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Abstract

A common finding of the optimal unemployment insurance literature is that the optimal UI replacement rate is around 50%, implying that current levels in the US are close to optimal. However, a key assumption in the existing literature is that unemployment benefits are the only government spending activity. In this paper I show that recommendations for optimal UI levels are dramatically reduced when one incorporates the fact that UI spending is a small part of overall government spending. This occurs because the negative impact of UI on income tax revenues implies added welfare costs, a mechanism that I refer to as a fiscal externality. Using both a calibrated structural job search model and a "sufficient statistics" method that relies on reduced-form elasticities, I find that the optimal replacement rate drops to zero once fiscal externalities are incorporated. However, I also consider the possibility that more generous UI could increase reservation wages and thus potentially increase the tax base, and I show that this second fiscal externality could have important effects on the results, with an optimal replacement rate which could rise above 70%.

Keywords: unemployment insurance, fiscal externality, job search, sufficient statistics, government spending

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1 Introduction

A large literature studies the optimal provision of unemployment insurance.[1] A common finding of this literature is that the optimal UI replacement rate is around 50%, implying that current levels in the US are close to optimal.[2] However, a key assumption in the existing literature is that unemployment benefits are the only spending activity of the government that needs to be financed with taxes on labour income. In this paper I show that recommendations for optimal UI levels are dramatically reduced when one incorporates the fact that UI spending is a small part of overall government spending.[3]

The economics behind this finding are simple and intuitive. When the government makes UI benefits more generous, the average duration of unemployment spells increases due to moral hazard, thereby lowering the steady-state level of employment and the tax base used to finance non-UI spending. Even if the tax rate is increased enough to cover the higher UI payments, the decrease in the tax base means that the revenues used to finance non-UI spending are no longer sufficient, implying either that this spending must decrease or that additional taxes must be levied. It follows that there is an added welfare cost associated with higher UI benefits relative to the case in which non-UI spending is zero. I refer to these effects on the non-UI budget as fiscal externalities, and the key message from my analysis is that these fiscal externalities are quantitatively important.[4]

I explore the quantitative implications of fiscal externalities for optimal UI using the two most popular approaches found in the literature. First, I calibrate a structural job search model to match a set of real-world moments under two scenarios: the first uses an estimate

[1]A variety of approaches have been used: for example, papers such as Hansen and İmrohoroğlu (1992) and Hopenhayn and Nicolini (1997) study the issue using calibrated structural models of unemployment, whereas Baily (1978) and Chetty (2008) are important examples of more reduced-form studies that derive equations for the welfare consequences of UI as functions of empirical statistics. The central tradeoff of the optimal UI problem is between the consumption-smoothing and risk-sharing benefits of UI, and the moral hazard costs of longer durations of unemployment which have been repeatedly found in papers such as Ehrenberg and Oaxaca (1976), Meyer (1990), and Chetty (2008).

[2]Acemoglu and Shimer (2000), Wang and Williamson (2002) and Chetty (2008) all argue that welfare gains from increasing UI from the current level are small.

[3]UI typically accounts for less than 1% of total government spending in the U.S.

[4]The interpretation of this phenomenon as an externality is due to the fact that individuals do not internalize the cost of reduced tax revenues when making their job search decisions. Fiscal externalities can also be thought of as an application of the Theory of the Second Best, in which optimal government policy must be considered in the context of a labour market, and in particular a job search decision, that is already distorted by income taxes. When UI lengthens durations of unemployment, this increases an already large distortion, and since the deadweight loss of a distortion is generally convex in the size of the distortion, the welfare costs of UI are amplified.

of total government spending, while the second ignores all government spending other than UI. I simulate the model and solve numerically for the optimal replacement rate in each case, permitting a comparison of the results across the two scenarios.

The second approach is a "sufficient statistics" method that relies on reduced-form elasticities: I solve a simple two-period model as in the seminal work of Baily (1978), deriving an expression for the optimal level of benefits which can be written in terms of a few empirical values, which are therefore sufficient statistics for welfare. I then use a statistical extrapolation of the sufficient statistics, approximating their values at alternative UI replacement rates, to calculate numerical results for the optimal replacement rate.

For my benchmark specification, I find that the effect of incorporating fiscal externalities into the analysis is large. Ignoring non-UI spending, I find the optimal UI replacement rate to be 37% using the structural approach and 46% using the sufficient statistics method, but these both drop to zero once fiscal externalities are recognized.

The benchmark specification uses the standard assumption that the wage distribution is degenerate, so that the only effect of UI is to lengthen unemployment durations. The logic of my analysis suggests that it is also important to consider the potential effects of UI on the wage received on a new job. If more generous UI increases reservation wages, encouraging individuals to find better matches and thereby increasing wages, this would increase the tax base and create offsetting effects to those emphasized above. I therefore extend the earlier analyses to consider this additional margin. While recent studies suggest that this wage effect is small, I nonetheless find that values at the high end of the empirically plausible range have dramatic quantitative effects on the optimal provision of UI: I now find that fiscal externalities can increase optimal replacement rates to 71% using the structural approach and 58% using the sufficient statistics method. A key implication is that better estimates of the effects of UI on re-employment wages are critical to the determination of optimal UI generosity.

The rest of the paper is organized as follows. Section 2 presents the dynamic job search model, and includes a discussion of the calibration and the results. In section 3, I describe a two-period model based on Baily (1978), and solve for the welfare derivative and optimal UI equation and present numerical results. Section 4 extends the analysis to a case in which UI has significant effects on subsequent wages, and section 5 concludes.

2 Calibrated Job Search Model

In this section, my analysis will be based on the model from Lentz (2009), which is a typical single-agent model, incorporating endogenous search intensity and private asset accumulation, as well as being intuitive and straightforward to simulate. My only modifications are the introduction of government spending outside of UI and a simplified functional form for the effort cost of job search. The first subsection describes the model, while the second explains the calibration; I then present the numerical results, and discuss the effects of fiscal externalities on the estimated optimal benefit level.

2.1 Model Setup

The model features a representative infinitely-lived risk-averse agent who makes stochastic transitions between states of employment and unemployment. When unemployed, the agent receives an after-tax UI benefit equal to b, with infinite potential duration,[5] and chooses search intensity s_t subject to a convex search cost function $e(s_t)$, where s_t is the probability of receiving a job offer.[6] All jobs have an identical wage y, and jobs end exogenously at a constant rate of δ per period. Agents receive utility from consumption $U(c_t)$ in each period, and τ will represent the tax rate. Finally, agents cannot borrow, but can make savings which earn interest at a rate of i per period, and face a discount factor of β, where a period represents a week.

In all periods and states, agents decide on their level of consumption, while unemployed agents also choose how hard to search for a job. It is convenient to formulate the worker's optimization problem using recursive methods; therefore, let $V_e(k)$ represent the maximum present value of being employed with assets equal to k, while $V_u(k)$ will be the analogous value of unemployment, and let k' represent next period's assets. The worker's problem can then be written as:

$$V_e(k) = \max_{k' \in \Gamma_{y(1-\tau)}(k)} [U((1+i)k + y(1-\tau) - k') + \beta[(1-\delta)V_e(k') + \delta V_u(k')]]$$

$$V_u(k) = \max_{k' \in \Gamma_b(k), s \geq 0} [U((1+i)k + b - k') - e(s) + \beta[sV_e(k') + (1-s)V_u(k')]]$$

[5] By assuming that benefits are constant and never expire, I keep the analysis simple and focus on only one dimension of the optimal UI problem, namely the optimal level of benefits.

[6] This simplified specification allows for a closed-form solution for the individual's search decision.

where $\Gamma_z(k) = (k' \in \mathbb{R} | 0 \leq k' \leq (1+i)k + z)$ is the set of permissable asset values.[7]

The agent is representative of a continuum of identical agents, and therefore I can consider the economy-wide steady-state, in which the government budget constraint is:[8]

$$(1-u)y\tau = ub + G$$

where u is the unemployment rate and G is the level of exogenous non-UI government spending.[9] The government chooses b and τ subject to this constraint to maximize steady-state expected utility,[10] and I will compare the results when the best estimate of G is used with the results obtained when I assume $G = 0$.

2.2 Calibration of the Model

Calibration requires choosing functional forms and parameter values, in order to be able to simulate the model. For functional forms, I assume constant relative risk-aversion utility with risk-aversion parameter R, so $U(c) = \frac{c^{1-R}}{1-R}$, and I borrow the search cost function from Chetty (2008): $e(s) = \frac{(\theta s)^{1+\kappa}}{1+\kappa}$, until $s = \bar{s}$, beyond which the marginal cost is infinite.[11]

Choosing a baseline value of UI benefits requires that I confront the fact that, while the model features infinite-potential-duration UI for simplicity, not all unemployed individuals actually receive UI, either because their benefits have expired or because they do not take up benefits. Simply assuming that everyone receives UI would imply a much larger size of the UI program than is the case in reality, and the relative size of UI spending and other programs defines the size of fiscal externalities. Therefore, I follow the approach of Fredriksson and Holmlund (2001) in deflating benefits in my model to be equal in expectation to real-world finite-duration benefits. That is, I multiply the replacement rate by 0.8, which is the approximate take-up rate over 1990-2005 found by Ebenstein and Stange (2010),[12] and by

[7] As in Lentz (2009), the numerical solutions always appear to yield concave value functions by asset level.

[8] Assuming a single proportional tax keeps the government budget constraint simple; the basic result will hold with a more complex tax system if taxes paid remain an increasing function of income.

[9] In principle, G could be made endogenous, but this would add considerable complexity while providing little new insight into the main point. If exogenous, G does not need to be accounted for in the individual's utility function.

[10] By maximizing steady-state welfare, I keep with the usual approach in the literature and do not account for transitional dynamics.

[11] Thus, $\bar{s}$ is the maximum feasible search intensity; in the simulations, this upper limit will be binding in very few instances.

[12] This accounts for the fact that the empirical quantities used later are defined for the entire population that is eligible for UI, regardless of whether they take up benefits; Gruber (1997) argues that this is in fact the policy-relevant population, because government can control benefit eligibility but not benefit receipt.

$\frac{15.8}{24.3}$, which is the ratio of mean compensated unemployment duration to mean total duration in the Mathematica sample of Chetty (2008).[13] Therefore, if r is the real-world replacement rate and τ_0 the baseline tax rate, $b = r(1-\tau_0)y(0.8)\left(\frac{15.8}{24.3}\right)$ is the corresponding value of the infinite-duration benefit in my model.[14] An alternative option is to explicitly model benefits that expire after 6 months, and while this is less simple to implement numerically, I show in appendix B.2 that the results are similar.

The selected parameter values are summarized in Table 1. The job separation rate is set to $\delta = \frac{1}{260}$ to correspond with a median job duration of 5 years measured by the Bureau of Labor Statistics in January 2006 for high-school graduates, a group which is a reasonable proxy for UI recipients. For the interest rate i, I follow the example of Hansen and İmrohoroğlu (1992) and Chetty (2008) in setting it to zero.[15] The wage y is normalized to one, and I use $R = 2$, which is a standard value for the coefficient of relative risk-aversion in studies of UI.[16] For the upper limit of search intensity, I use $\bar{s} = 0.5$, which means that it is not possible for an individual to guarantee finding a job immediately.

Table 1: Parameters

Parameter	Definition	Value(s)
δ	job separation rate	$\frac{1}{260}$
i	real interest rate	0
y	per-period wage	1
r_0	baseline replacement rate	0.46
τ_0	baseline tax rate	0.23
R	coefficient of relative risk-aversion	2
$\bar{s}$	maximum search intensity	0.5
θ	search cost parameter	TBD
κ	search cost parameter	TBD
β	discount factor	TBD

I also select initial values of $r_0 = 0.46$ and $\tau_0 = 0.23$; the former is the mean effective replacement rate over 1988-2010 reported by the U.S. Department of Labor, while the latter

[13] Fredriksson and Holmlund (2001) make a slightly different adjustment, finding the average replacement rate for all unemployed individuals whether eligible for UI or not, across both UI and social assistance, accounting for UI benefit exhaustion.

[14] I assume that when r changes, benefits continue to be taxed at the baseline rate.

[15] Chetty (2008) finds that unemployed individuals tend to have little in the way of long-term savings, and Hansen and İmrohoroğlu (1992) argues that previous findings of near-zero average real returns on "highly liquid short-term debt" justify the assumption of a non-interest-bearing asset.

[16] For example, Chetty and Saez (2010) use $R = 2$, and Lentz (2009) estimates a value of 2.21. Several sensitivity analyses in appendix B.1 consider alternative values of R.

incorporates a 15% federal rate of the typical UI recipient, 5% for a typical state income tax, and 3% for the Medicare tax.[17] In the case where I assume $G = 0$, meanwhile, the tax rate is that which pays for UI benefits, and I assume that the payroll tax is paid only by employees, so benefits are untaxed.

The remaining parameters, θ, κ and β, are set to make the model match a set of moments from the real world. The moments used are the unemployment rate u, the percentage gap between average consumption when employed and unemployed $\frac{E(c_e)-E(c_u)}{E(c_e)}$, and the elasticity of the unemployment rate with respect to benefits, which I denote as $E_b^u = \frac{b}{u}\frac{du}{db}$.

The unemployment rate is often used to calibrate job search models, and while all three moments jointly determine the parameter values, u is especially informative about the level of the search cost function, which is primarily determined by θ. E_b^u, meanwhile, is informative about the curvature parameter κ. Finally, the consumption gap is closely related to workers' ability to maintain a buffer stock of assets, so $\frac{E(c_e)-E(c_u)}{E(c_e)}$ primarily identifies the discount factor.[18] These three moments are also related to the sufficient statistics that will be used later in the paper.

The specific values used for the moments are summarized in Table 2, and are as follows. The unemployment rate u is set to 0.054 to match the average unemployment rate among high-school graduates during 1992-2010; combined with r_0 and τ_0, this implicitly defines $G = 0.208$. Gruber (1997) estimates a relationship of $\frac{E(c_e)-E(c_u)}{E(c_e)} = 0.222 - 0.265r$,[19] which implies a value of 0.1001 at baseline.[20] Finally, Chetty (2008) estimates an elasticity of

[17] The employee and employer shares of the Medicare tax add up to 2.9%. 23% is meant to represent a best approximation of a single tax rate applying to earned income and UI. In terms of the tax rate applying to UI, FICA taxes are not applicable to UI benefits, and some state taxes also do not apply to UI benefits, whereas I also ignore the possibility of some individuals being in higher tax brackets, due to their own income or that of a spouse. A marginal earned income tax rate of $\tau_0 = 0.23$, meanwhile, likely represents a conservative estimate, as I ignore the Social Security tax on the grounds that it is more of a pension contribution than a tax, and I also ignore the fact that some UI recipients may be on the downward-sloping part of the EITC, which would significantly increase the marginal tax on earned income.

[18] My decision to set the discount factor to match a moment is unusual, as the standard approach is simply to choose a "reasonable" value; however, there is no definite consensus on the right "reasonable" value. An annual discount rate of around 4-5% is typical, but to take opposite extremes, Acemoglu and Shimer (2000) use an annual rate of nearly 11%, whereas Coles (2008) produces results for a zero discount rate. Lentz (2009) uses a value of 5.1%, but finds that his results are very sensitive to the gap between the interest and discount rates, motivating my attempt to use real-world data to pin down this parameter.

[19] Given the difficulties of obtaining good quality data on consumption across states of employment and unemployment, Gruber's data is on food consumption from the PSID; he estimates the year-to-year drop in consumption for individuals who were employed in year $t-1$ and unemployed in t, which is a reasonable approximation to the consumption gap between average unemployed and employed individuals.

[20] In this model, it is difficult to generate a consumption gap which declines with b, because I consider the steady-state, and in steady-state agents accumulate fewer assets as b increases. The variances of $U(c)$

unemployment durations with respect to benefits of 0.53,[21] though this estimate is based on a sample of UI recipients, whereas the consumption estimates in Gruber (1997) are from a sample of unemployed workers who were initially eligible for UI, regardless of whether they were actually receiving benefits. Therefore, as with the benefit level, I need to adjust for benefit non-receipt, and I follow Gruber's recommendation and multiply the elasticity by 0.48, the derivative of benefit receipt to benefit eligibility in his sample.[22] If the average duration of unemployment is D, this gives $E_b^D = \frac{b}{D}\frac{dD}{db} = 0.2544$, and the fact that $u = \frac{D}{D+\frac{1}{\delta}}$ means that $E_b^u = (1-u)E_b^D = 0.946 \times 0.2544 = 0.2407$.

Table 2: Moments

Parameter	Definition	Value
u	unemployment rate	0.054
$\frac{E(c_e)-E(c_u)}{E(c_e)}$	consumption gap between employment and unemployment	$0.222 - 0.265r_0 = 0.1001$
$E_b^u = \frac{b}{u}\frac{du}{db}$	elasticity of u wrt b	$0.946 \times 0.48 \times 0.53 = 0.2407$

The model will be calibrated twice, once for $G = 0$ and once for the true $G = 0.208$. In this way, given each set of starting assumptions about the size of government, I find the parameters that match the real-world moments, and then find the level of UI that is optimal in each case,[23] allowing me to compare the results with fiscal externalities to those from the usual approach.

2.3 Numerical Results

To numerically solve the model, I use value function iteration, and the parameters used are displayed in Table 3, where I report the annual discount rate ρ instead of the weekly discount factor $\beta = \left(\frac{1}{1+\rho}\right)^{\frac{1}{52}}$; the details of the numerical methods are explained in detail in appendix

and $U'(c)$ do, however, decrease with b. Since my later sufficient statistics analysis uses Gruber's estimated relationship directly in the welfare derivative, including the negative relationship between benefits and the consumption gap, this strengthens my claim to be testing the robustness of my conclusions to different assumptions.

[21] This estimate is close to the middle of the typical range of estimates in the literature; Chetty (2008) describes the usual range of estimates as 0.4 to 0.8, while Fredriksson and Holmlund (2001) claim that their own value of 0.5 is "in the middle range of the available estimates."

[22] 0.48 is also very close to my value of $0.8 \times \frac{15.8}{24.3} \simeq 0.52$ for the fraction of time that initially eligible unemployed individuals receive benefits, a closely related quantity.

[23] This is not a comparative statics exercise; I am studying the effect of fiscal externalities on optimal UI *calculations*, not the impact on optimal UI of increasing the size of government. Governments' fiscal activities have always been much more extensive than just UI, but this has been ignored by the optimal UI literature, and I want to know how much the estimated optimum changes if this fact is no longer ignored.

A.1, which also contains the resulting moments.[24] The results for the optimal replacement rates and estimated welfare gains can be found in Table 4; the column labelled "Welf. Gain" expresses the gain from moving from $r = 0.46$ to the optimum as percentage points of initial spending on UI, whereas the next column, labelled "Diff.", displays the welfare gain from moving between the replacement rate believed to be optimal when $G = 0$ and the "true" optimum.[25]

Table 3: Calibrated Parameters

	$G = 0.208$	$G = 0$
ρ	0.01090	0.01076
θ	32.0	26.3
κ	0.527	0.560

Table 4: Optimal Replacement Rates & Welfare Gains

	r	Welf. Gain	Diff.
$G = 0.208$	0.00	11.81	6.80
$G = 0$	0.37	0.45	6.33

When G is assumed to be zero, the optimal replacement rate is 37%. Previous results from the existing literature which implicitly make this same assumption are summarized in Table 5, which shows that 37% is within the existing range of estimates, though towards the low end. However, a positive value of G dramatically reduces the optimal replacement rate to zero,[26] as the added welfare cost of providing UI completely outweighs the gains from consumption-smoothing. The welfare gain obtained from moving to the optimum is nearly 12% of current spending on UI, and since during 2001-2010, the average size of UI spending

[24] An over-identifying moment can be generated from the asset distribution, by comparing my simulated steady-state distribution to the asset distribution in the SIPP data of Chetty (2008). My model does not contain motives for saving other than self-insurance against unemployment, and I also restrict assets to be non-negative, so I cannot match the long left and right tails of a real-world distribution, but I can consider the median of my distribution. In the four cases I consider, the median level of assets generated by the model amounts to between 40% and 62% of a year's pre-tax labour income. If I assume that half of housing equity can be counted as liquid wealth, I find that the median liquid wealth in Chetty's sample is about 40% of mean annual income; if all of housing equity is counted, then median wealth is about 70% of mean income. Therefore, the centre of the asset distribution is on the right order of magnitude.

[25] I estimate the welfare gain per worker per period, convert this to dollars using a base of mean consumption while unemployed, divide by per-period spending on UI and multiply by 100.

[26] More accurately, the optimum is very close to zero, as close as computationally feasible; an actual zero benefit level, along with a zero interest rate and an upper limit to search intensity below one, would mean that zero consumption in a period occurs with a probability that is bounded away from zero, with expected utility equal to negative infinity.

was \$59 billion, this implies a welfare gain of about \$7 billion per year. Meanwhile, the welfare gain of moving from the $G = 0$ optimum to the $G = 0.208$ optimum is nearly 7% of UI spending.

Table 5: Numerical Results from Structural Studies of Optimal UI

Paper	Optimal Replacement Rate
Hansen and İmrohoroğlu (1992)	0.15^* (with moral hazard)/0.65 (without)
Davidson and Woodbury (1997)	$0.66^*/1.30^{**}$
Hopenhayn and Nicolini (1997)	$> 0.94^*$ (with optimal tax)
Acemoglu and Shimer (2000)	$> 0.4^{**}$
Fredriksson and Holmlund (2001)	$0.38 - 0.42^*$
Wang and Williamson (2002)	$0.24^*/0.56^{**}$
Coles and Masters (2006)	0.76^*
Lentz (2009)	$0.43 - 0.82^*$

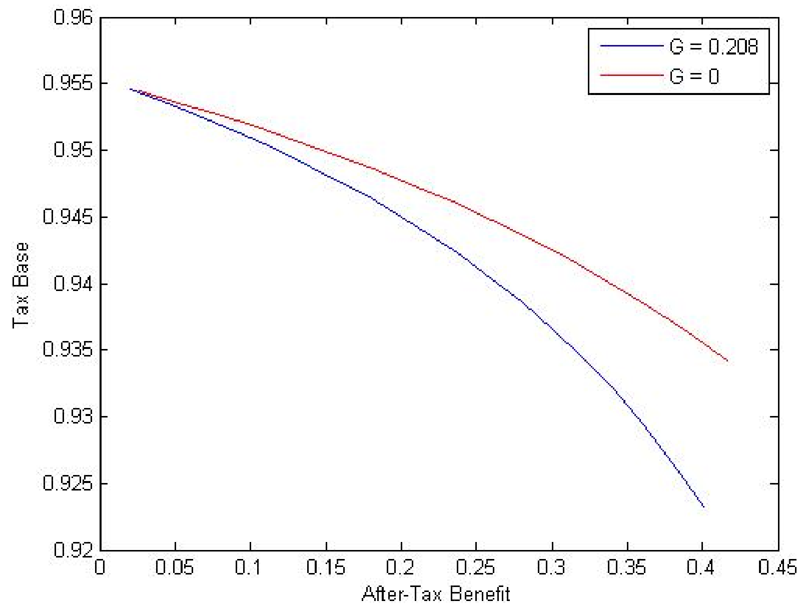
* corresponds to infinite-duration UI, ** to finite-duration (typically 26 weeks)

Figures 1 through 3 illustrate graphically the implications of changing UI generosity. Figure 1 shows how the tax base $1 - u$ varies with the after-tax benefit b (which takes a baseline value of 0.184 when $G = 0.208$ and 0.239 when $G = 0$),[27] while Figure 2 shows total UI spending as a function of b. UI is a small program, one that would still account for a small fraction of overall spending even if benefits were greatly increased, as illustrated in Figure 2. However, UI has an impact on the tax base that is significant, and which is considerably larger when G is large. These results are combined in Figure 3, which shows the effect of UI spending on the budget-balancing tax rate: taxes rise much more rapidly with b in the $G = 0.208$ case than when I abstract from G. Each dollar given to an unemployed person costs more to the government when G is large, and this impact on the tax base and therefore on the equilibrium tax rate is the reason behind the optimal level of UI being reduced to zero when G is large.

I have also performed an extensive series of sensitivity analyses, with results that are displayed in appendix B.1. First of all, I present results with $R = 5$, a value that Chetty (2008) finds to be consistent with large income effects of UI, and I show that although the optimal replacement rates are higher, the basic result is robust to higher risk-aversion: the optimal replacement rate drops significantly from 72% to 46% due to fiscal externalities.

[27] Since b is the after-tax benefit, a given value of b corresponds to a different replacement rate in the $G = 0.208$ and $G = 0$ cases, because an analysis that assumes $G = 0$ ignores income taxes. In this sense, the results in Table 4 understate the effect of fiscal externalities in after-tax UI dollars.

Figure 1: Tax Base $= 1 - u$



I then consider different values of a wide variety of parameters and moments used in the baseline analysis, for both $R = 2$ and $R = 5$, and show that the results are strongly robust to these modifications, with the partial exception of the case with a positive interest rate, which significantly reduces the optimal benefit level, all the way to near zero if $R = 2$ even in the absence of fiscal externalities. In appendix B.2, I have also extended the model to include benefits that expire after 6 months; the results are very similar to the baseline, indicating that my conclusions hold with a more realistic (but less simple) approach to modelling unemployment benefits.

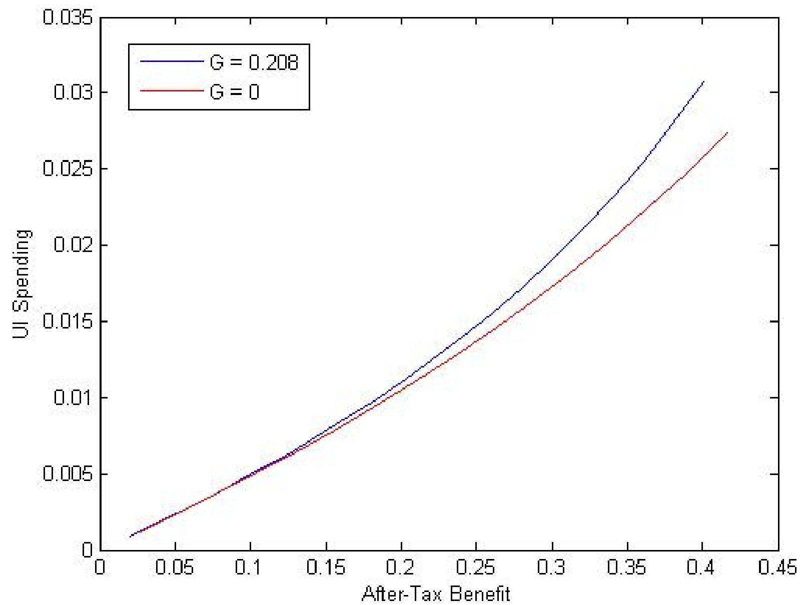
Thus, I can conclude that the overall results of the analysis are robust, simple and striking: the minor modification of a standard search model to include non-UI spending can substantially affect optimal UI calculations, with the baseline finding that the optimal replacement rate drops from around 40% to zero.

3 Sufficient Statistics Approach

The structural approach has numerous strengths, but it is difficult to present a clear demonstration of the mechanisms at work in this framework,[28] and so I now switch my focus to a a

[28]Shimer and Werning (2007) state that structural models "rely heavily on the entire structure of the model and its calibration, which sometimes obscures the economic mechanisms at work and their empirical

Figure 2: UI Spending



simpler and more reduced-form model of unemployment in the sufficient statistics tradition. This approach will permit a more detailed, step-by-step analysis of fiscal externalities in the context of UI, and will generate an analytical solution as an intuitive function of observable empirical quantities. The sufficient statistics approach has been used recently by Chetty (2008) and Shimer and Werning (2007), but I will focus on Martin Baily's (1978) original seminal paper in optimal UI; the formula generated by his two-period model of unemployment is used by Gruber (1997), and Chetty (2006) demonstrates that it applies to a wide range of job search models.

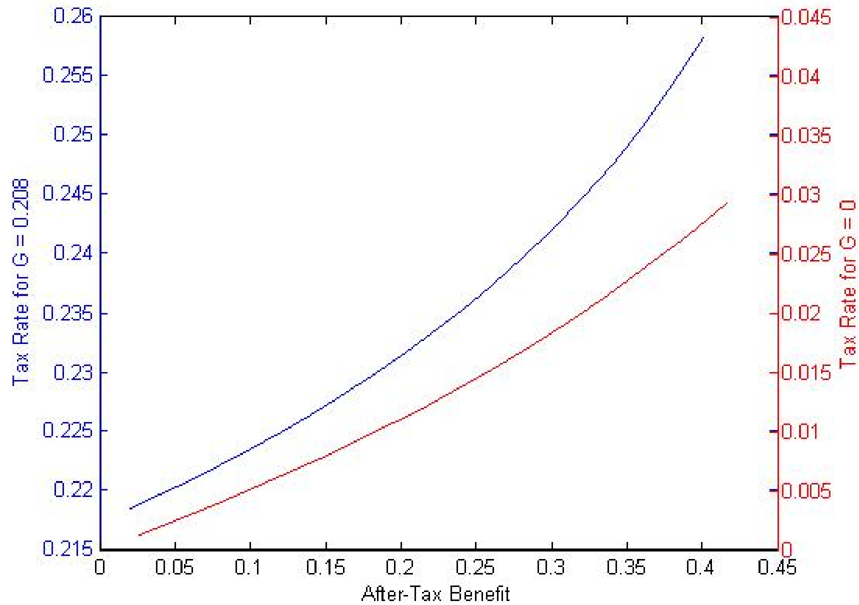
The first subsection presents the model and derives a general version of the optimal benefit equation, while the second explores this equation in further detail and provides the equations needed to perform the numerical analysis, and the third subsection presents numerical results.

3.1 Baily (1978) Model & Optimal Benefit Equation

The only modification that I make to Baily's model is to add G to the government budget constraint; the notation from Baily's paper is also altered to make it more compatible with the notation used earlier. The model is more reduced-form than the structural model earlier,

validity."

Figure 3: Budget-Balancing Tax Rates



but captures many of the same features, and the simplicity of the model makes it well suited to an exposition of the effects of fiscal externalities.

In Baily's model, time is finite and consists of two periods,[29] with the interest and discount rates both set to zero. In the first period, the representative worker is employed at an exogenous wage y,[30] and between periods they face a risk of unemployment: with exogenous probability δ the worker loses their job and becomes unemployed, whereas they keep their initial job at the same wage for the entire second period with probability $1 - \delta$. If the worker becomes unemployed, they choose search effort e (normalized into income units) and a desired wage y_n. They will then spend a fraction $1 - s$ of the second period unemployed and the remaining $s \in (0, 1)$ at a new job at wage y_n,[31] where s is a deterministic function of e and y_n:[32]

$$s = s(e, y_n), \quad \frac{\partial s}{\partial e} > 0, \quad \frac{\partial s}{\partial y_n} < 0.$$

Individuals receive utility from consumption in each period according to the continuous

[29]Baily meant this to represent a two-year time horizon, but the model could also stand for a world with a longer time horizon divided into two halves.

[30]In an extension in appendix I.4, I consider the effect of allowing choice over initial labour supply.

[31]y_n is assumed to be deterministic, such that a worker defines the type of job (i.e. wage level) that they will search for, and will eventually find such a job, with it taking longer to find high-wage jobs.

[32]In appendix I.1, I examine the consequences of a stochastic s.

function $U(c)$, where $U' > 0$ and $U'' < 0$. k represents first-period savings, so overall expected utility is given by:

$$V = U(c_1) + (1-\delta)U(c_e) + \delta U(c_u) \tag{1}$$

where $c_1 = y(1-\tau) - k$, $c_e = y(1-\tau) + k$ and $c_u = (1-s)(b-e) + sy_n(1-\tau) + k$,[33] and τ remains the income tax rate and b the after-tax UI benefit.

The government budget constraint over the two periods is:

$$[(2-\delta)y + \delta s y_n]\tau = \delta(1-s)b + 2G \tag{2}$$

where G again represents per-period exogenous government expenditures.

The next step is to evaluate the derivative of social welfare with respect to UI benefits. From (1), the worker's lifetime expected utility can be written generally as $V = V(e, y_n, k; b, \tau)$; the government sets the values of b and τ, and the worker chooses $\{e, y_n, k\}$ to maximize V taking $\{b, \tau\}$ as given. The government planner has equally-weighted utilitarian preferences, and therefore wants to maximize V at the individual's optimum, choosing b and τ so that the government budget constraint is satisfied in equilibrium. Since the individual chooses $\{e, y_n, k\}$ to maximize V, the partial derivatives with respect to these individual choices are zero: $\frac{\partial V}{\partial e} = \frac{\partial V}{\partial y_n} = \frac{\partial V}{\partial k} = 0$. In other words, for a small change in b, behavioural responses have no first-order effect on welfare, and the envelope theorem implies that the welfare derivative can be written as a function of the two partial derivatives $\frac{\partial V}{\partial b}$ and $\frac{\partial V}{\partial \tau}$ and the derivative of the government budget constraint:

$$\frac{dV}{db} = \frac{\partial V}{\partial b} + \frac{\partial V}{\partial \tau}\frac{d\tau}{db}. \tag{3}$$

Loosely speaking, $\frac{\partial V}{\partial b}$ represents the marginal benefit of increased UI, which is equivalent in utility terms to a marginal increase in consumption while unemployed. The second term, meanwhile, represents the marginal cost in the form of higher taxes, with $\frac{d\tau}{db}$ identifying the size of the tax increase needed to pay for higher benefits and $\frac{\partial V}{\partial \tau}$ the welfare cost of higher taxes in terms of lost consumption. The partial derivatives are:

$$\frac{\partial V}{\partial b} = \delta(1-s)U'(c_u) \tag{4}$$

[33]The assumption that, if the worker loses their job, utility in the second period is defined over total consumption implies no credit constraints within a period: the worker can borrow as much as necessary to smooth consumption during the second period. I consider a relaxation of this assumption in appendix I.2.

$$\frac{\partial V}{\partial \tau} = -yU'(c_1) - (1-\delta)yU'(c_e) - \delta s y_n U'(c_u). \tag{5}$$

Therefore, the welfare derivative is:

$$\frac{dV}{db} = \delta(1-s)U'(c_u) - [yU'(c_1) + (1-\delta)yU'(c_e) + \delta s y_n U'(c_u)]\frac{d\tau}{db}. \tag{6}$$

At the optimal level of UI, the right-hand side of (6) must be equal to zero. As in Baily's analysis, the goal is to express the right-hand side in terms of observable empirical quantities, to provide a simple mapping from these empirical quantities into the optimal value of the replacement rate.

As in Baily, I begin by replacing $U'(c_1)$ and $U'(c_e)$ using the individual's first-order condition for saving and a first-order Taylor series expansion of first-period marginal utility $U'(c_1)$ around $U'(c_u)$, specifically $U'(c_1) = U'(c_u) + \Delta c U''(\theta)$, where $\Delta c = c_1 - c_u$ and θ is between c_u and c_1. As demonstrated in appendix C.1, this allows $\frac{dV}{db}$ to be written in terms of $U'(c_u)$ and $U''(\theta)$:

$$\frac{dV}{db} = -2y\Delta c U''(\theta)\frac{d\tau}{db} - [(2-\delta)y + \delta s y_n]\, U'(c_u)\left[\frac{d\tau}{db} - \omega\right] \tag{7}$$

where $\omega = \frac{\delta(1-s)}{(2-\delta)y+\delta s y_n}$. To further simplify this expression, I follow Baily in making two additional assumptions, which are summarized below.

Assumption 1. *In the analysis in this section, I assume that the wage distribution is degenerate with $y_n = y$.*

Assumption 2. *In the analysis in this section, I assume that $c_1 U''(\theta) = c_u U''(c_u)$.*

Given these two assumptions,[34] and setting $\frac{dV}{db}$ equal to zero, the following proposition characterizes the optimal level of UI benefits.

Proposition 1. *Given assumptions 1 and 2, the equation for the optimal value of b is:*

$$\frac{\Delta c}{c_1} R = (1-u)\frac{\frac{d\tau}{db} - \omega}{\frac{d\tau}{db}}. \tag{8}$$

[34] Assumption 1 replicates the standard assumption of no effects of UI on wages, which I also made in section 2. Assumption 2, meanwhile, is necessary if I am to incorporate the second derivative of utility terms into an observable empirical quantity, specifically a coefficient of relative risk-aversion. Appendix C.1 describes this assumption in greater detail.

where $R = \frac{-c_u U''(c_u)}{U'(c_u)}$ *is the coefficient of relative risk-aversion and* $u = \frac{\delta(1-s)}{2}$ *is the unemployment rate. Equivalently, using elasticities, the optimal UI equation is:*

$$\frac{\Delta c}{c_1} R = (1-u)\frac{E_b^\tau - \psi}{E_b^\tau}. \tag{9}$$

where $E_b^\tau = \frac{b}{\tau}\frac{d\tau}{db}$ *is the elasticity of* τ *with respect to* b*, and* $\psi = \frac{\omega b}{\tau} = \frac{ub}{ub+G}$ *is the fraction of total government expenditures allocated to UI.*

Proof. The proof of this result can be found in appendix C.1. □

These equations state the condition that must hold at the optimum; even though b does not appear explicitly, only a value of b that causes (12) to hold can be optimal.

3.2 Analysis of Optimal Benefit Equation

In order to be able to use either of the equations derived above, I need to evaluate the response of taxes to benefits. I begin by evaluating $\frac{d\tau}{db}$, and Assumption 1 tells us not only that $y_n = y$, but also that $\frac{dy_n}{db} = 0$; therefore, total differentiation of the government budget constraint (2) gives:

$$\frac{d\tau}{db} = \frac{\delta(1-s) - \delta b\frac{ds}{db} - \delta\tau y\frac{ds}{db}}{(2-\delta+\delta s)y}. \tag{10}$$

The three terms in the numerator represent three separate components of the response of taxes to benefits. I will call the first the "mechanical effect": even if there is no behavioural response to UI, if b increases, the tax rate must increase to compensate, and $\frac{\delta(1-s)}{(2-\delta+\delta s)y}$ represents the size of this increase. The second component will be referred to as the "duration effect," and captures the fact that, if higher benefits increase the duration of unemployment, this increases the total amount of benefits received over time, requiring a further tax increase. Finally, I will refer to the third component as the "revenue effect": longer unemployment durations also reduce the amount of taxes paid on labour income, raising the required tax increase still further. Notice that while the magnitude of the duration effect doesn't depend on the size of government, the revenue effect is multiplied by τ; this highlights the importance of the standard assumption that τ is a small payroll tax, rather than a large income tax, as that has led to a significant understatement of the revenue effect of UI.

Writing in terms of elasticities, I can solve for E_b^τ:

$$E_b^\tau = \psi + \left(\psi + \frac{u}{1-u}\right) E_b^D \tag{11}$$

where $E_b^D = \frac{b}{1-s}\frac{d(1-s)}{db}$ is the elasticity of unemployment durations with respect to b. The three components of the tax response are apparent here as well: the first ψ is the mechanical effect, while the ψ and $\frac{u}{1-u}$ multiplying E_b^D represent the duration effect and the revenue effect respectively.

I can now explain (9) as an intuitive representation of the tradeoff between marginal benefits and costs of increased UI (the interpretation of (8) is exactly equivalent). The left-hand side represents the welfare gain from increased UI in the form of consumption-smoothing, which is increasing both in the magnitude of risk-aversion and the consumption shock upon unemployment. To interpret the right-hand side, note that since ψ is exactly equal to what I labelled as the mechanical effect, $\frac{E_b^\tau - \psi}{E_b^\tau}$ is the fraction of the total response of taxes to benefits generated by the duration and revenue effects. Because the mechanical effect represents a lump-sum transfer of income between employed and unemployed states, it is not a cost to society, and so $\frac{E_b^\tau - \psi}{E_b^\tau}$ represents the cost of increased UI in terms of behavioural distortions. This in turn is weighted by $1-u$, which reflects the size of the tax base.

Therefore, the equation for the optimum is:

$$\frac{\Delta c}{c_1}R = (1-u)\frac{\left(\psi + \frac{u}{1-u}\right)E_b^D}{\psi + \left(\psi + \frac{u}{1-u}\right)E_b^D}. \tag{12}$$

This is the equation which I will use to solve for the optimal value of b, and the quantities that appear in (12) are the sufficient statistics.

However, even without a numerical analysis, I can draw some important lessons from (12). First of all, the standard assumption that $G = 0$ means $\psi = 1$; $\frac{u}{1-u}$ is likely to be a small number, so the mechanical and duration effects are large compared to the revenue effect, because the taxes that induce the revenue effect are so small.[35] However, if G is large, ψ will be small, meaning that the revenue component will be at least on the same order of magnitude as the mechanical and duration components.

[35] Baily implicitly assumes $G = 0$, and adds an additional assumption which is equivalent to assuming that $\psi + \left(\psi + \frac{u}{1-u}\right)E_b^D = 1$ in the denominator of (12), and finds that the result simplifies to:

$$\frac{\Delta c}{c_1}R = (1-u)\left(1 + \frac{u}{1-u}\right)E_b^D = E_b^D.$$

The same result is found in Chetty (2006), motivating the conclusion that $\frac{\Delta c}{c_1}$, R and E_b^D are the three sufficient statistics for optimal unemployment insurance.

The significance of a larger revenue component can be shown formally by examining the effect of G on the welfare derivative. In appendix C.2, I perform some simple algebra on a derivative normalized into dollar terms, $\frac{dW}{db} = \frac{\frac{dV}{db}}{U'(c_u)}$, and show that if I assume that $\frac{\Delta c}{c_1}R < 1-u$,[36] then for $G > 0$, $\frac{dW}{db}(b;G) - \frac{dW}{db}(b;0)$ has the same sign as $-E_b^D$. The empirical literature overwhelmingly finds that $E_b^D > 0$, which means that if two researchers use (12) to estimate the baseline welfare derivative, one using $G = 0$ and the other a positive value of G, the latter will necessarily find a less positive welfare gain from increasing b.

The researcher with the positive G will also necessarily find a lower level of optimal UI b^* if they assume strict quasi-concavity of welfare and use the method of statistical extrapolation recommended by Chetty (2009) and previously used by Baily (1978) and Gruber (1997): for each sufficient statistic in the optimal benefit equation, the available data is used to select the best functional form of that statistic with respect to b, allowing for an extrapolation of $\frac{dW}{db}$ out of sample to find the optimum. If a statistical extrapolation is used to find $b^*(0)$, and the same statistical extrapolation is used for $G > 0$, then $\frac{dW}{db}(b^*(0);G)$ takes the same sign as $-E_b^D$, and if that sign is positive, then by strict quasi-concavity $b^*(G) > b^*(0)$.

3.3 Numerical Results

In this subsection, I will numerically evaluate (12) to find the optimal replacement rate r, by selecting baseline values for the sufficient statistics and then using the method of statistical extrapolation mentioned above. Appendix D provides a detailed explanation of the precise method used for extrapolation; Chetty (2009) also provides further discussion, but the basic idea is to select functional forms and parameters for each of the quantities in (12) with respect to r, to provide the best approximation to how each will change as r is changed.

Many of the quantities for the sufficient statistics have already been used earlier in the paper, so the discussion will be kept brief. Starting with the functional form of $\frac{\Delta c}{c_1}$, I use the estimate of $\frac{\Delta c}{c_1} = 0.222 - 0.265r$ from Gruber (1997). I also continue to use $E_b^D = 0.48 \times 0.53 = 0.2544$ and $R = 2$. The baseline value of r is set to 0.46 as before, and I again use an initial unemployment rate of $u_0 = 0.054$. Then, let $u = \phi r^{E_b^D}$, and it is easy to solve for $\phi = 0.0658$, allowing me to extrapolate u out of sample. Finally, at baseline values, $\psi = \frac{ub}{(1-u)\tau y} = \frac{u}{1-u}\frac{1-\tau_0}{\tau}(0.8)\left(\frac{15.8}{24.3}\right)r$, and the baseline tax rate is $\tau_0 = 0.23$, so

[36] It is clear from (5) that $\frac{\partial V}{\partial \tau} < 0$, and then $\frac{\Delta c}{c_1}R < 1-u$ follows immediately if Assumptions 1 and 2 are accurate, as those assumptions imply that $\frac{\partial V}{\partial \tau} = -2yU'(c_u)\left[(1-u) - \frac{\Delta c}{c_1}R\right]$.

$\psi = 0.77\frac{12.64}{24.3}\frac{ur}{(1-u)\tau}$, which equals 0.0457 at baseline. The parameter values are summarized in Table 6.[37]

Table 6: Sufficient Statistics & Parameters

Statistic	Value/Extrapolation
$\frac{\Delta c}{c_1}$	$0.222 - 0.265r$
E_b^D	0.2544
R	2
r_0	0.46
u_0	0.054
ϕ	0.0658
ψ	$0.77\frac{12.64}{24.3}\frac{ur}{(1-u)\tau}$

I am now prepared to solve the non-linear first-order condition (12) defining the optimal replacement rate. I solve for the optimal value of $r \in [0, 2]$,[38] and Table 7 below presents the optimal r for my parameter values, as well as the results when I set $G = 0$. The latter case does not perfectly reproduce Baily's results; to do so, I also need to make an extra assumption made by Baily,[39] in which case the result is $r = 0.3577$.

Table 7: Optimal Replacement Rates & Welfare Gains

	r	Welf. Gain	Diff.
$G = 0.208$	0.0000	46.39	46.34
$G = 0$	0.4595	0.00	11.26

The results for the optimal replacement rate are remarkably similar to those from the structural analysis: once again, fiscal externalities cause the optimal replacement rate to drop to zero, and the drop is more dramatic now, from about 46%. In other words, the usual analysis would indicate that the current average generosity of UI in the US is optimal, but adding fiscal externalities is enough to make abolishing UI the optimal policy. Meanwhile, the estimated welfare gains from the sufficient statistics method tend to be larger than those from the structural model, largely due to differences in models and assumptions about elasticities; the estimated welfare gain of moving from $r = 0.46$ to $r = 0$ of nearly 50% of initial UI

[37] These quantities encode all relevant information from underlying structural parameters, making the welfare equations robust to many modifications and modelling decisions; Chetty (2006) shows that the results from Baily (1978) are applicable to a far more general class of models.

[38] I assume that the government is not interested in extracting payments from unemployed workers, so a zero represents a corner solution.

[39] Specifically, I need to assume that $\psi + \left(\psi + \frac{u}{1-u}\right) E_b^D = 1$ in the denominator of (12).

spending may be implausibly large, as this is due to the assumption that unemployment goes to zero as r approaches zero.[40]

The results, however, confirm that fiscal externalities can alter the nature of the optimal UI problem and significantly change the numerical results; the efficiency costs of UI are larger than previously recognized, and taking that into account can significantly reduce the optimal level of UI. In appendix H, I consider the case of $R = 5$, and show that the results are less dramatic in this case, though fiscal externalities continue to reduce the optimal replacement rate by a significant amount.

4 Results with Effects of UI on Wages

To this point, the analysis has proceeded on the assumption that UI has no effect on the wages received upon finding a new job, and that therefore the only labour market impact of UI is a lengthening of unemployment durations. This has long been the standard assumption in the literature; only Acemoglu and Shimer (2000) consider effects of UI on wages in a welfare analysis of unemployment insurance, and their analysis features a parametrized structural model with a wage distribution containing only two mass points. However, the idea that more generous unemployment benefits should raise reservation wages comes out of many job search models, and the existing empirical literature studying the responsiveness of post-unemployment wages to unemployment benefits is fairly sparse and reports a wide range of results, which are summarized in Table 8.[41]

[40] If, instead of allowing $\frac{dW}{db}$ to become very negative as $r \to 0$, I hold it constant at its $r = 0.288$ value (which is the peak of $\frac{dW}{db}$), which is likely a conservative assumption, the welfare gain drops to about 20% of UI spending.

[41] None of the papers listed report coefficients in the form of an elasticity, so their coefficients have been transformed into approximate elasticities using mean values of wages and benefit levels. Classen (1977) and Holen (1977) do not provide summary statistics, so I use mean values from Burgess and Kingston (1976), who use a smaller version of the dataset used by Holen (1977). Additionally, the estimate listed for Meyer (1989) is from one of 10 individual regressions; the author does not designate a preferred estimate, so the basic difference-in-differences is used. I do not list papers answering questions other than the effect of higher benefit levels on wages; I therefore omit Blau and Robins (1986), who find a moderately large but not significant effect of UI benefits on the wage offer distribution, plus a positive effect of UI on reservation wages, Fitzenberger and Wilke (2007), who perform a Box-Cox quantile regression and do not arrive at a single estimate, and McCall and Chi (2008), whose findings correspond to an initial elasticity of 0.10 which declines over the spell of unemployment. I also omit Gaure, Roed, and Westlie (2008), who find a positive effect of benefit duration on wages, and Lalive (2007) and Schmieder, von Wachter, and Bender (2012), who do not (the latter paper finds a negative effect); Centeno (2004), Centeno and Novo (2006), and Tatsiramos (2009), who find that more generous UI leads to greater subsequent job duration, and Portugal and Addison (2008), who do not. This list of omitted papers, therefore, also leads to an ambiguous conclusion about the effect of UI on job characteristics.

Table 8: Results of Empirical Literature on Benefit Elasticity of Wages

Paper	Approx. Elasticity	95% Confidence Interval
Ehrenberg and Oaxaca (1976)	0.27 for older men	(0.12,0.43)
	0.06 for older women	(0.03,0.09)
	0.04 for young men	(-0.04,0.12)
	0.02 for young women	(-0.06,0.10)
Burgess and Kingston (1976)	0.45	(0.26,0.64)
Classen (1977)	0.03	(-0.16,0.21)
Holen (1977)	0.64	(0.55,0.72)
Meyer (1989)	-0.17	(-1.03,0.69)
Maani (1993)*	0.11	(0.02,0.20)
Addison and Blackburn (2000)	-0.05	(-0.14,0.05)

*Maani (1993) uses data from New Zealand; all other papers in this table use American data.

The more recent literature has tended to indicate small or zero effects of UI on wages; Chetty (2008), in particular, focusses on two recent papers, Card, Chetty, and Weber (2007) and van Ours and Vodopivec (2008), which use natural-experiment methodologies to test for an effect of the potential duration of unemployment benefits on wages, using European data (from Austria and Slovenia respectively), and which find no significant effects.[42] However, since the literature covers a wide range of values, in this section of the paper I will extend both of my approaches to allow for effects of UI benefits on reservation wages and therefore on observed post-unemployment wages. I begin with an extension of the structural model, and then I will consider how wage effects can alter the conclusions from the sufficient statistics approach.

4.1 Structural Model with Wage Effects

Welfare analysis of UI in the presence of a wage distribution has only been attempted once before to my knowledge, in the aforementioned paper by Acemoglu and Shimer (2000); this subsection, therefore, represents the first attempt to do so using a wage distribution covering more than two wages. Job offers now contain a wage y drawn from a distribution $F(y)$, and an unemployed worker receiving such an offer decides whether to accept or to remain unemployed. Denoting the reservation wage by $\bar{y}$, the individual's recursive decision

[42] These findings are at least suggestive, but may not be definitive in a North American context, given the different labour market structures and institutions found in Europe, such as higher union coverage, as acknowledged by Card, Chetty, and Weber (2007).

problem is:

$$V_e(k,y) = \max_{k' \in \Gamma_{y(1-\tau)}(k)} [U((1+i)k + y(1-\tau) - k') + \beta[(1-\delta)V_e(k',y) + \delta V_u(k')]]$$

$$V_u(k) = \max_{k' \in \Gamma_b(k), \bar{y}, s \geq 0} \Big[U((1+i)k + b - k') - e(s) + \beta[s\tilde{V_e}(k',\bar{y}) + (1 - s(1-F(\bar{y})))V_u(k')]\Big]$$

where $\tilde{V_e}(k',\bar{y}) = \int_{y \geq \bar{y}} V_e(k,y)dF(y)$.

To calibrate the model, I now allow for a constant in the search cost function: $e(s) = d + \frac{(\theta s)^{1+\kappa}}{1+\kappa}$, where d can be thought of as direct disutility from being unemployed; this is necessary in order to obtain the desired order of magnitude for the effect of UI on wages. Meanwhile, the wage is defined as $y = \underline{y} + y_{LN}$, where $\underline{y}$ is a constant and $y_{LN} \sim \ln N(\mu, \sigma^2)$; for the purpose of simulations, a discretized approximation is used with intervals of 0.002 over a central portion of the distribution and mass points at each end containing the remainder of the mass, at the mean value for said mass. The parameters are set to match the previous moments, as well as a mean wage of 1 at baseline and a wage elasticity $\frac{d\ln(E(y))}{d\ln(b)} = 0.02$. To put this in the context of the estimates in Table 8, recall the two-period model of section 3: if $s = 0.8$ and so individuals who lose their jobs spend 20% of the second period unemployed, this would correspond to a post-unemployment wage elasticity of 0.0926, and dividing by 0.48, the equivalent elasticity among individuals who are receiving UI benefits would be 0.1929, a number that is roughly in the middle of the estimates in Table 8. The parameters and moments can be found in Tables 12 and 13 in appendix A.2, and the numerical results are in Table 9.

Table 9: Optimal Replacement Rates & Welfare Gains with Wage Effects

	r	Welf. Gain	Diff.
$G = 0.208$	0.71	4.27	1.65
$G = 0$	0.55	0.48	1.64

Allowing for this positive effect on wages leads to dramatically different results from those observed in Table 4; while the optimal replacement rates are higher in both cases, this is especially the case for $G = 0.208$, sufficiently so that fiscal externalities actually lead to an increase in the optimal benefit level. Therefore, if UI benefits increase reservation wages and thus subsequent wages by an amount that is conceivable given the existing empirical estimates, the resulting positive effect on income tax revenues can overturn the earlier conclusion of lower optimal UI. A similarly dramatic set of results can be found in appendix B.3

for the case in which $R = 5$. This indicates that the results are sensitive to this parameter and that further empirical work would be beneficial in determining whether or not we should in fact be ignoring this mechanism.

4.2 Sufficient Statistics with Wage Effects

Compared to the structural approach, accounting for effects of UI on wages is easy in the sufficient statistics approach. I begin by returning to (7), and while I maintain Assumption 2, I replace Assumption 1 with 1.A:

Assumption 1.A. *In the analysis in this subsection, I assume that y_n comes from a non-degenerate distribution, but that in equilibrium y_n will be approximately equal to y.*

This assumption states that, although y_n is chosen by the worker from a non-degenerate distribution, the value that is chosen in equilibrium is not very different from y. In general, the optimal level of UI depends both on the level of y_n (as that determines the weight placed on income lost from unemployment) and the elasticity of y_n with respect to UI benefits; Assumption 1.A allows the elasticity to impact the results, as I do not assume $\frac{dy_n}{db} = 0$, but fixes the level of y_n by assuming it to be equal to y. As in the case of Assumption 2, this can only be an approximation, since the worker is now allowed to choose y_n. Assumption 1.A then allows me to simplify the expression from (7) to (9) as before, and I find the new expression for the derivative of the government budget constraint:

$$\frac{d\tau}{db} = \frac{\delta(1-s) - \delta b\frac{ds}{db} - \delta\tau y\frac{ds}{db} - \delta s\tau\frac{dy_n}{db}}{(2-\delta+\delta s)y}$$

where the fourth term in the numerator is a second "revenue effect" capturing the gain in tax revenues if higher UI increases y_n, reducing the tax increase needed to finance higher benefits. Equations (11) is now:

$$E_b^\tau = \psi + \left(\psi + \frac{u}{1-u}\right) E_b^D - \frac{\delta s}{2(1-u)} E_b^y \tag{13}$$

where $E_b^y = \frac{b}{y_n}\frac{dy_n}{db}$ is the elasticity of post-unemployment wages y_n with respect to b, and so the equation for the optimum is:

$$\frac{\Delta c}{c_1} R = (1-u)\frac{\left(\psi + \frac{u}{1-u}\right) E_b^D - \frac{\delta s}{2(1-u)} E_b^y}{\psi + \left(\psi + \frac{u}{1-u}\right) E_b^D - \frac{\delta s}{2(1-u)} E_b^y}. \tag{14}$$

This equation can be utilized to provide numerical results as before, except that there are three new statistics to take into account: E_b^y, s and δ. I attempt to capture the wide range of values in Table 8 by using $E_b^y = 0.48 \times \{-0.17, 0, 0.1, 0.2, 0.4, 0.64\}$.[43] Meanwhile, the starting value s_0 depends on the way the structure of the model is interpreted. If the two periods are taken literally to represent two years, then the finding of Chetty (2008) that the mean unemployment duration in his sample is 18.3 weeks implies an estimate of $s_0 = \frac{52-18.3}{52} = 0.648$. If, however, the model represents a larger portion of an individual's working life, perhaps its entirety, then the fact that Farber (1999) finds that 20.9% of workers aged 45-64 had at least 20 years of tenure in 1996 can be interpreted to mean that $\delta = 0.791$, so $s_0 = 1 - \frac{2u_0}{\delta} = 0.863$. To cover this range of possibilities, I use the set of values given by $s_0 = \{0.648, 0.725, 0.8, 0.863\}$. Then, in each case, the definition $u_0 = \frac{\delta(1-s_0)}{2}$ implies a fixed value of δ.

Table 10 presents the optimal r for my parameter values, as well as the results when I set $G = 0$, and I report a numerical check on the second-order conditions in appendix E.

Table 10: Optimal Replacement Rates Calculated from (14)

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.4500	0.4459	0.4390	0.4275
	0	0.4595	0.4595	0.4595	0.4595
E_b^y	0.048	0.4651	0.4675	0.4717	0.4789
	0.096	0.4707	0.4756	0.4842	0.4987
	0.192	0.4821	0.4921	0.5095	0.5399
	0.3072	0.4959	0.5122	0.5410	0.5922

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0	0	0	0
	0	0	0	0	0
E_b^y	0.048	0.0301	0.2633	0.3933	0.5237
	0.096	0.3737	0.4711	0.5959	0.7591
	0.192	0.5650	0.6835	0.8514	1.0841
	0.3072	0.7152	0.8638	1.0789	1.3788

The difference between the $G = 0$ and $G = 0.208$ cases is substantial; as already seen earlier, for low values of E_b^y, fiscal externalities from the income tax cause the optimal replacement rate to drop to zero, but now we see that for higher values of E_b^y, the replacement rate increases significantly, perhaps even above one. For comparison to the case studied with the structural model in the previous subsection, I find that the optimal replacement rate with $s_0 = 0.8$ and $E_b^y = 0.0926$ is 0.5845. In appendix H, I report results for $R = 5$ as well, and show that although the effect of fiscal externalities is less dramatic, the optimal replacement rates spread out as seen above.

[43] Since estimates of E_b^y are based on samples of UI recipients, I once again multiply by 0.48.

For comparison with results from Chetty (2008), who only reports the baseline value of the welfare derivative, I also report the baseline values of $\frac{dW}{db}$ in appendix F. A direct comparison to that paper, however, is limited by the fact that the models are different, as well as by the different ways marginal welfare is normalized into dollars; Chetty divides by marginal utility when re-employed, while I divide by $U'(c_u)$. The results are qualitatively similar to those in the tables above, as values of $\frac{dW}{db}$ cluster around zero when $G = 0$ but range from -0.08 to 0.18 when $G > 0$.

I also examine equation (14) and the underlying equation for $\frac{dW}{db}$ analytically in appendix G, where I produce a series of results which are more general than the specific numerical estimates above. To summarize the main result, I show that fiscal externalities increase the welfare derivative and b^* if and only if $\frac{d(sy_n)}{db} > 0$, and since s and y_n are the only non-exogenous components of total earnings, this means that optimal UI increases when G is accounted for if and only if higher UI leads to higher total income and thus a larger tax base. I also show that higher E_b^y increases both $\frac{dW}{db}$ and b^*, and that E_b^y has a larger impact on the welfare derivative when G is larger; in my final result, I show that $b^*(G) - b^*(0)$ follows a single crossing property in E_b^y, so that $b^*(G) < b^*(0)$ when E_b^y is small and vice-versa when E_b^y is large.

The robustness of the numerical results are also examined through the use of a number of sensitivity analyses, which can be found in appendix H. First, I try alternative values of E_b^D, specifically, $0.48 \times \{0.3, 0.8\}$, and unsurprisingly the optimal replacement rates move up in the former case and down in the latter; the effects of fiscal externalities remain significant in both cases.[44] I then try two alternative values of G, 0.131 and 0.275, and unsurprisingly the effects of fiscal externalities are less severe in the former case and more so in the latter; zero optimal UI remains for $E_b^y < 0$ even in the low G case. I also try three other sensitivity analyses: I consider complete take-up of benefits, I use a larger value of the initial unemployment rate, specifically $u_0 = 0.064$, and I allow for different tax rates applying to UI benefits and earned income, specifically 0.15 for the former and 0.262 for the latter. The first two of these changes reduce the size of the difference between optimal replacement rates with $G = 0$ and $G > 0$, but I still obtain zeros for $E_b^y = 0$, and the analysis with alternative tax rates results in slightly increased effects of fiscal externalities.

[44] $E_b^D = 0.144$ is small enough to generate a positive local maximum for $R = 2$ and $E_b^y = 0$; to obtain a positive global maximum, I need E_b^D to drop below 0.096.

Finally, given the simplicity of the Baily model, I also perform a number of extensions to the model, which can be found in appendix I. I allow for stochastic duration of unemployment, and restrictions on borrowing during unemployment, which both tend to move the optimal replacement rate closer to one, and I use a second-order Taylor series expansion of marginal utility, and allow for variable labour supply on the initial job, which both reduce the optimal benefit level. However, although the numerical results do change somewhat, the results are still quite similar, and the qualitative conclusions are unchanged: the pairwise comparisons of optimal replacement rates given the two values of G under consideration are nearly identical in each case.

Therefore, the analysis of this subsection confirms the findings from the structural analysis: although the optimal replacement rate is zero under standard assumptions, it is highly sensitive to the effect of UI on wages over the empirically plausible range, and better estimates of the wage effects of UI are therefore critical to the determination of the optimal generosity of UI.

5 Conclusion

The optimal UI literature has explored many of the aspects of the design and generosity of unemployment insurance systems, but there has not yet been any effort to account for the role of fiscal externalities, and I have demonstrated in this paper that this is an important omission. My results demonstrate how substantial an impact fiscal externalities resulting from income taxes can have on optimal UI calculations, while also indicating the previously unrecognized importance of parameters such as the elasticity of post-unemployment wages with respect to UI benefits.

I present results from both of the main approaches in the existing optimal UI literature, specifically the macro-based structural approach and the sufficient statistics method, and I consider a wide range of parameter values. The baseline results from both approaches, using the most typical set of parameters including a zero elasticity of wages with respect to benefits, feature an optimal replacement rate of zero. However, a value of E_b^y that is on the upper end of recent estimates would be sufficient to offset the negative fiscal effects of increased durations of unemployment and increase the estimated optimal replacement rate.

The baseline results indicate that the efficiency costs of UI are likely to be more severe than has previously been recognized, but given the remaining uncertainty about the appropriate parameter values, specifically the effect of UI on wages and the coefficient of relative risk-aversion, it would be premature to conclude that UI should be abolished; further empirical work is needed.

This paper also raises a number of new questions, about how past work on UI policy over the business cycle may be affected by fiscal externalities, and about the role of active labour market programs in reducing durations of unemployment and facilitating better matches. One lesson of this paper is that relatively small improvements in labour market efficiency can provide significant benefits when the labour market is already highly distorted, suggesting that the benefits provided by labour market programs considered in Card, Kluve, and Weber (2010) might be larger than previously realized.

Finally, the insights in this paper can also be generalized into other areas of government policy. In Lawson (2013a), I apply to post-secondary education an analysis that is similar to the current paper. Meanwhile, substantial literatures examine the effects of social insurance programs on labour market outcomes, not just unemployment insurance but also disability insurance (see the employment disincentive effects documented in Bound (1989), Gruber (2000), and Chen and van der Klaauw (2008), for example) and old age security (e.g. Blau (1994), Rust and Phelan (1997), and Coile and Gruber (2007)). Government policies regarding health insurance have also been shown to affect incentives for retirement, job transitions, and entrepreneurship, as documented by Gruber and Madrian (1994), Boyle and Lahey (2010), and Fairlie, Kapur, and Gates (2011). However, as in the area of UI, welfare analyses of these programs typically abstract away from other roles of government (for example, Feldstein (1985) and İmrohoroğlu, İmrohoroğlu, and Joines (1995) on social security, Golosov and Tsyvinski (2006) on disability insurance, and the analysis of public health insurance in Chetty and Saez (2010)), thus ignoring fiscal externalities and leaving room for future work which addresses their consequences.[45] Additionally, the "Second Best" character of the fiscal externality problem should motivate us to consider more carefully the

[45] Parry and Oates (2000) recognize the importance of interactions between environmental policies and the tax system, and argue that this will apply to other programs and institutions that raise the cost of living, but they do not consider any of the programs mentioned above, restricting their discussion to areas of trade, agriculture, occupational licensing and monopolies. Some studies of the programs I discuss do take income taxes that pay for other spending into account, including İmrohoroğlu, İmrohoroğlu, and Joines (2003) and Laitner and Silverman (2012) in the area of social security, and Gruber (1996) and Bound, Cullen, Nichols, and Schmidt (2004) on disability insurance, but even in these cases, there is no mention of the importance of this component or the fact that including it represents an important departure from the rest of the literature.

second-best nature of other aspects of optimal social program design, and in Lawson (2013b) I examine how the welfare analysis of state-contingent transfer programs is complicated not only by fiscal externalities but also by substitution of individuals between various programs.

A Technical Appendix for Structural Calibration

A.1 Baseline Case without Wage Effects

To numerically solve the model for any given set of parameters and a value of b, I begin by making a guess for the tax rate and performing value function iteration: an initial guess is chosen for the value functions, and the maximization problem is solved for a range of asset values, which then provides a new guess for the value functions. This process is repeated until the value functions converge, and I only evaluate the maximization problem for a subset of the asset value grid on each iteration, and then use cubic spline interpolation to fill in intermediate points of the value functions, as done by Lentz. Next, the transition process of agents between states is iterated to calculate the steady-state distribution. I then evaluate the government budget surplus, and then re-set the tax rate and repeat the above steps until the budget is balanced, except in the baseline where I know the tax rate is $\tau_0 = 0.23$.

In order to calculate E_b^u, the model must be solved at baseline, and again for a different level of b; since the numerical procedure involves discretizing the asset distribution, the results are slightly "lumpy" at high magnification, so I use a replacement rate of $r = 0.56$, and compute the resulting arc elasticity.[46] With both sets of numerical results in hand, I then estimate the moments of interest in the simulated data and compare them to their real-world counterparts. Due to the "lumpiness" of the results, a precise numerical search for the minimum-distance parameters is not feasible; instead, I find values for the parameters that match the moments as closely as is practical. Finally, in each case, once the parameters have been calibrated, I perform a grid search over r to find the optimal level.

Table 11 displays the moments calculated from the simulated data. In each case, the upper limit of the asset distribution was chosen so as to not be binding for all relevant cases, while the number of knots in the cubic spline was chosen so that increasing it further made no difference to the results. The spacing of the asset distribution was set at 0.005; if average UI benefits are about \$300 per week, then a 46% replacement rate implies weekly wages of about \$650, and the asset distribution spacing corresponds to about \$3.25. Tests were made of all convergence parameters to ensure that further tightening had no non-negligible effect on results.

Table 11: Calculated Moments

	$G = 0.208$	$G = 0$
u	0.0539	0.0542
E_b^u	0.2405	0.2405
$\frac{E(c_e)-E(c_u)}{E(c_e)}$	0.1001	0.0998

A.2 Parameters and Moments with Wage Effects

Tables 12 and 13 display the parameters and moments when there are effects of UI on wages for $R = 2$.

[46]This variation is comparable to that studied in the empirical literature; for example, Addison and Blackburn (2000) estimate a mean replacement rate of 0.44 with a standard deviation of 0.12 in their data.

Table 12: Calibrated Parameters with Wage Distribution

	$G = 0.208$	$G = 0$
ρ	0.01148	0.01147
θ	9.94	9.335
κ	2.94	2.96
d	0.72	0.575
$\underline{y}$	0.8041	0.80335
μ	-2.344	-2.337
σ	0.776	0.774

Table 13: Calculated Moments with Wage Distribution

	$G = 0.208$	$G = 0$
u	0.0541	0.0540
E_b^u	0.2392	0.2397
$\frac{E(c_e)-E(c_u)}{E(c_e)}$	0.1001	0.1001
E_b^y	0.0199	0.0200
$E(w)$	1.000	1.001

B Sensitivity Analyses and Extension of Structural Model

B.1 Sensitivity Analyses

I have performed a wide range of sensitivity analyses, to examine how the results change when the parameters or moments used in calibration are altered. To begin with, as Chetty (2008) states that his results imply a value of about $R = 5$ in the context of unemployment,[47] I have done the calculations again using that value of risk-aversion. The parameters used can be found in Table 14, the resulting moments are in Table 15, and the results are displayed in Table 16. With more risk-averse individuals, optimal UI is more generous, but the effect of fiscal externalities remains dramatic, with the optimal replacement rate dropping from 72% to 46%.

Table 14: Calibrated Parameters with $R = 5$

	$G = 0.208$	$G = 0$
ρ	0.0485	0.0473
θ	92.0	40.3
κ	0.395	0.465

The rest of the sensitivity analyses have been performed for both $R = 2$ and $R = 5$, and the results for each of them can be found in Table 17; the complete sets of resulting parameters and moments are available upon request.

It is striking how robust the baseline results are to the changes considered. A larger job separation rate and a larger unemployment rate both leave the results essentially unchanged, as does a modification of the tax system to allow for a tax rate of 0.15 applying to UI benefits and a rate of 0.262 for earned income.[48]

[47] Chetty (2006) argues that such a parameter must be chosen to be consistent with the context in which it is being considered, and that "empirical studies that have identified large income effects on labor supply for the unemployed" are inconsistent with low values of R.

[48] Since Medicare benefits and some state taxes do not apply to UI benefits, I use a conservative estimate of

Table 15: Calculated Moments with $R = 5$

	$G = 0.208$	$G = 0$
u	0.0538	0.0541
E_b^u	0.2408	0.2407
$\frac{E(c_e)-E(c_u)}{E(c_e)}$	0.1001	0.0999

Table 16: Optimal Replacement Rates & Welfare Gains with $R = 5$

	r	Welf. Gain	Diff.
$G = 0.208$	0.46	0.00	6.08
$G = 0$	0.72	5.53	5.53

Allowing for perfect take-up or utility from leisure when unemployed modestly reduces the effect of fiscal externalities, as does matching 0.1001 to the drop in consumption at the moment of job loss rather than the average consumption gap, though the latter also leads to significantly higher optimal benefit levels; however, in each of these cases the effect of fiscal externalities remains substantial.

The only case in which the results are dramatically altered is when a positive interest rate is used, as this reduces optimal replacement rates, perhaps so much as to eliminate the gap in optimal policy in the $R = 2$ case, but even then the welfare gain is much larger when fiscal externalities are considered, suggesting that the zero lower bound is "more binding" in that case. Allowing for lower and higher values of E_b^u also shifts the results, unsurprisingly leading to higher and lower optimal replacement rates respectively, but the effect of fiscal externalities remains significant; the same is true when lower and higher values of G are used. Finally, intermediate values of R lead to results in between those for $R = 2$ and $R = 5$, and it appears that a risk-aversion coefficient just above two is sufficient to eliminate the zero-optimal-UI result.

B.2 Extension to Finite-Duration Benefits

I now attempt to model more realistically the finite duration of UI benefits; specifically, a period now represents a month rather than a week, and benefits expire after 6 months. I assume that individuals receive outside income of 0.1 per period, to ensure that consumption never reaches zero in the uninsured state. I ignore the question of take-up and simply define the benefit level for an individual receiving benefits as $b = r(1 - \tau_0)$; this will tend to bias downwards the importance of fiscal externalities, and means that my results may be more comparable to those with perfect takeup in Table 17. I also use a different functional form for the effort cost of search, specifically $e_t(s) = d - \frac{\theta_t}{1-\kappa}(1-s)^{1-\kappa} - \theta_t\left(s - \frac{1}{1-\kappa}\right)$. To capture duration dependency and ensure that a reasonable proportion of unemployment spells are of long duration and involve exhaustion of benefits, I allow θ_t to increase with time t spent out of work according to experimental results in Kroft, Lange, and Notowidigdo (2012);[49] for the same reason, I also set $\delta = \frac{1}{84}$, corresponding to an expected job duration of 7 years rather than 5. Parameters and moments are in Tables 18 and 19, and the numerical results are in Table 20.

This more realistic modelling choice makes relatively little difference to the results. The results with $R = 2$ are quite similar to those from the baseline, except that the optimal replacement rate in the $G = 0$

0.15 as the tax rate on UI. Meanwhile, Cushing (2005) uses the Social Security Administration's projections of future mortality rates to compute estimated marginal OASDI tax rates, and finds a rate of about 3.2% for 37-year-olds, which is the mean age of individuals in the SIPP sample of Chetty (2008), implying a total tax rate of 0.262 on earned income.

[49] Kroft, Lange, and Notowidigdo (2012) find that the interview-finding rate drops from about 7% to about 4% over the first six months of an unemployment spell, then remains roughly constant. Given a θ_1 and a target search intensity s_1, I therefore find the θ_t for $t \in \{2, 3, 4, 5, 6, 7\}$ (where 7 represents all periods of benefit exhaustion) that generates the same effort cost for $s_t = s_1 - (t-1)s_1/14$.

Table 17: Optimal Replacement Rates & Welfare Gains

		$R=2$			$R=5$		
		r	Welf. Gain	Diff.	r	Welf. Gain	Diff.
$(1+i)^{52}=0.03$	$G=0.208$	0.01	41.59	0.00	0.00	11.42	7.64
	$G=0$	0.01	24.17	0.00	0.36	0.26	1.36
$\delta=\frac{1}{364}$	$G=0.208$	0.00	11.90	7.27	0.46	0.00	6.72
	$G=0$	0.38	0.44	6.44	0.73	5.54	5.54
$t_b=0.15, t_y=0.262$	$G=0.237$	0.00	12.76	7.49	0.44	0.11	8.47
	$G=0$	0.37	0.45	6.43	0.72	5.53	6.28
$E_b^u=0.1362$	$G=0.208$	0.12	4.60	3.98	0.67	2.06	3.91
	$G=0$	0.46	0.02	4.97	0.89	10.31	3.23
$E_b^u=0.3633$	$G=0.208$	0.00	19.90	8.96	0.30	2.55	9.07
	$G=0$	0.32	1.64	6.01	0.59	1.82	7.11
perfect take-up	$G=0.205$	0.04	9.09	3.75	0.48	0.03	3.32
	$G=0$	0.34	0.80	4.37	0.66	3.43	2.84
$u=0.07$	$G=0.201$	0.01	11.70	6.78	0.42	0.19	6.71
	$G=0$	0.37	0.54	6.16	0.70	4.41	5.74
consumption drop	$G=0.208$	0.30	2.25	6.30	0.70	4.65	6.50
	$G=0$	0.59	1.13	5.01	0.90	18.05	4.83
utility from leisure	$G=0.208$	0.05	9.91	5.96	0.48	0.09	2.80
	$G=0$	0.39	0.35	6.20	0.65	3.57	2.90
different values of G	$G=0.131$	0.15	5.82	2.43	0.58	1.01	1.92
	$G=0.275$	0.00	19.04	12.24	0.35	1.09	12.64
		$R=2.5$			$R=3$		
different risk-aversion	$G=0.208$	0.09	7.82	6.98	0.18	4.62	7.00
	$G=0$	0.45	0.01	6.13	0.52	0.24	5.95

Table 18: Calibrated Parameters with Finite-Duration Benefits

	$R=2$		$R=5$	
	$G=0.203$	$G=0$	$G=0.207$	$G=0$
ρ	0.0092	0.00915	0.0951	0.0918
θ_1	10.45	8.90	0.789	0.3705
κ	1.1	1.1	3.7	3.7
d	-0.708	-0.576	-2.266	-0.94346

case is lower, because perfect take-up is assumed here (a 0.28 drop in optimal UI due to fiscal externalities compares very closely to the 0.30 drop in the perfect take-up case in Table 17). Meanwhile, the results with $R=5$ are even more dramatic, primarily because I allow for outside income of 0.1, so that the consumption-smoothing effects at lower levels of UI are less dramatic. Thus, given the qualitative similarity of my results to those with deflated infinite-duration benefits, I conclude that my results are robust to alternative specifications of UI benefits.

B.3 Results with Wage Effects and $R=5$

In this subsection, I present the optimal UI results when unemployment benefits have a positive effect on wages for $R=5$. Tables 21 and 22 display the calibrated parameters and moments, and Table 23 presents the optimal replacement rates. As before, allowing for a positive effect on wages significantly alters the

Table 19: Calculated Moments with Finite-Duration Benefits

	$R=2$		$R=5$	
	$G=0.203$	$G=0$	$G=0.207$	$G=0$
u	0.0538	0.0540	0.0539	0.0541
E_b^u	0.2410	0.2403	0.2411	0.2407
$\frac{E(c_e)-E(c_u)}{E(c_e)}$	0.1002	0.1002	0.0999	0.1000

Table 20: Optimal Replacement Rates & Welfare Gains with Finite-Duration Benefits

	$R=2$			$R=5$		
	r	Welf. Gain	Diff.	r	Welf. Gain	Diff.
$G=0.203/0.207$	0.00	10.62	3.84	0.25	1.09	9.41
$G=0$	0.28	1.39	2.69	0.72	3.11	7.58

results, with much higher optimal replacement rates of over 100%,[50] and a slight increase in optimal UI due to fiscal externalities.

Table 21: Calibrated Parameters with Wage Distribution and $R=5$

	$G=0.208$	$G=0$
ρ	0.0525	0.0534
θ	14.54	11.60
κ	2.31	2.04
d	0.98	0.39
$\underline{y}$	0.5475	0.500
μ	-1.101	-0.968
σ	0.3823	0.3532

C Proofs and Algebra

C.1 Proof of Proposition 1

The individual's first-order condition for saving is:

$$\frac{\partial V}{\partial k} = -U'(c_1) + (1-\delta)U'(c_e) + \delta U'(c_u) = 0$$

and I also use a first-order Taylor series expansion of $U'(c_1)$ around $U'(c_u)$:

$$U'(c_1) = U'(c_u) + \Delta c U''(\theta)$$

where θ is between c_u and c_1, and $\Delta c = c_1 - c_u$. Combining these allows me to rewrite (6) as:

$$\frac{dV}{db} = -2y\Delta c U''(\theta)\frac{d\tau}{db} - \left[(2-\delta)y + \delta s y_n\right] U'(c_u) \left[\frac{d\tau}{db} - \omega\right]$$

where $\omega = \frac{\delta(1-s)}{(2-\delta)y+\delta s y_n}$.

[50]The replacement rate applies to a real-world finite-duration benefit, so this does not correspond to a universal benefit that is always higher than wages.

Table 22: Calculated Moments with Wage Distribution and $R = 5$

	$G = 0.208$	$G = 0$
u	0.0540	0.0542
E_b^u	0.2418	0.2398
$\frac{E(c_e)-E(c_u)}{E(c_e)}$	0.1002	0.1002
E_b^y	0.0201	0.0196
$E(w)$	1.002	1.004

Table 23: Optimal Replacement Rates & Welfare Gains with Wage Effects and $R = 5$

	r	Welf. Gain	Diff.
$G = 0.208$	1.08	51.98	0.06
$G = 0$	1.07	41.05	0.29

Next, I make two assumptions that are also found in Baily (1978); they are listed in subsection 3.1 as Assumptions 1 and 2. The first is that the wage distribution is degenerate with $y_n = y$, so all wages can be written in terms of y. The second assumption is that $c_1 U''(\theta) = c_u U''(c_u)$, which permits the second derivative of utility to be incorporated into a coefficient of relative risk-aversion. The validity of this assumption depends on the functional form of utility and on the magnitude of risk-aversion; in general it can only be an approximation. If I assume constant relative risk-aversion, then for my baseline risk-aversion coefficient of 2, this assumption will tend to overstate the consumption smoothing benefit implied by $U''(\theta)$ (that is, $-c_1 U''(\theta) < -c_u U''(c_u)$), and therefore the estimated optimal replacement rate will be too high. However, simulations (available upon request) confirm that it is a much more accurate assumption than the usual assumption of $U'(c_1) = U'(c_u) + \Delta c U''(c_u)$.

Combining these two assumptions, and dividing by $U'(c_u)$ to put the welfare derivative in dollar terms, I find:

$$\frac{dW}{db} \equiv \frac{\frac{dV}{db}}{U'(c_u)} = 2y\frac{\Delta c}{c_1} R \frac{d\tau}{db} - 2(1-u)y\left[\frac{d\tau}{db} - \omega\right] \tag{15}$$

where $R = \frac{-c_u U''(c_u)}{U'(c_u)}$ is the coefficient of relative risk-aversion, and $u = \frac{\delta(1-s)}{2}$ is the unemployment rate. At the optimum, $\frac{dW}{db} = 0$, and this will be a unique optimum if W is strictly quasi-concave; thus, the expression for the optimum is:

$$\frac{\Delta c}{c_1} R = (1-u)\frac{\frac{d\tau}{db} - \omega}{\frac{d\tau}{db}}.$$

Using elasticities, the marginal value of increased benefits is also equal to:

$$\frac{dW}{db} = \frac{2u}{(1-u)\psi}\left[\frac{\Delta c}{c_1} R E_b^\tau - (1-u)\left(E_b^\tau - \psi\right)\right] \tag{16}$$

where $E_b^\tau = \frac{b}{\tau}\frac{d\tau}{db}$ is the elasticity of τ with respect to b, and $\psi = \frac{\omega b}{\tau} = \frac{ub}{ub+G}$ is the fraction of total government expenditures allocated to UI; set equal to zero, this gives the following expression for the optimum:

$$\frac{\Delta c}{c_1} R - (1-u)\frac{E_b^\tau - \psi}{E_b^\tau}.$$

C.2 Algebraic Analysis of $\frac{dW}{db}$

In order to prove that $\frac{dW}{db}$ increases with G if and only if E_b^D is negative, (16) can also be written as:

$$\frac{dW}{db}(b; G) = \frac{2u}{1-u}\left[\frac{\Delta c}{c_1} R \frac{E_b^\tau}{\psi} - (1-u)\left(\frac{E_b^\tau}{\psi} - 1\right)\right]$$

and therefore we can compare the welfare derivatives when two different values of G are used, 0 and $G > 0$:

$$\frac{dW}{db}(b;G) - \frac{dW}{db}(b;0) = \frac{2u}{1-u}\left[\frac{\Delta c}{c_1}R - (1-u)\right]\left[\left(\frac{E_b^\tau}{\psi}\right)_{G>0} - \left(\frac{E_b^\tau}{\psi}\right)_{G=0}\right].$$

Using (13) and the definition of ψ:

$$\frac{E_b^\tau}{\psi} = 1 + E_b^D + \frac{ub+G}{ub}\left[\frac{\delta(1-s)}{2(1-u)}E_b^D\right]$$

and thus the welfare derivative difference becomes:

$$\frac{dW}{db}(b;G) - \frac{dW}{db}(b;0) = \frac{\delta(1-s)}{(1-u)^2 b}\left[\frac{\Delta c}{c_1}R - (1-u)\right]E_b^D G.$$

Since I assume that $\frac{\Delta c}{c_1}R < 1-u$, this right-hand side will be positive if and only if E_b^D is negative.

D Summary of Statistical Extrapolation Procedure

In this appendix, I describe the procedure of statistical extrapolation used to numerically evaluate (12) to find the optimal benefit level. First of all, the optimal UI literature overwhelmingly solves for an optimal replacement rate rather than a dollar value of UI, so as in the structural analysis earlier, I will do the same. As before, I define the replacement rate as $r = \frac{b}{(0.8)(\frac{15.8}{24.3})y(1-\tau_0)}$, where τ_0 is the baseline real-world tax rate, 0.8 is the take-up rate, and $\frac{15.8}{24.3}$ is the ratio of mean compensated unemployment duration to mean total duration. The steps in the procedure used to solve (12) for the optimal replacement rate are as follows:

- select an equation for $\frac{\Delta c}{c_1}$ as a function of r
- select fixed values of E_b^D and R
- select current values of r and u
- use the fixed value of E_b^D to define a functional form for u with respect to r: $u = \phi r^{E_b^D}$, and use the current values of u and r to solve for ϕ
- select the current value of ψ, and specify the relationship of ψ to r
- solve the resulting non-linear equation in r

E Second-Order Conditions

For the optimal UI equation (14) to identify the unique maximum, strict quasi-concavity is required, and I can test this assumption in my numerical analysis by plotting the estimated value of $\frac{dW}{db}$ at intervals of 0.01 for $r \in [0.01, 2]$, for each set of parameter values, and for the initial model as well as all sensitivity analyses and extensions. I can then see if any failures of quasi-concavity appear over that range; beyond $r = 2$, failures of quasi-concavity might be expected on the grounds that Assumptions 1.A and 2 become especially poor approximations. All plots are available upon request.

For the baseline model, quasi-concavity always appears to be satisfied. However, in the extension to $R = 5$, quasi-concavity fails in several cases when $G = 0.208$: for $E_b^y \geq 0.048$, there appears to be a local minimum at low values of r (always less than 0.08). It is not surprising, however, that these violations of strict quasi-concavity occur at low values of r when R is large, as that is exactly when $\frac{\Delta c}{c_1}R < 1-u$ may fail to hold and my assumptions will tend to be most inaccurate, and at which the estimated E_b^τ could turn negative. Over the vast majority of the range of r that I consider, however, $\frac{dW}{db}$ behaves normally and consistent with quasi-concavity.

In each of the sensitivity analyses in appendix H, similar local minima are found at low r for $R = 5$ and $E_b^y \geq 0.048$. Additionally, in the case with $E_b^D = 0.144$, for $R = 2$, $E_b^y = 0$ and all s_0, I observe local maxima at positive values of r with the global maximum at $r = 0$, as labelled in the results. Finally, in the perfect-take-up case, for $R = 2$, $E_b^y = 0.048$ and $s_0 = 0.648$, there is a second local maximum to the right of the global maximum of $r = 0.0393$.

Further failures of quasi-concavity are observed in each of the extensions in appendix I; in the first, second, and fourth extensions, local minima are observed for $R = 5$ and $E_b^y \geq 0.048$. The first extension presents cases for $R = 2$, $E_b^y = 0$ and $s_0 \leq 0.8$ where local maxima are observed at positive values of r but the global maximum is at $r = 0$. Similar cases are observed thrice in the second extension at high values of E_b^y and s_0 for $R = 5$, though as discussed in appendix I.2, these are anomalous as the optimal replacement rate should logically be close to one. A second local maximum also occurs in the second extension for $R = 2$, $E_b^y = 0.048$ and $s_0 = 0.648$. In the third extension, local maxima are also observed for $R = 5$, $E_b^y = 0$, and each s_0, but once again the global maximum is at zero.

F Baseline Values of $\frac{dW}{db}$

Equation (16), when combined with (13), provides a way of evaluating $\frac{dW}{db}$, and I present in Tables 24 and 25 the values of this derivative at the baseline value of $r = 0.46$, for both $R = 2$ and $R = 5$. The results are conceptually similar to those in Tables 10 and 27, in that a positive value of G causes the values in the table to "spread out."

Table 24: Baseline Values of $\frac{dW}{db}$ Calculated from (16) and (13) for $R = 2$

Baseline $\frac{dW}{db}$ for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	-0.0008	-0.0011	-0.0016	-0.0025
	0	-0.0000	-0.0000	-0.0000	-0.0000
	0.048	0.0004	0.0006	0.0009	0.0014
	0.096	0.0008	0.0012	0.0018	0.0029
	0.192	0.0017	0.0024	0.0037	0.0058
	0.3072	0.0027	0.0039	0.0059	0.0094

Baseline $\frac{dW}{db}$ for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	-0.0418	-0.0487	-0.0605	-0.0805
	0	-0.0258	-0.0258	-0.0258	-0.0258
	0.048	-0.0165	-0.0124	-0.0054	0.0063
	0.096	-0.0071	0.0011	0.0150	0.0384
	0.192	0.0117	0.0280	0.0558	0.1027
	0.3072	0.0343	0.0602	0.1048	0.1798

Table 25: Baseline Values of $\frac{dW}{db}$ Calculated from (16) and (13) for $R = 5$

Baseline $\frac{dW}{db}$ for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.0430	0.0428	0.0425	0.0420
	0	0.0435	0.0435	0.0435	0.0435
	0.048	0.0437	0.0438	0.0440	0.0443
	0.096	0.0440	0.0442	0.0446	0.0452
	0.192	0.0445	0.0449	0.0457	0.0470
	0.3072	0.0451	0.0458	0.0470	0.0491

Baseline $\frac{dW}{db}$ for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.0185	0.0144	0.0073	-0.0046
	0	0.0280	0.0280	0.0280	0.0280
	0.048	0.0337	0.0361	0.0402	0.0472
	0.096	0.0393	0.0441	0.0524	0.0664
	0.192	0.0505	0.0602	0.0768	0.1048
	0.3072	0.0640	0.0795	0.1061	0.1509

G Analytical Results

In this section, I will further analyze the equations derived in subsection 4.2, and specifically I will present a series of analytical results about equations for $\frac{dW}{db}$ and for the optimal level of UI benefits. I will discuss $\frac{dW}{db}(b; G)$, the estimated welfare derivative at a particular value of b given an estimated value of G, and $b^*(G)$, the estimated optimal value of b for a given value of G. I consider how the results change when estimated quantities like G and E_b^y are changed.

It should be emphasized that this is not a comparative statics exercise, as I am not considering a change to a primitive parameter of the model; rather, I consider how the numerical results should be expected to change when the estimated value of G used in the calculations is altered. This represents a change in assumptions about the model, not a change in parameters, and so the values of the sufficient statistics are unaltered, since they reflect the unchanged real world to which the model is calibrated. A helpful thought experiment is that of the “two researchers”: one who assumes that the true value of G is zero, and another who has estimated a positive value of G from some real-world data, while they agree on all other sufficient statistics necessary to calculate the optimum. My analysis answers the question: who will estimate a larger optimal b, and by how much?

Throughout this section, I maintain two additional assumptions, as in the discussion in subsection 3.2; the first is that $\frac{\Delta c}{c_1} R < 1 - u$, and the second is that W (the integral of $\frac{dW}{db}$) is strictly quasi-concave in b. I begin with an analysis of how the results change when I alter the selected value of G. The first result concerns the value of the welfare derivative at a given value of b, and is described in the proposition below.

Proposition 2. *For $G > 0$, $\frac{dW}{db}(b; G) - \frac{dW}{db}(b; 0)$ has the same sign as $sE_b^y - (1-s)E_b^D$, or equivalently the same sign as $\frac{d(sy_n)}{db}$.*

Proof. Starting from (16):

$$\frac{dW}{db}(b; G) = \frac{2u}{1-u}\left[\frac{\Delta c}{c_1} R \frac{E_b^\tau}{\psi} - (1-u)\left(\frac{E_b^\tau}{\psi} - 1\right)\right]$$

and therefore the difference in welfare derivatives is:

$$\frac{dW}{db}(b; G) - \frac{dW}{db}(b; 0) = \frac{2u}{1-u}\left[\frac{\Delta c}{c_1} R - (1-u)\right]\left[\left(\frac{E_b^\tau}{\psi}\right)_{G>0} - \left(\frac{E_b^\tau}{\psi}\right)_{G=0}\right].$$

Using (13) and the definition of ψ:

$$\frac{E_b^\tau}{\psi} = 1 + E_b^D + \frac{ub + G}{ub}\left[\frac{\delta(1-s)}{2(1-u)} E_b^D - \frac{\delta s}{2(1-u)} E_b^y\right]$$

and thus the welfare derivative difference becomes:

$$\frac{dW}{db}(b; G) - \frac{dW}{db}(b; 0) = \frac{\delta}{(1-u)^2 b}\left[\frac{\Delta c}{c_1} R - (1-u)\right]\left[(1-s)E_b^D - sE_b^y\right] G.$$

Since I assume that $\frac{\Delta c}{c_1} R < 1 - u$, this right-hand side will be positive if and only if $sE_b^y - (1-s)E_b^D$ is positive. The latter expression can also be written as:

$$sE_b^y - (1-s)E_b^D = \frac{sb}{y_n}\frac{dy_n}{db} + b\frac{ds}{db} = \frac{b}{y_n}\frac{d(sy_n)}{db}.$$

and therefore $\frac{dW}{db}(b; G) - \frac{dW}{db}(b; 0)$ has the same sign as $\frac{d(sy_n)}{db}$. □

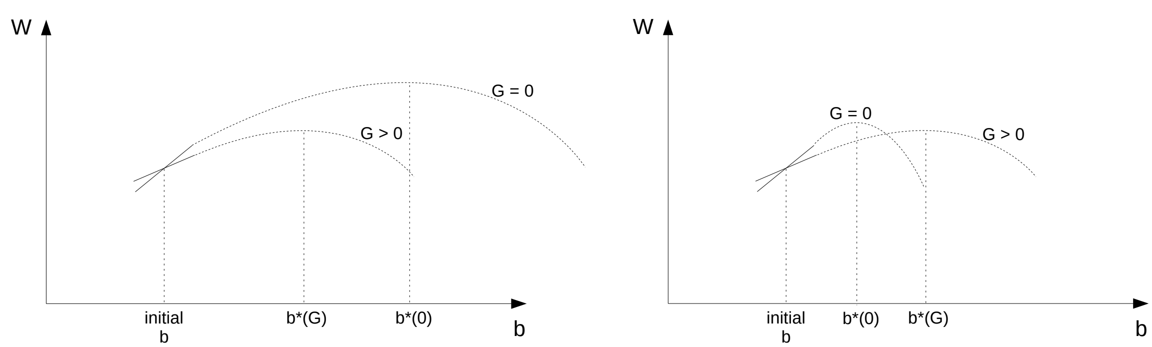
Therefore, if two researchers use (16) to estimate the baseline welfare derivative, one using $G = 0$ and the other a positive value of G, the latter will find a larger welfare gain from increasing b if and only if $\frac{d(sy_n)}{db}$ is positive. Ignoring G greatly understates the revenue effects of changing b, and while higher UI is expected to increase durations of unemployment, it may also increase wages. If this wage effect is so large as to lead to an increase in total post-unemployment earnings sy_n, which is the only non-exogenous component of total earnings in the model, the overall revenue effect is positive and welfare-increasing. Therefore, using a positive value of G, which implies higher taxes, amplifies this positive revenue effect and increases the welfare gain from raising benefits. If $\frac{d(sy_n)}{db}$ is negative, the reverse holds.

An immediate corollary arising from quasi-concavity is that, if the baseline welfare derivative is zero for $G = 0$, and thus the current level of b is estimated to be optimal in that case, then the optimum for the true G will be larger or smaller according to the sign of $\frac{d(sy_n)}{db}$. For example, if $\frac{d(sy_n)}{db} > 0$, $\frac{dW}{db}(b; G) > 0$ and quasi-concavity means that the optimum must be found at a higher b. A similar logic applies if $\frac{dW}{db}(b; G) = 0$; if one of the welfare derivatives is zero, I only need to know the other to make a comparison. This result is summarized by the following corollary.

Corollary 1. *If, for the current value of b,* $\frac{dW}{db}(b;0) = 0$ *or* $\frac{dW}{db}(b;G) = 0$*,* $b^*(G) > b^*(0)$ *if and only if* $sE_b^y - (1-s)E_b^D > 0$*, or equivalently if and only if* $\frac{d(sy_n)}{db} > 0$*.*

Furthermore, if the welfare derivative is of opposite signs for $G = 0$ and $G > 0$, then a comparison of the estimated optimal values of b is simple; if, for example, $\frac{dW}{db}(b;0) > 0$ and $\frac{dW}{db}(b;G) < 0$, then clearly $b^*(0) > b^*(G)$. For a more general result, however, I need to go beyond the local welfare derivative and make out-of-sample assumptions; as an illustration, consider Figure 4, which displays graphically how knowledge of a local welfare derivative doesn't permit unambiguous conclusions about the optimum.[51] Chetty (2009) recommends the method of statistical extrapolation that has been used by Baily (1978) and Gruber (1997), in which each sufficient statistic is extrapolated out of sample as described at the end of subsection 3.2. For this purpose, I define $\chi = \{\frac{\Delta c}{c_1}, R, s, E_b^D, E_b^y\}$ as the vector of sufficient statistics, the underlying quantities in (14) which are not exogenously fixed, and let $\chi(b)$ denote a particular vector of extrapolated values of these quantities.[52] This leads to the following corollary.

Figure 4: Two Possible Welfare Functions



Corollary 2. *For statistical extrapolations that do not depend on the estimated value of G, i.e.* $\chi(b;G) = \chi(b)$*,* $b^*(G) > b^*(0)$ *if and only if* $sE_b^y - (1-s)E_b^D > 0$*, or equivalently if and only if* $\frac{d(sy_n)}{db} > 0$*, in between* $b^*(0)$ *and* $b^*(G)$*.*

Proof. If a statistical extrapolation is used to find $b^*(0)$, and the same statistical extrapolation is used for the case of $G > 0$, then $\frac{dW}{db}(b^*(0);G)$ takes the same sign as $\frac{d(sy_n)}{db}$ at $b^*(0)$. If that sign is positive, then by strict quasi-concavity $b^*(G) > b^*(0)$, and $\frac{d(sy_n)}{db}$ will continue to be positive at least until $b^*(G)$. If the sign is negative, the opposite is true. □

Therefore, if two researchers using $G = 0$ and $G > 0$ use the same statistical extrapolations of the sufficient statistics, then the second researcher's estimated optimal value $b^*(G)$ will be the larger of the two if and only if $\frac{d(sy_n)}{db} > 0$ in between the optimal values of b; the proof explains why the sign of $\frac{d(sy_n)}{db}$ will not change in that region.[53] This is arguably the most important result in this section, and provides

[51]The values of W in the diagram are normalized to be equal at the initial b, but while it is clear that $\frac{dW}{db}(b;0) > \frac{dW}{db}(b;G)$ in the diagram, the dotted lines further to the right are meant to indicate that the sufficient statistics alone give no definite answer about the shape of these curves.

[52]Strict quasi-concavity of W, when the latter is estimated out of sample using statistical extrapolations, implicitly places some restrictions on the extrapolations allowed.

[53]This is not, however, a restrictive assumption relying on quasi-concavity. If everything is continuous, then a marginal increase in the estimated value of G will lead to a marginal change in the optimal b according to the sign of $\frac{d(sy_n)}{db}$. Supposing that $\frac{d(sy_n)}{db} > 0$, b^* will only increase with G as long as it stays in a range where $\frac{d(sy_n)}{db} > 0$, so it can never increase out of this range, and thus a change in the estimated G can

general intuition about fiscal externalities, as well as explaining my numerical results. If UI benefits increase total earnings, this welfare-increasing fiscal externality will appear larger when I account for larger taxes, and the optimal benefit level will increase. If, on the other hand, the effect of UI on wages is zero, as has commonly been assumed, the only behavioural effect of benefits will be to increase unemployment, reducing total earnings, and the fiscal externality will be negative. As demonstrated in the numerical results, the reduction in the optimal benefit level in this case can be substantial.

Next, I present results on the role of E_b^y, to demonstrate that the value of this parameter could be important;[54] these results are straightforward, and begin with the following proposition.

Proposition 3. *For* $E_b^{y2} > E_b^{y1}$, $\frac{dW}{db}(b; G, E_b^{y2}) > \frac{dW}{db}(b; G, E_b^{y1})$.

Proof. Combining (16) and (13), I immediately get:

$$\frac{dW}{db}(b; G, E_b^{y2}) - \frac{dW}{db}(b; G, E_b^{y1}) = \frac{2u}{(1-u)\psi}\left[\frac{\Delta c}{c_1}R - (1-u)\right]\left[\frac{-\delta s}{2(1-u)}\right]\left[E_b^{y2} - E_b^{y1}\right]. \tag{17}$$

Given that $E_b^{y2} > E_b^{y1}$, and since the middle two terms are both negative, this expression is always positive. □

A higher value of E_b^y means that b has a more positive effect on wages, meaning a smaller tax increase to pay for benefits, and thus a larger welfare gain from higher UI. The two following simple corollaries follow the pattern of the previous two.

Corollary 3. *For the current value of* b*, if* $\frac{dW}{db}(b; G, E_b^{y1}) = 0$ *or* $\frac{dW}{db}(b; G, E_b^{y2}) = 0$*, or if* $\frac{dW}{db}(b; G, E_b^{y1}) < 0$ *and* $\frac{dW}{db}(b; G, E_b^{y2}) > 0$*,* $b^*(G, E_b^{y2}) > b^*(G, E_b^{y1})$.

Corollary 4. *For statistical extrapolations of* $\chi_1 = \{\frac{\Delta c}{c_1}, R, s, E_b^D\}$ *that do not depend on the estimated value of* E_b^y*, i.e.* $\chi_1(b; E_b^y) = \chi_1(b)$*,* $b^*(G, E_b^{y2}) > b^*(G, E_b^{y1})$.

If the current b is the estimated optimal value for one of the values of E_b^y under consideration, or if the signs of $\frac{dW}{db}$ are opposite, then I can make an unambiguous statement. More generally, once I define a statistical extrapolation that does not depend on E_b^y, I can state that a researcher choosing a larger value of E_b^y will always find a larger optimal b.

I have now shown that higher E_b^y increases the optimal value of b, and found the conditions under which a higher value of G increases or decreases the optimal b; the final analytical results concern the interaction of G and E_b^y. As already mentioned, Baily (1978) is among the few papers that acknowledge the fact that a parameter like E_b^y could enter into optimal UI calculations, but he ultimately drops this parameter from his equation on the grounds that, since the UI payroll tax is quite small, it will have little effect on the results. However, E_b^y is more important when G is large, both to social welfare and to the calculation of the optimal value of b; a demonstration of this begins with the following proposition.

Proposition 4. *For* $E_b^{y2} > E_b^{y1}$, $\frac{dW}{db}(b; G, E_b^{y2}) - \frac{dW}{db}(b; G, E_b^{y1}) > \frac{dW}{db}(b; 0, E_b^{y2}) - \frac{dW}{db}(b; 0, E_b^{y1})$.

Proof. I start with (17), and from the definition of ψ:

$$\left[\frac{dW}{db}(b; G, E_b^{y2}) - \frac{dW}{db}(b; G, E_b^{y1})\right] - \left[\frac{dW}{db}(b; 0, E_b^{y2}) - \frac{dW}{db}(b; 0, E_b^{y1})\right]$$

$$= \frac{-\delta s}{(1-u)^2 b}\left[\frac{\Delta c}{c_1}R - (1-u)\right]\left[E_b^{y2} - E_b^{y1}\right]G.$$

This equation is clearly positive. □

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never move the estimated optimal b enough to change the sign of $\frac{d(sy_n)}{db}$. If there is a value of b for which $\frac{d(sy_n)}{db}$ takes the opposite sign, there must be no value of G such that this b would be optimal. Furthermore, this prediction is supported for all sets of parameter values in the numerical results: when $b^*(G) > b^*(0)$, $sE_b^y - (1-s)E_b^D$ is found to be positive for all $b \in [b^*(0), b^*(G)]$, and vice-versa when $b^*(G) < b^*(0)$.

[54] The notation is now slightly altered to allow E_b^y to enter $\frac{dW}{db}$ and b^* as an argument.

This proposition says that the effect of E_b^y on the welfare derivative is increasing in G. Thus, it may be true that a researcher who ignores G will find that the value of E_b^y is relatively unimportant to their calculations, but when G is large, the tax rate will also be large, and E_b^y will matter far more to the value of the welfare derivative. Proposition 4 can also be interpreted as saying that the importance of G to the welfare derivative is increasing in E_b^y.

The final analytical result addresses the question of whether E_b^y is more important in determining the value of the optimal b when G is large. The results so far make it logical to suspect that $b^*(G; E_b^y) - b^*(0; E_b^y)$ is increasing in E_b^y, i.e. that the increase in b^* caused by G is more positive when E_b^y is larger; after all, I have proved that b^* is increasing in E_b^y, and that the difference in the welfare derivative for different values of E_b^y is increasing in G. This suspicion, however, cannot be turned into proof without unusual and unintuitive assumptions; I can, however, prove a somewhat weaker result, as summarized below.

Proposition 5. *For continuous statistical extrapolations that do not depend on the estimated values of G and E_b^y, if $\frac{dE_b^D}{db} \geq 0$, $\frac{dR}{db} = 0$, $E_b^D > -1$, and $\frac{d}{db}\left(\frac{\frac{\Delta c}{c_1}}{1-u}\right) < 0$, the following is true:*

- *if $\exists$ an E_b^{y*} such that $b^*(G, E_b^{y*}) = b^*(0, E_b^{y*})$, then $b^*(G, E_b^{y2}) > b^*(0, E_b^{y2})$ for $E_b^{y2} > E_b^{y*}$ and $b^*(G, E_b^{y1}) < b^*(0, E_b^{y1})$ for $E_b^{y1} < E_b^{y*}$.*

Proof. I begin with the fact that, at the optimum, $\frac{\Delta c}{c_1} R E_b^\tau = (1-u)\left(E_b^\tau - \psi\right)$; using (13) and rearranging, this becomes:

$$\frac{\Delta c}{c_1} R\psi(1+E_b^D) - (1-u)\psi E_b^D = \frac{\delta}{2(1-u)}\left[sE_b^y - (1-s)E_b^D\right]\left[\frac{\Delta c}{c_1}R - (1-u)\right].$$

Observe that, because $\frac{\Delta c}{c_1}R < 1-u$, $sE_b^y - (1-s)E_b^D > 0$ at the optimum if and only if $\frac{\Delta c}{c_1}R(1+E_b^D) - (1-u)E_b^D < 0$. I wish to show that $b^*(G, E_b^{y2}) > b^*(0, E_b^{y2})$ and $b^*(G, E_b^{y1}) < b^*(0, E_b^{y1})$ for $E_b^{y2} > E_b^{y*} > E_b^{y1}$, so Corollary 2 says that $sE_b^y - (1-s)E_b^D$ must be positive for E_b^{y2} and negative for E_b^{y1}. Then, if I define $X(b) = \frac{\Delta c}{c_1}R(1+E_b^D) - (1-u)E_b^D$, I want $X < 0$ at the optimum for E_b^{y2} and $X > 0$ for E_b^{y1}; given that I am considering continuous statistical extrapolations, a sufficient and necessary condition is that $\frac{dX}{db} < 0$ at $X = 0$. The derivative is:

$$\frac{dX}{db} = R(1+E_b^D)\frac{d\left(\frac{\Delta c}{c_1}\right)}{db} + \frac{\Delta c}{c_1}(1+E_b^D)\frac{dR}{db} + \frac{\Delta c}{c_1}R\frac{dE_b^D}{db} - E_b^D\frac{d(1-u)}{db} - (1-u)\frac{dE_b^D}{db}$$

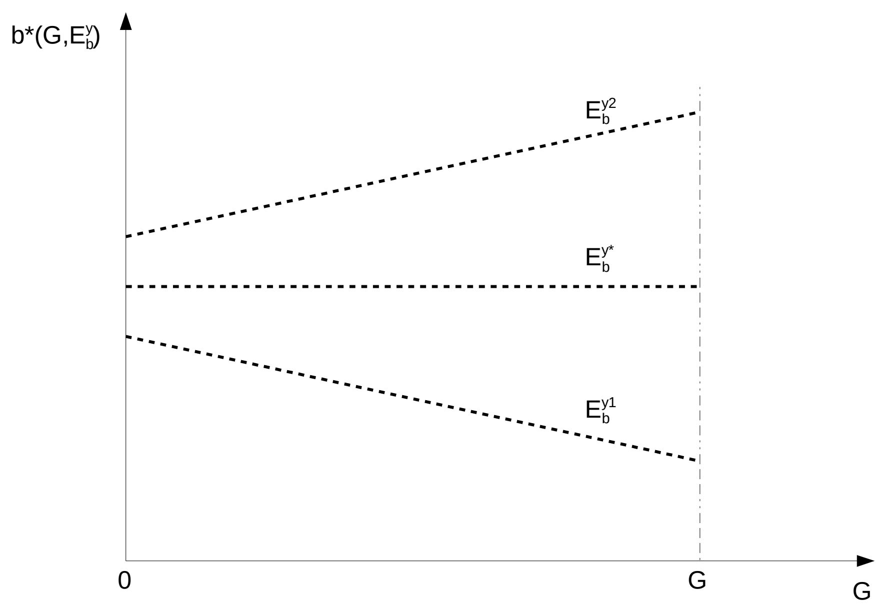
and at $X = 0$, $\frac{\Delta c}{c_1}R(1+E_b^D) = (1-u)E_b^D$, and thus:

$$\frac{dX}{db}|_{X=0} = \frac{\Delta c}{c_1}(1+E_b^D)\frac{dR}{db} + \left[\frac{\Delta c}{c_1}R - (1-u)\right]\frac{dE_b^D}{db} + \frac{R(1+E_b^D)}{(1-u)}\left[(1-u)\frac{d\left(\frac{\Delta c}{c_1}\right)}{db} - \frac{\Delta c}{c_1}\frac{d(1-u)}{db}\right].$$

Sufficient conditions for this to be negative are that $\frac{dR}{db} = 0$, $\frac{dE_b^D}{db} \geq 0$, $1 + E_b^D > 0$ and $(1-u)\frac{d\left(\frac{\Delta c}{c_1}\right)}{db} < \frac{\Delta c}{c_1}\frac{d(1-u)}{db}$. The first two assumptions are standard, and I make them in my numerical analysis; $\frac{dR}{db}$ is commonly assumed to equal zero, as it would with a CRRA utility function, and Chetty (2006) states that estimates of $\frac{dE_b^D}{db}$ "are broadly similar across studies with different levels of benefit generosity." The third assumption is a formality, as a nearly universal finding of the empirical literature is that E_b^D is positive. The final assumption requires a bit more explanation; it is easiest to understand when written as $\frac{d}{db}\left(\frac{\frac{\Delta c}{c_1}}{1-u}\right) < 0$. The consumption gap $\frac{\Delta c}{c_1}$ is likely to be much smaller than $1-u$, and to decline faster, as the former is always less than one and could reach or even drop below zero, whereas $1-u$ is always at least as large as $1 - \frac{\delta}{2}$. Therefore, this condition is likely to be satisfied in nearly every case of interest, and this assumption is strongly supported by the numerical results in subsection 4.2 in all cases in which the optimal replacement rate is above zero; my functional form assumptions cause $\frac{d(1-u)}{db}$ to become unboundedly large and negative as benefits approach zero. □

This proposition says that, although I cannot prove the stronger condition that $b^*(G; E_b^y) - b^*(0; E_b^y)$ is increasing in E_b^y, I can state that for small values of E_b^y, $b^*(G, E_b^y) < b^*(0, E_b^y)$, and vice-versa for sufficiently large values of E_b^y;[55] this result is summarized by the diagram in Figure 5. Therefore, at least locally around E_b^{y*}, the stronger condition will hold, and I will show in my numerical results that the stronger condition does describe the general behaviour of b^* for the parameters and functional forms that I use.

Figure 5: Consequences of Proposition 5



The results derived in this section apply in particular to UI, but similar results will also apply in the context of other government programs with impacts on the labour market. The idea that the direction of the change in optimal policy caused by fiscal externalities depends only on the direction of the program's effect on total taxable income is intuitive and more general than the current context, as is the result that effects of a program on wages are more important when the full size of government is taken into account.

H Sensitivity Analyses in the Baily Model

In this appendix, I present results from a number of sensitivity analyses in the sufficient statistics approach. I begin by extending the results in Table 7 to a case of $R = 5$, with results as displayed in Table 26. The decline in optimal UI due to fiscal externalities is smaller in this case; the difference is generated partly by the fact that E_b^D increases with b in the dynamic job search model, whereas it is held fixed during statistical extrapolations, but also because of other differences in the models, such as the fact that the steady-state asset distribution changes with b in the structural model.

Next, I again use $R = 5$ and present results when there may be effects of UI on subsequent wages. Table 27 presents results analogous to those in Table 10. The numerical results are less extreme than they were for $R = 2$, but the same pattern of findings is present there as well: when $G > 0$, the optimal replacement rates spread out noticeably, becoming more sensitive to both E_b^y and s.

[55]In the unlikely case that an increase in E_b^y causes an increase in the optimal b which makes s decrease sufficiently quickly, the critical value E_b^{y*} may not exist; in that case, $b^*(G) - b^*(0)$ is always negative as long as $E_b^u > 0$.

Table 26: Optimal Replacement Rates & Welfare Gains with $R = 5$

	r	Welf. Gain	Diff.
$G = 0.208$	0.5996	4.03	1.68
$G = 0$	0.6866	10.32	1.58

Table 27: Optimal Replacement Rates Calculated from (14) for $R = 5$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.6831	0.6815	0.6787	0.6741
	0	0.6866	0.6866	0.6866	0.6866
	0.048	0.6887	0.6897	0.6914	0.6942
	0.096	0.6909	0.6928	0.6962	0.7020
	0.192	0.6951	0.6991	0.7061	0.7183
	0.3072	0.7004	0.7069	0.7184	0.7390

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.5518	0.5307	0.4953	0.4387
	0	0.5996	0.5996	0.5996	0.5996
	0.048	0.6276	0.6402	0.6620	0.6987
	0.096	0.6554	0.6806	0.7240	0.7970
	0.192	0.7102	0.7601	0.8452	0.9853
	0.3072	0.7741	0.8523	0.9831	1.1914

I then spend the rest of this appendix considering the sensitivity of my results to a different set of parameters, for both $R = 2$ and $R = 5$. First, I use different values of E_b^D over a wide range; I try $E_b^D = 0.48 \times 0.3$, with results in Tables 28 and 29, and $E_b^D = 0.48 \times 0.8$, with results in Tables 30 and 31. Not surprisingly, the optimal replacement rates move up in the former case and down in the latter; the effects of fiscal externalities remain sizable in both cases. A value of E_b^D around 0.144 is where I begin to observe a positive local maximum when $R = 2$ and $E_b^y = 0$; for a positive global maximum, I need E_b^D to drop below 0.096.

Then I try two alternative values of G; in particular, I consider cases with baseline tax rates of 0.15 and 0.30, which imply values of G equal to 0.131 and 0.275 respectively, with results in Tables 32 and Tables 33. The results are unsurprising, as a lower value of G leads to results that are closer to the $G = 0$ case, while higher values imply more extreme effects of fiscal externalities.

Next, I ignore the question of take-up of benefits and only deflate benefits by the ratio of compensated to total unemployment duration; the ensuing results can be found in Tables 34 and 35. This tends to reduce the size of the difference between optimal replacement rates with $G = 0$ and $G > 0$, but I still observe zeros for $R = 2$ and $E_b^y = 0$.

I then try a larger value of the initial unemployment rate, specifically $u_0 = 0.064$. This leads to the results displayed in Tables 36 and 37. The optimal replacement rates spread out for $G = 0$, but the effects are more modest for $G > 0$, meaning a small reduction in the effect of fiscal externalities.

Finally, instead of using a tax rate of 0.23 to apply to both UI and earned income, I allow for one tax rate applied to UI benefits and another for earned income; as explained in appendix B.1, I use 0.15 as the tax rate on UI and 0.262 as the tax rate on earned income. The results are displayed in Tables 38 and 39, and the effects of fiscal externalities are slightly increased.

I Extensions to Baily Model

This appendix will analyze a variety of extensions to the Baily model, including stochastic duration of unemployment, within-period borrowing constraints, use of a second-order Taylor series expansion of marginal utility, and variable labour supply on the initial job. I will present results with both the baseline value of $R = 2$ and $R = 5$, and I will demonstrate that, although the formulas change somewhat in each case, as do the specific numerical results, the qualitative effects of fiscal externalities change very little.

Table 28: Optimal Replacement Rates for $R = 2$ and $E_b^D = 0.144$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.5908	0.5859	0.5776	0.5637
	0	0.6020	0.6020	0.6020	0.6020
	0.048	0.6087	0.6116	0.6167	0.6254
	0.096	0.6154	0.6213	0.6317	0.6494
	0.192	0.6289	0.6411	0.6624	0.6997
	0.3072	0.6454	0.6653	0.7007	0.7640

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0	0	0	0
	*0	0	0	0	0
	0.048	0.5311	0.5824	0.6585	0.7678
	0.096	0.6392	0.7181	0.8358	1.0062
	0.192	0.8028	0.9254	1.1095	1.3767
	0.3072	0.9598	1.1257	1.3739	1.7299

*Local Maximum of 0.3451 for this row

Table 29: Optimal Replacement Rates for $R = 5$ and $E_b^D = 0.144$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.7391	0.7371	0.7338	0.7283
	0	0.7435	0.7435	0.7435	0.7435
	0.048	0.7461	0.7473	0.7493	0.7528
	0.096	0.7487	0.7511	0.7552	0.7623
	0.192	0.7540	0.7589	0.7675	0.7824
	0.3072	0.7606	0.7686	0.7828	0.8081

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.6311	0.6066	0.5656	0.5005
	0	0.6874	0.6874	0.6874	0.6874
	0.048	0.7208	0.7358	0.7618	0.8058
	0.096	0.7543	0.7846	0.8368	0.9253
	0.192	0.8212	0.8819	0.9860	1.1588
	0.3072	0.9008	0.9968	1.1586	1.4177

Table 30: Optimal Replacement Rates for $R = 2$ and $E_b^D = 0.384$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.3129	0.3096	0.3039	0.2943
	0	0.3213	0.3213	0.3213	0.3213
	0.048	0.3262	0.3283	0.3317	0.3376
	0.096	0.3312	0.3352	0.3422	0.3541
	0.192	0.3411	0.3493	0.3636	0.3882
	0.3072	0.3531	0.3664	0.3897	0.4310

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0	0	0	0
	0	0	0	0	0
	0.048	0.0257	0.0544	0.1477	0.3031
	0.096	0.1503	0.2551	0.3873	0.5457
	0.192	0.3675	0.4800	0.6326	0.8357
	0.3072	0.5148	0.6460	0.8312	1.0832

Table 31: Optimal Replacement Rates for $R = 5$ and $E_b^D = 0.384$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.6292	0.6279	0.6256	0.6219
	0	0.6320	0.6320	0.6320	0.6320
	0.048	0.6337	0.6345	0.6359	0.6382
	0.096	0.6354	0.6370	0.6398	0.6445
	0.192	0.6389	0.6421	0.6478	0.6576
	0.3072	0.6430	0.6483	0.6576	0.6741

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.4764	0.4581	0.4273	0.3777
	0	0.5182	0.5182	0.5182	0.5182
	0.048	0.5422	0.5529	0.5713	0.6021
	0.096	0.5659	0.5870	0.6232	0.6836
	0.192	0.6117	0.6530	0.7228	0.8365
	0.3072	0.6642	0.7279	0.8337	1.0009

Table 32: Optimal Replacement Rates for $R = 2$

Optimal r for $G = 0.131$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0	0	0	0
	0	0	0	0	0
E_b^y	0.048	0.3124	0.3622	0.4254	0.5071
	0.096	0.4133	0.4728	0.5562	0.6709
	0.192	0.5359	0.6186	0.7389	0.9089
	0.3072	0.6428	0.7500	0.9076	1.1316

Optimal r for $G = 0.275$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0	0	0	0
	0	0	0	0	0
E_b^y	0.048	0.0253	0.1354	0.3625	0.5387
	0.096	0.3368	0.4696	0.6308	0.8367
	0.192	0.5900	0.7397	0.9496	1.2386
	0.3072	0.7771	0.9623	1.2287	1.5966

Table 33: Optimal Replacement Rates for $R = 5$

Optimal r for $G = 0.131$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.6071	0.5941	0.5719	0.5353
	0	0.6362	0.6362	0.6362	0.6362
E_b^y	0.048	0.6532	0.6610	0.6743	0.6968
	0.096	0.6702	0.6857	0.7124	0.7575
	0.192	0.7039	0.7349	0.7879	0.8766
	0.3072	0.7438	0.7928	0.8762	1.0128

Optimal r for $G = 0.275$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.4953	0.4668	0.4201	0.3508
	0	0.5621	0.5621	0.5621	0.5621
E_b^y	0.048	0.6014	0.6190	0.6493	0.7007
	0.096	0.6403	0.6754	0.7359	0.8372
	0.192	0.7165	0.7856	0.9028	1.0933
	0.3072	0.8044	0.9114	1.0886	1.3643

Table 34: Optimal Replacement Rates for $R = 2$ and Perfect Take-Up

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.4500	0.4459	0.4390	0.4275
	0	0.4595	0.4595	0.4595	0.4595
E_b^y	0.048	0.4651	0.4675	0.4717	0.4789
	0.096	0.4707	0.4756	0.4842	0.4987
	0.192	0.4821	0.4921	0.5095	0.5399
	0.3072	0.4959	0.5122	0.5410	0.5922

Optimal r for $G = 0.205$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0	0	0	0
	0	0	0	0	0
E_b^y	0.048	0.0393	0.3148	0.4086	0.5160
	0.096	0.3925	0.4719	0.5775	0.7184
	0.192	0.5516	0.6537	0.7996	1.0034
	0.3072	0.6821	0.8116	1.0001	1.2649

Table 35: Optimal Replacement Rates for $R = 5$ and Perfect Take-Up

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.6831	0.6815	0.6787	0.6741
	0	0.6866	0.6866	0.6866	0.6866
E_b^y	0.048	0.6887	0.6897	0.6914	0.6942
	0.096	0.6909	0.6928	0.6962	0.7020
	0.192	0.6951	0.6991	0.7061	0.7183
	0.3072	0.7004	0.7069	0.7184	0.7390

Optimal r for $G = 0.205$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.5786	0.5614	0.5322	0.4847
	0	0.6173	0.6173	0.6173	0.6173
E_b^y	0.048	0.6400	0.6503	0.6679	0.6978
	0.096	0.6626	0.6831	0.7184	0.7779
	0.192	0.7071	0.7479	0.8176	0.9332
	0.3072	0.7595	0.8237	0.9319	1.1064

Table 36: Optimal Replacement Rates for $R = 2$ and $u_0 = 0.064$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.4491	0.4443	0.4362	0.4227
	0	0.4603	0.4603	0.4603	0.4603
E_b^y	0.048	0.4670	0.4698	0.4748	0.4833
	0.096	0.4736	0.4794	0.4895	0.5069
	0.192	0.4871	0.4989	0.5198	0.5562
	0.3072	0.5034	0.5228	0.5574	0.6195

Optimal r for $G = 0.203$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0	0	0	0
	0	0	0	0	0
E_b^y	0.048	0.0306	0.2725	0.3978	0.5265
	0.096	0.3784	0.4743	0.5981	0.7610
	0.192	0.5672	0.6853	0.8532	1.0868
	0.3072	0.7170	0.8656	1.0813	1.3827

Table 37: Optimal Replacement Rates for $R = 5$ and $u_0 = 0.064$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.6828	0.6809	0.6777	0.6723
	0	0.6870	0.6870	0.6870	0.6870
E_b^y	0.048	0.6895	0.6906	0.6926	0.6960
	0.096	0.6920	0.6943	0.6984	0.7053
	0.192	0.6971	0.7018	0.7101	0.7247
	0.3072	0.7033	0.7111	0.7249	0.7499

Optimal r for $G = 0.203$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.5554	0.5347	0.5003	0.4454
	0	0.6023	0.6023	0.6023	0.6023
E_b^y	0.048	0.6299	0.6424	0.6639	0.7003
	0.096	0.6574	0.6824	0.7254	0.7981
	0.192	0.7116	0.7613	0.8462	0.9863
	0.3072	0.7753	0.8532	0.9840	1.1924

Table 38: Optimal Replacement Rates for $R = 2$ and Multiple Tax Rates

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.4500	0.4459	0.4390	0.4275
	0	0.4595	0.4595	0.4595	0.4595
E_b^y	0.048	0.4651	0.4675	0.4717	0.4789
	0.096	0.4707	0.4756	0.4842	0.4987
	0.192	0.4821	0.4921	0.5095	0.5399
	0.3072	0.4959	0.5122	0.5410	0.5922

Optimal r for $G = 0.237$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0	0	0	0
	0	0	0	0	0
E_b^y	0.048	0.0294	0.2543	0.3909	0.5249
	0.096	0.3708	0.4710	0.5987	0.7653
	0.192	0.5670	0.6880	0.8592	1.0963
	0.3072	0.7202	0.8717	1.0908	1.3960

Table 39: Optimal Replacement Rates for $R = 5$ and Multiple Tax Rates

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.6831	0.6815	0.6787	0.6741
	0	0.6866	0.6866	0.6866	0.6866
E_b^y	0.048	0.6887	0.6897	0.6914	0.6942
	0.096	0.6909	0.6928	0.6962	0.7020
	0.192	0.6951	0.6991	0.7061	0.7183
	0.3072	0.7004	0.7069	0.7184	0.7390

Optimal r for $G = 0.237$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.5476	0.5259	0.4896	0.4317
	0	0.5968	0.5968	0.5968	0.5968
E_b^y	0.048	0.6256	0.6387	0.6610	0.6989
	0.096	0.6543	0.6802	0.7249	0.8000
	0.192	0.7106	0.7620	0.8495	0.9935
	0.3072	0.7764	0.8567	0.9911	1.2046

I.1 Stochastic Duration of Unemployment

I first consider the effect of allowing the duration of unemployment to be stochastic. I follow Baily's approach of defining the actual duration of unemployment $(1-\tilde{s})$ as:

$$(1-\tilde{s}) = [1-s(e,y_n)] + v$$

where s is deterministic, and v is a stochastic term with mean zero which is uncorrelated with s.[56]

If I now denote second-period consumption if the worker loses their job as $\tilde{c_u}$, then:

$$\begin{aligned}\tilde{c_u} &= (1-\tilde{s})(b-e) + \tilde{s}y_n(1-\tau) + k \\ &= c_u - v\Delta y\end{aligned}$$

where c_u is defined as before, and $\Delta y = y_n(1-\tau) - (b-e)$. Utility can now be written as:

$$V = U[y(1-\tau)-k] + (1-\delta)U[y(1-\tau)+k] + \delta E_v[U(\tilde{c_u})].$$

(4) and (5) now have to be replaced by:

$$\frac{\partial V}{\partial b} = \delta E_v[U'(\tilde{c_u})(1-s+v)]$$

$$\frac{\partial V}{\partial \tau} = -yU'(c_1) - (1-\delta)yU'(c_e) - \delta y_n E_v[U'(\tilde{c_u})(s-v)].$$

A first-order Taylor series expansion of $U'(c_1)$ gives $U'(c_1) = U'(c_u) + \Delta c U''(\theta)$ as before, and I perform a similar expansion of $U'(\tilde{c_u})$:

$$\begin{aligned}U'(\tilde{c_u}) &= U'(c_u) + U''(\gamma)(\tilde{c_u} - c_u) \\ &= U'(c_u) - v\Delta y U''(\gamma)\end{aligned}$$

where γ is somewhere between c_u and $\tilde{c_u}$. Upon reaching this point in the calculations, Baily (1978) implicitly makes an assumption that he does not state explicitly, which is that $U''(\gamma)$ is uncorrelated with v and v^2, capturing an intuition that the average first and second derivatives shouldn't be too far from the respective derivatives at the average c_u, as well as greatly simplifying the algebra. I make the same assumption, and therefore:

$$E_v[U'(\tilde{c_u})] = U'(c_u)$$

$$E_v[U'(\tilde{c_u})v] = -\Delta y E_v[U''(\gamma)]Var(v).$$

As a result, the individual's first-order condition for savings can now be written as:

$$\frac{\partial V}{\partial k} = -U'(c_1) + (1-\delta)U'(c_e) + \delta U'(c_u) = 0$$

As before, I make the assumptions that $y_n = y$ and $c_1 U''(\theta) = c_u U''(c_u)$, and to this I add the assumption that $E[U''(\gamma)] = U''(\theta)$, which will tend towards underestimating the welfare gain from raising b. I then combine the results above and write the welfare derivative as:

$$\frac{dW}{db} = 2y\frac{\Delta c}{c_1}R\frac{d\tau}{db} + \delta\frac{\Delta y}{c_1}RVar(v) + \delta y\frac{\Delta y}{c_1}RVar(v)\frac{d\tau}{db} - 2(1-u)y\left[\frac{d\tau}{db} - \omega\right].$$

which can also be written as:

$$\frac{dW}{db} = \frac{2u}{(1-u)\psi}\left[\frac{\Delta c}{c_1}R + \frac{\Delta y}{c_1}\frac{RVar(v)}{1-s}\right]E_b^\tau - \frac{2u}{\psi}\left[1 + \frac{\Delta y}{c_1}\frac{RVar(v)}{1-s}\right][E_b^\tau - \psi]$$

[56] As noted by Baily, this can only be an approximation given that $(s-v)$ is constrained to lie in $(0,1)$.

The budget constraint takes an expectation over all workers, and so is unchanged, and the equation for the optimum is:

$$\frac{\Delta c}{c_1}R + \frac{\Delta y}{c_1}\frac{RVar(v)}{1-s} = (1-u)\left[1+\frac{\Delta y}{c_1}\frac{RVar(v)}{1-s}\right]\frac{E_b^\tau - \psi}{E_b^\tau}. \quad (18)$$

If I make the same assumptions as Baily, then this formula collapses to that used in his extension to stochastic unemployment durations. Most of the terms in (18) have exactly the same interpretation as before, or, as in the case of u and s, still work as averages or expectations, but there are two new terms to consider: $\frac{\Delta y}{c_1}$ and $Var(v)$. The latter is also the variance of the duration of unemployment $(1-s)$, and to evaluate this parameter, I turn to Chetty (2008), who estimates a mean duration of unemployment of 18.3 weeks, and a standard deviation of 14.2, so I normalize the standard deviation by the mean and write $std(v) = \frac{14.2}{18.3}(1-s_0)$, and therefore $Var(v) = \left(\frac{14.2}{18.3}\right)^2 (1-s_0)^2$.[57] Meanwhile, in the absence of any better evidence, I will use Baily's assumption that $\frac{\Delta y}{c_1} = 1-r$. Evaluation of (18) then gives the results displayed below in Tables 40 and 41.

Table 40: Optimal Replacement Rates Calculated from (18) for $R = 2$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.6529	0.6208	0.5812	0.5367
	0	0.6590	0.6303	0.5969	0.5630
	0.048	0.6626	0.6360	0.6063	0.5790
	0.096	0.6662	0.6417	0.6157	0.5954
	0.192	0.6735	0.6532	0.6351	0.6292
	0.3072	0.6824	0.6673	0.6590	0.6720

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0	0	0	0
	0	0	0	0	0
	0.048	0.5444	0.5334	0.5455	0.6008
	0.096	0.6034	0.6225	0.6788	0.7921
	0.192	0.7003	0.7611	0.8779	1.0739
	0.3072	0.7962	0.8945	1.0658	1.3354

Table 41: Optimal Replacement Rates Calculated from (18) for $R = 5$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.7961	0.7779	0.7560	0.7322
	0	0.7986	0.7819	0.7626	0.7432
	0.048	0.8002	0.7843	0.7665	0.7499
	0.096	0.8017	0.7867	0.7705	0.7567
	0.192	0.8048	0.7916	0.7786	0.7708
	0.3072	0.8085	0.7975	0.7887	0.7887

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.7138	0.6756	0.6202	0.5423
	0	0.7427	0.7215	0.6969	0.6722
	0.048	0.7597	0.7486	0.7428	0.7519
	0.096	0.7767	0.7758	0.7888	0.8317
	0.192	0.8107	0.8300	0.8803	0.9877
	0.3072	0.8510	0.8942	0.9868	1.1623

As can be seen, allowing for an uncertain duration of unemployment tends to make the optimal rate closer to one, since the desire to provide full insurance is made greater by the uncertainty; this means a decrease in cases where the optimal rate was above one, as it is no longer as desirable to "over-insure" when unemployed individuals face uncertainty about duration. The qualitative conclusion remains the same, however, regarding the effect of the fiscal externality from income taxes: the optimal replacement rate decreases for lower values of s and especially E_b^y, whereas it increases for higher values. Indeed, the pairwise comparisons between the side-by-side tables are identical in the sense that, if the optimal replacement rate is higher for $G = 0$ than for $G = 0.208$ in Table 10 or 27, the same is true in Table 40 or 41, and vice-versa.

[57] There are potential offsetting biases in these calculations; Chetty (2008) uses a sample in which the duration of unemployment is truncated at 50 weeks, suggesting I may be underestimating $Var(v)$, but on the other hand, Chetty's is an unconditional variance, some of which may be explained by individual characteristics, which means $Var(v)$ may be an overestimate.

I.2 Within-Period Borrowing Constraints

Another unrealistic feature of the basic two-period model is the assumption that individuals can not only save or borrow as much as they want across periods, but that they can also perfectly smooth consumption within the second period. Recent work, in particular that of Chetty (2008), has emphasized the importance of liquidity constraints among the unemployed and the beneficial role of UI in loosening these constraints. I will therefore consider the case of no borrowing during unemployment; I assume that utility is additively time-separable within the second period, so that second period utility of a worker who loses their job is $(1-s)U(c_u) + sU(c_n)$, where c_u is now per-period consumption while unemployed and c_n is per-period consumption when re-employed in a new job.[58] I also assume that, if a worker loses their job, any savings from the first period are completely consumed while unemployed; none of those savings are kept for consumption when re-employed.[59] I can therefore write total utility as:

$$V = U[y(1-\tau) - k] + (1-\delta)U[y(1-\tau) + k] + \delta\left[(1-s)U\left((b-e) + \frac{k}{1-s}\right) + sU\left(y_n(1-\tau)\right)\right].$$

(4) still holds, and (5) is now replaced by:

$$\frac{\partial V}{\partial \tau} = -yU'(c_1) - (1-\delta)yU'(c_e) - \delta s y_n U'(c_n).$$

I replace $U'(c_e)$ using the first-order condition for saving, as before, and I assume that $c_n = c_1$, which is generally consistent with the finding in Gruber (1997) that workers who lose their job in one year but are re-employed in the following year see their consumption return to within 4% of their pre-unemployment consumption. Combining this with the usual Taylor series expansion of $U'(c_1)$:

$$\frac{\partial V}{\partial \tau} = -[(2-\delta)y + \delta s y_n]U'(c_u) - [2y + \delta s y_n]\Delta c U''(\theta).$$

Putting this together with (4):

$$\frac{dW}{db} = [2+\delta s]y\frac{\Delta c}{c_1}R\frac{d\tau}{db} - 2(1-u)y\left(\frac{d\tau}{db} - \omega\right)$$

and therefore the equation for the optimum is:

$$\frac{\Delta c}{c_1}R = \frac{2(1-u)}{2+\delta s}\frac{E_b^\tau - \psi}{E_b^\tau}. \qquad (19)$$

E_b^τ is the same as before, so this equation is almost identical to (9), and it is easy to introduce the extra δs term into the calculations. Solving for the optimal replacement rate generates the results found in Tables 42 and 43.

The pattern of the results changes a little, as the optimal replacement rates generally tend to move closer to one (or more precisely, closer to $\frac{0.222}{0.265}$). In a few cases with $R = 5$ and high E_b^y and s_0, anomalous results occur in which the local maximum obtained at a replacement rate near one is not estimated to be the global maximum, which appears to occur at zero. In these cases, the assumption that unemployment goes to zero as benefits go to zero is partly responsible, as is a failure of an assumption that $\frac{\Delta c}{c_1}R < \frac{2(1-u)}{2+\delta s}$.[60] However, aside from these cases, the changes tend to be quite small, surprisingly so given the shift in the

[58]Chetty (2006) argues that, in his model, the nature of borrowing constraints does not change the optimal UI formula, as this effect will simply show up in the magnitude of the consumption drop. In a sense, this is correct in my analysis as well, but how I interpret borrowing constraints changes what I call the value of consumption during unemployment; I previously defined c_u as the total consumption in the second period if a worker experiences a spell of unemployment, whereas I now define c_u to be consumption while unemployed.

[59]Given that unemployment durations are deterministic, as long as y_n is not too far from y, there is no reason for a worker to save more than they would want to consume in a spell of unemployment.

[60] $\frac{\Delta c}{c_1}R > \frac{2(1-u)}{2+\delta s}$ implies $\frac{\partial V}{\partial \tau} > 0$, while I also estimate that $\frac{d\tau}{db} < 0$.

Table 42: Optimal Replacement Rates Calculated from (19) for $R = 2$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.4850	0.4947	0.5098	0.5316
	0	0.4934	0.5063	0.5263	0.5551
E_b^y	0.048	0.4985	0.5133	0.5362	0.5694
	0.096	0.5035	0.5203	0.5463	0.5841
	0.192	0.5137	0.5344	0.5670	0.6146
	0.3072	0.5260	0.5518	0.5926	0.6534

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0	0	0	0
	0	0	0	0	0
E_b^y	0.048	0.2384	0.3621	0.4716	0.5905
	0.096	0.4156	0.5095	0.6274	0.7723
	0.192	0.5821	0.6956	0.8500	1.0509
	0.3072	0.7216	0.8621	1.0577	1.3160

Table 43: Optimal Replacement Rates Calculated from (19) for $R = 5$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.6963	0.7002	0.7063	0.7151
	0	0.6995	0.7047	0.7128	0.7244
E_b^y	0.048	0.7014	0.7074	0.7167	0.7301
	0.096	0.7034	0.7102	0.7207	0.7359
	0.192	0.7073	0.7157	0.7289	0.7481
	0.3072	0.7120	0.7225	0.7390	0.7636

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.5825	0.5775	0.5721	0.5697
	0	0.6229	0.6318	0.6456	0.6649
E_b^y	0.048	0.6470	0.6651	0.6929	0.7324
	0.096	0.6713	0.6991	0.7422	0.8057
	0.192	0.7202	0.7681	0.8442	*0/0.9604
	0.3072	0.7786	0.8510	*0/0.9669	*0/1.1440

*Global Maximum/Local Maximum

nature of borrowing constraints, as zeros still occur for $R = 2$ and low values of E_b^y, and the qualitative comparisons are similar to those from the basic model. One explanation for this is that I still allow for unrestricted savings in the first period, so workers take into account the borrowing constraints in the second period when they make their savings decision, and the desire for within-period consumption smoothing may be fairly small. Additionally, using the same expression for $\frac{\Delta c}{c_1}$ when I have redefined c_u to be consumption while unemployed will tend to shift the results downwards, offsetting the tendency of optimal benefit levels to increase.

I.3 Second-Order Taylor Series Expansion of Marginal Utility

Chetty (2006) argues that ignoring third and higher derivatives of the utility function may be a mistake; he reports that, for simulation exercises using a CRRA utility function, using a first-order expansion of marginal utility can sometimes lead to an underestimate of the true optimal replacement rate on the order of 30%, whereas a revised welfare equation using a second-order expansion reduces this error to less than 4%. The model used by Chetty (2006) is somewhat different from mine, and he writes all marginal utilities in terms of consumption while employed rather than $U'(c_u)$, so the results are not directly comparable.[61] However, I will now explore how the results change when I use a second-order Taylor series expansion of marginal utility.

To do so, I must follow the approach of Chetty (2006) and rewrite my expression in terms of $U'(c_1)$ rather than $U'(c_u)$, although this will hinder the comparability of my results with those of the baseline case. I begin with (4) and (5), and the standard first-order condition for saving. Next, I use a new Taylor series

[61]The first-order Taylor series used in my paper is in fact an exact equality, not an approximation; it is the assumption that $c_1U''(\theta) = c_uU''(c_u)$ which generates the potential for error. As already discussed, that assumption tends to be a liberal one, but Chetty's effective assumption that $U''(\theta)$ is equal to U'' at the average level of consumption while employed is a conservative one in his context, which explains why this leads to an underestimate in his paper. Baily's assumption that the E_b^τ in the denominator of the right-hand side of (9) is equal to one is, in the context of his model, a significant reason for underestimation of the optimal b.

expansion of $U'(c_u)$ around $U'(c_1)$:

$$U'(c_u) = U'(c_1) - \Delta c U''(c_1) + \Delta c^2 \frac{U'''(\theta)}{2}$$

where θ is not necessarily the same value as before, but is still between c_1 and c_u. Using this to replace $U'(c_u)$ in both (4) and (5):

$$\frac{dV}{db} = 2u \left[U'(c_1) - \Delta c U''(c_1) + \Delta c^2 \frac{U'''(\theta)}{2} \right]$$
$$- \left[[(2-\delta)y + \delta s y_n] U'(c_1) + \delta(y - s y_n) \left(\Delta c U''(c_1) - \Delta c^2 \frac{U'''(\theta)}{2} \right) \right] \frac{d\tau}{db}.$$

I make the usual assumption that $y_n = y$, and add the modified assumption that $\theta = c_1$, and then a bit of rearranging gives:

$$\frac{dW_1}{db} \equiv \frac{\frac{dV}{db}}{U'(c_1)} = 2y \left(\frac{\Delta c}{c_1} R + \frac{1}{2} \left(\frac{\Delta c}{c_1} \right)^2 RP \right) \frac{d\tau}{db} - 2(1-u)y \left(1 + \frac{\Delta c}{c_1} R + \frac{1}{2} \left(\frac{\Delta c}{c_1} \right)^2 RP \right) \left[\frac{d\tau}{db} - \omega \right]$$

where $P = \frac{-c_1 U'''(c_1)}{U''(c_1)}$ is the coefficient of relative prudence. Therefore, the equation for the optimum is:

$$\left(\frac{\Delta c}{c_1} R + \frac{1}{2} \left(\frac{\Delta c}{c_1} \right)^2 RP \right) = (1-u) \left(1 + \frac{\Delta c}{c_1} R + \frac{1}{2} \left(\frac{\Delta c}{c_1} \right)^2 RP \right) \frac{E_b^\tau - \psi}{E_b^\tau} \qquad (20)$$

where E_b^τ is unchanged thus and still given by (13).

I can use parameter values and functions as before, but there is one additional parameter to select: the coefficient of relative prudence. In a CRRA utility function $U(c) = \frac{c^{1-R}}{1-R}$, $P = \frac{-cU'''(c)}{U''(c)} = R + 1$, so one possibility is to set $P = R + 1$. However, previous studies have tended to find low estimates of relative prudence; Merrigan and Normandin (1996) are on the high end of the results in the literature when they find estimates ranging from 1.78 to 2.33.[62] I will therefore use a value of $P = 2$, and the results from evaluation of the optimal replacement rate are displayed in Tables 44 and 45.

Table 44: Optimal Replacement Rates Calculated from (20) for $R = 2$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0.4000	0.3947	0.3858	0.3707
	0	0.4124	0.4124	0.4124	0.4124
E_b^y	0.048	0.4196	0.4227	0.4281	0.4371
	0.096	0.4269	0.4331	0.4438	0.4620
	0.192	0.4414	0.4539	0.4755	0.5124
	0.3072	0.4588	0.4788	0.5138	0.5741

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
	-0.0816	0	0	0	0
	0	0	0	0	0
E_b^y	0.048	0.0238	0.0963	0.3317	0.5013
	0.096	0.3073	0.4384	0.5843	0.7581
	0.192	0.5502	0.6794	0.8513	1.0768
	0.3072	0.7128	0.8637	1.0720	1.3510

The results this time are somewhat different quantitatively, in that optimal replacement rates are zero for low values of E_b^y even for $R = 5$. However, the qualitative comparison remains the same, right down to nearly the exact same pairwise comparisons: at low values of s and especially E_b^y, the fiscal externality term considerably reduces the optimal replacement rate, whereas at higher values it considerably increases it.

[62] Eisenhauer and Ventura (2003) are an exception in finding values of R and P in the 7 to 8 range, but they base their estimation on answers regarding willingness to pay for a security from a Bank of Italy survey of Italian households.

Table 45: Optimal Replacement Rates Calculated from (20) for $R = 5$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.6522	0.6499	0.6549	0.6392
	0	0.6574	0.6574	0.6574	0.6574
	0.048	0.6604	0.6618	0.6641	0.6682
	0.096	0.6634	0.6662	0.6709	0.6790
	0.192	0.6695	0.6750	0.6846	0.7008
	0.3072	0.6768	0.6857	0.7011	0.7274

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0	0	0	0
	0	0	0	0	0
	0.048	0.5565	0.5817	0.6201	0.6760
	0.096	0.6095	0.6499	0.7101	0.7955
	0.192	0.6922	0.7543	0.8452	0.9717
	0.3072	0.7704	0.8521	0.9700	1.1313

I.4 Variable Labour Supply

To this point, I have assumed that y is fixed, and thus ignored any distortionary effects of taxes on labour supply among the employed. Chetty (2006) points out that, with a lump-sum tax on workers, the envelope condition means that whether or not individuals can change the amount of their labour supply while employed is irrelevant to the optimal UI calculation. However, with a proportional tax, changes in y have an effect though the government budget constraint. Saez (2002) argues that much of the responsiveness of modest-income workers is on the extensive margin, which is already largely captured here by the decision about whether or not to seek work, but all the same I will examine how significant this effect could be. I begin by rewriting the utility function to allow for choice of y, assuming that the worker must make the same choice of y in both the first and second periods if they retain their job. If disutility from work effort, which I denote as $h(y)$, is separable from consumption, (1) becomes:

$$V = U[y(1-\tau)-k] + (1-\delta)U[y(1-\tau)+k] - (2-\delta)h(y) + \delta U[(1-s)(b-e) + sy_n(1-\tau)+k] - \delta h(sy_n).$$

Because (4) and (5) are unchanged, both (8) and (9) remain valid; the only change is to the derivative of the government budget constraint. The latter now becomes:

$$\frac{d\tau}{db} = \frac{\delta(1-s) - \delta b\frac{ds}{db} - \delta\tau y_n\frac{ds}{db} - \delta s\tau\frac{dy_n}{db} - (2-\delta)\tau\frac{dy}{db}}{(2-\delta)y + \delta sy_n}.$$

and rewritten in terms of elasticities, this is equivalent to:

$$E_b^\tau = \psi + \psi E_b^u + \frac{u}{1-u}E_b^D - \frac{\delta s}{2(1-u)}E_b^y - \frac{2-\delta}{2(1-u)}\varepsilon_b^y \qquad (21)$$

where $\varepsilon_b^y = \frac{b}{y}\frac{dy}{db}$.

I now have to decide on a value for ε_b^y. When b increases, τ increases as well - unless E_b^y is so large as to actually lead to increased tax revenues, which cannot be the case in equilibrium - so some version of an elasticity of taxable income is required. Gruber and Saez (2002) find an elasticity of taxable income with respect to the net-of-tax rate of 0.4;[63] using this, and assuming that the only effect of changes in b and τ on y go through the channel of taxes:

$$\varepsilon_b^y = \frac{dy}{db}\frac{b}{y} = \frac{dy}{d\tau}\frac{d\tau}{db}\frac{b}{y} = -0.4\frac{b}{1-\tau}\frac{d\tau}{db}.$$

To simplify the calculations, I replace $\frac{d\tau}{db}$ with the partial derivative:

$$\varepsilon_b^y \simeq -0.4\frac{b}{1-\tau}\left[\frac{\delta(1-s)}{((2-\delta)y+\delta sy_n}\right] \simeq -0.4\frac{\tau}{1-\tau}\psi.$$

Finally, I use the baseline tax rate of $\tau = 0.23$, so my estimate of the elasticity is $\varepsilon_b^y = \frac{-0.092}{0.77}\psi$; I do not need to multiply this by 0.48, as this estimate is meant to apply to the entire universe of workers. The ensuing numerical results are displayed in Tables 46 and 47.

Table 46: Optimal Replacement Rates Calculated from (9) and (21) for $R = 2$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.3412	0.3428	0.3456	0.3503
	0	0.3496	0.3548	0.3638	0.3791
	0.048	0.3546	0.3619	0.3746	0.3966
	0.096	0.3596	0.3691	0.3856	0.4144
	0.192	0.3697	0.3835	0.4080	0.4513
	0.3072	0.3818	0.4012	0.4356	0.4978

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0	0	0	0
	0	0	0	0	0
	0.048	0.0270	0.1511	0.3125	0.4579
	0.096	0.2854	0.3886	0.5185	0.6897
	0.192	0.4809	0.5989	0.7670	1.0039
	0.3072	0.6279	0.7733	0.9855	1.2867

Table 47: Optimal Replacement Rates Calculated from (9) and (21) for $R = 5$

Optimal r for $G = 0$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.6399	0.6406	0.6417	0.6435
	0	0.6430	0.6450	0.6486	0.6547
	0.048	0.6448	0.6477	0.6527	0.6615
	0.096	0.6466	0.6504	0.6569	0.6684
	0.192	0.6503	0.6558	0.6656	0.6828
	0.3072	0.6548	0.6625	0.6763	0.7011

Optimal r for $G = 0.208$:

		s_0			
		0.648	0.725	0.8	0.863
E_b^y	-0.0816	0.5198	0.5020	0.4719	0.4229
	0	0.5639	0.5656	0.5685	0.5735
	0.048	0.5898	0.6033	0.6266	0.6669
	0.096	0.6156	0.6408	0.6846	0.7598
	0.192	0.6666	0.7149	0.7983	0.9385
	0.3072	0.7263	0.8012	0.9282	1.1345

The inclusion of the labour supply elasticity tends to lower the optimal replacement rate, though in many cases not by much; in the $R = 5$ case, the effect is often almost negligible, whereas in the $R = 2$ case the effect can be somewhat larger for moderate values of E_b^y. However, the essential point remains that consideration of fiscal externalities can greatly change the results, either in a positive or negative direction; the pairwise comparisons are again nearly identical to the baseline.

I.5 Summary of Extensions

In each of the four extensions considered, altering the model generally does change the numerical results; allowing for a stochastic duration of unemployment or second-period borrowing constraints tends to move the optimal replacement rates closer to one, whereas allowing for variable y or using the third derivative of marginal utility tends to reduce the estimated optimal replacement rate. These results, however, are all remarkable similar in terms of what they tell us about the importance of fiscal externalities; even the pairwise comparisons of optimal replacement rates given $G = 0$ versus $G = 0.208$ are nearly identical in each case.

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[63] Their estimated elasticities for lower and moderate income individuals, who are more likely to end up on UI, are smaller, so this is likely to exaggerate the distortionary effects of taxation.

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