.979	0.000	0.057
	(0.016)	(0.012)	(0.657)	(1.104)	(0.068)	(0.020)