Heterogeneity matters: labour productivity differentiated by age and skills

Patrick Aubert[1]
Muriel Roger[2]
Malgorzata Wasmer[3][4]

December 2013

(Work in Progress)

[1] SG-COR and CREST-INSEE
[2] Banque de France and PSE(INRA)
[3] CNRS GATE Lyon, Office fédéral de la santé public Suisse
[4]The data analysis has been completed during the research stay of Malgorzata Wasmer at the Institut National de la Statistique et des Etudes Economique in Paris. We would also like to thank Didier Blanchet, Anthony Briant and Sébastien Roux for their fruitful remarks as our work was progressing and to Virginie Régnier for her assistance in the dataset construction.

1 Introduction

An ageing population bears important consequences for labour markets. Policy makers and an increasing number of individual organisations as well as private firms realise that early retirement of older employees deprives companies of valuable expertise and creates a shortage of qualified employees. Thus, the trend towards early retirement policies starts being reversed.

Nevertheless, the available evidence indicates that in many countries older workers continue to confront unfavourable labour market conditions compared to other age groups. Despite the increasing need for working life extension as well as the age diversity enabling the transmission of skills and know-how, the employment and hiring rates of older workers stay at a very low level in many developed countries. Employers' negative perceptions about the adaptability and productivity of older employees are among the reasons. An increase in the workforce's average age is frequently associated with higher labour costs as well as greater resistance to technological developments and only rarely with an expected increase in productivity (Remery and al., 2003). Although employers appreciate older employees' experience, loyalty and low turnover, nevertheless younger workers are preferred when it comes to actual hiring decisions (Guest and Shacklock, 2005).

Actually, the distribution of productivity across the age groups is not a priori determined. It can be higher for older workers due to longer experience acquired and higher level of job-specific knowledge. On the other hand, the older workers' efficiency might become deteriorated as a result of the structural changes in the labour market, e.g. accelerating technological progress and less training offered (Daveri and Maliranta, 2007; Maurer, 2001). Interestingly, age-productivity profiles might differ between occupations. Older employees can remain highly productive in the domain that they know well and where relatively long experience is important. Thanks to tacit knowledge, older managers may perform as well as younger ones (Colonia-Willner, 1998). On the other hand, reduction in productivity of seniors appears to be the strongest in jobs where speed, spatial orientation and learning new materials are important (Skirbekk, 2004).

In particular, the situation of older workers on the labour market may be undermined when aging drives a negative wedge between the workers' productivity and earnings. The increasing age-earnings pattern characterised by lower wages for young workers and higher wages for older workers is widely observed (Loewenstein and Sicherman, 1991). This fact combined with doubts about the true productivity of older workers raises an important question on the relations between productivity and earnings across the age groups. There exists some empirical evidence against the paradigm of wage and marginal productivity equality. Frank (1984) as well as Campbell and Kamlani (1997) have found that wage rates vary substantially less than do individual productivity values. Even if a wide discrepancy in productivity among individual workers exists, many firms continue to follow strict remuneration schemes based on education, experience and tenure length. As a result of rigid remuneration systems, in response to a negative productivity shock, employers instead of adjusting wages, adjust their employment structure. Consequently, the least productive workers are the first to become redundant. Likewise, the employment opportunities of workers whose wages exceed their productivity levels become reduced.

The present study aims at evaluating the actual profile of marginal productivity across the age classes within the workforce. The comparison with earnings profiles allow us to analyse the relative productivity and test whether differences in wage shares across groups of workers are justified by proportional productivity contribution. Since, as mentioned above, age-productivity profiles might differ between occupations, the workforce has been differentiated not only by age (young, middle-aged, old), but also by skills (low-skilled, high-skilled). The simultaneous differentiation by age and by skills is of high interest in the perspective of possible dissimilarities among different categories of workers with respect to the sensitivity to work effort incentives, training offered, etc.

Although there is a growing research interest in the relation between age and productivity, the empirical analyses so far have often been focused on the estimation of Cobb-Douglas production functions specification in capital and labour. The firm-level labour productivity itself is treated as a simple summation of productivities of individual workers (Hellerstein et al., 1999; Crépon et al., 2003; Aubert and Crépon, 2003). Thus, the existing studies are characterised by an assumption of perfect substitutability between different categories of workers. In this study, we refer to the production function estimation as well. However, in contrast to the previous studies, the use of the less restrictive, constant-elasticity-of-substitution (CES) functional form is proposed at the level of labour input. This more general form, thanks to smaller number of constraints imposed on the production technology, allows the imperfect substitution between different categories of workers.

The dataset used in this study (DADS-BRN) covers the French manufacturing, services and trade sectors. French data are particularly interesting in the perspective of our study. Actually, among all OECD countries, France is characterised by the highest employment rate of people aged 25-54 (83 % in 2008) and at the same time one of the lowest employment rate of people over 55 (38 % in 2008). In fact, workers over 50 are often affected by long-term unemployment. In particular, the low-skilled workers face problems to stay employed and once unemployed hardly find a new job.

Differentiating the workforce simultaneously by age and skills allows us to observe the differences in the age-productivity and age-earnings profiles separately within each skill group. We find that this differentiation is, in fact, very important. The productivity profile observed across different age groups seems actually to depend on the skill level. Among the main findings of this study, labour productivity is found to be the lowest for the oldest low-skilled workers. In the high-skilled manufacturing labour, the mean productivity stays quite stable across the age groups, being the highest for the workers over 50. In trade, the high-skilled oldest employees are clearly the most productive group. Moreover, we observe a very similar age-earnings pattern across the sectors. Wage rates vary considerably less than productivity in both skill groups. The wage profile is however steeper for the high-skilled workers.

The results for the manufacturing sector show that the age-productivity and age-earnings profiles are compatible with a deferred compensation system. It might indicate that the effort incentive problem has been regulated in practice by many firms by offering at the start of the career wages under the workers' marginal productivity and compensating this difference in the later periods. On the other hand, in services and in trade, we observe the combined relevance of specific human capital and deferred compensation.

Though, the most interesting aspect is the workers' productivity in relation to their cost. It is particularly important as it may present for the employers an incentive to exclude some age groups from the labour market and to give preference to the others. In our study, the relative productivity over cost in manufacturing sector has been found to represent a similar pattern in both skill groups, being the highest for the young, followed by middle-aged and old workers. In both skill groups in services and for low-skilled trade employees the productivity/earnings ratio is the highest for the middle-aged, followed by young and senior workers. This discrepancy between productivity and wage can be a source of employment difficulties particularly for the older low-skilled workers.

The remainder of the paper is organised as follows. Section 2 discusses the existing theoretical literature as well as empirical evidence on the relation between age, wage and productivity. Section 3 shows the model. In the fourth part an econometric estimation method is presented. The dataset used in this study has been described in part 5. The estimation results are analysed in section 6, followed by the conclusions.

2 Age, wage and productivity: a review

2.1 Theoretical background

From an employer's perspective, the actual distribution of productivity and earnings across different age groups of the current workforce is very important. While deciding on the production level, a firm has to choose the optimal level of labour input needed to generate the given output. From the economic point of view, "there is an incentive to find the age mix of the workforce that can produce a given output at the least cost. This will be the age mix that yields the highest labour productivity and is described as the optimal age mix of the firm's workforce" (Guest and Shacklock, 2005). Although the first aspect, dispersion of earnings, is usually easy to verify, there exists still large uncertainties regarding the productivity distribution across age categories.

Over the 70s and 80s, the relationship between workers' age, wage rates and their productivity has attracted the attention of many researchers. Observations of wage tending to grow with worker's seniority in the firm brought questions on the link between this phenomenon and the evolution of worker's productivity. There exists two important approaches that predict a possible relation between age, wage and productivity: 1) human capital theory suggesting that wage profiles are either equivalent to or flatter than productivity growth over the life cycle and 2) deferred compensation models justifying the need for a wage profile steeper than productivity.

An explanation yielded by human capital theory is based on the idea that wages increase over time due to investments in human capital, particularly investments in the job training (Mincer, 1974; Becker, 1975). Older workers are therefore paid more since they have accumulated more firm-specific human capital and thus they are more productive. The general human capital theory is founded on the assumption that at any point in time workers' wage indicates their productivity. Wages rise over the life cycle at a decreasing rate until depreciation exceeds the level of skill acquisition, yielding a concave earnings profile. According to specific human capital theory, the firm and the worker are assumed to share the investment in worker's training during an initial period. While being trained, workers receive a wage that is lower than wages offered otherwise but still higher than their productivity. Thanks to training, workers become more productive and in later periods gain the returns from the investment through higher marginal products and higher wages. The second period wage, although higher than in the previous period, lies below the value of marginal productivity. In fact, the employer and the employee set the second period wage so as to split the quasi-rents generated by specific training. In this case, the resulting wage profiles will be flatter than the productivity path (Hashimoto, 1981).

On the other hand, deferred compensation models underline the possibility of incentive based compensation schemes. A good example of such model is the agency model by Becker and Stigler (1974) and Lazear (1979, 1981). In order to discourage workers' shirking, the firm pays young workers below their marginal productivity and later in their career remunerates them over their marginal product. Senior workers receive high salaries, not due to relatively higher productivity but because it creates the appropriate wage incentives for them and for their younger co-workers (Lazear, 1981). Consequently, a steeper wage profile increases workers' effort. In particular, the young workers who hope to stay in the firm are induced to perform at the optimal level. The efficiency wage models (Yellen, 1984; Shapiro and Stiglitz, 1984; Bulow and Summers, 1986) point out the importance of the payment above market clearing wages as a mechanism to elicit more effort from the worker when it cannot be fully observable. The fact that wages do not reflect the actual productivity but increase with seniority may result in raised employment difficulties for older workers. Indeed, the deferred compensation contracts may constitute a form of fixed costs for the employer. We observe that for certain jobs many firms employ, but tend not to hire older workers (Hutchens, 1986).

According to the arguments mentioned above, it takes time for workers to accumulate education, experience and skill through learning-by-doing. Thus, older workers seem to be more productive than younger ones. However, searching for an optimal age composition of a given workforce, one should not forget that younger cohorts of workers today have much higher education levels than their predecessors. Moreover, within the given enterprise, workers of different age might be less than perfectly substitutable as it is usually assumed. Hence, the optimum age composition of a given workforce might in fact depend on two elements: relative marginal productivity and the degree of substitutability between workers of different age (Lam, 1989).

2.2 Empirical evidence

In the empirical literature, there is relatively little research aiming at productivity and wage data comparison and their correlation with workers' age. Among the recent studies, the contribution of Hellerstein and al. (1999) is worth being mentioned. Using a cross-section plant-level matched employer-employee dataset, the authors analysed the relationship between productivity and wage differentials among manufacturing workers distinguished by different demographic characteristics such as gender, race, marital status, age, education and occupation. In this purpose, a translog production function was jointly estimated with earnings equation. Different categories of workers were assumed to be perfectly substitutable but have potentially different marginal products. The condition of perfectly competitive market was not imposed. The authors allowed possible inequality between relative marginal productivity and relative wage for different groups of workers, which could be then interpreted as an indicator of long-term incentive contracts or discrimination. As a result of their analysis, the authors claimed that in fact wage differentials reflect actual differences in marginal products for most types of workers, particularly for the age category. Consequently, and as underlined by authors, this finding is coherent with the general human capital model by Mincer (1974) mentioned above.

Opposite results have been obtained by Crépon and al. (2003) who expanded the approach by Hellerstein et al. Using the French matched employer-employee panel data set they estimated the Cobb-Douglas production function. The assumption of perfect substitutability between different types of workers was also made. However, instead of parallel estimation of production function and earnings equation as had done Hellerstein et al., only one equation was estimated here. The authors made use of disaggregated data on wages that were not available to Hellerstein et al. The production function was modified in a way to contain directly a ratio of hourly productivity to wage for different workers categories. Among the findings, the authors stated the existence of a wage productivity gap which tends to expand with age. The wages continue to increase with workers' age whereas the productivity stops rising at one point or even declines. It is though unclear whether the old workers are overpaid or the young ones underpaid, or if both events take place. However, the authors pointed out that increase in wages for workers over 35 cannot be interpreted as reflecting human capital accumulation.

Expanding on the previous methodologies, Aubert and Crépon (2003) made use of the French panel data decomposing the labour force into thinner age groups. Through the estimation of the Cobb-Douglas production function, they found that productivity tends to grow with age up to age of 40 and stabilises afterwards. In all sectors, workers aged 35-39 appear to be slightly less productive than those over 40 and around 15 to 20% more productive than young workers under 30 years old. At the same time, the authors found no evidence of a significant difference between wage and productivity that could explain the lower employability of older workers. Although for workers older than 55 a slight decrease in productivity is observed, this result is not statistically significant.

Again, quite different results have been obtained by Hellerstein and Neumark (2007) who extend the analysis of their previous article (Hellerstein et al., 1999). They used the same specifications and sample selection criteria but used larger and more representative dataset. The estimated age profiles suggest that the most productive group are prime-age workers (35-54), followed by the younger ones and the seniors (over 55) as the least productive. Furthermore, the wage profile appears steeper than productivity profile, a result consistent with the deferred compensation model à la Lazear. Finally, the authors strongly rejected the hypothesis of productivity and wage differentials equality.

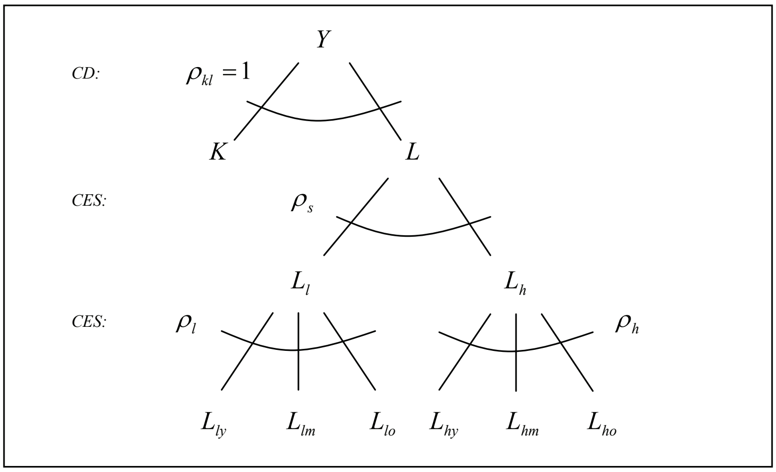
Overall, it is evident from this literature review that no clear conclusion can be drawn from earlier research. The current paper continues to investigate the productivity and wage relationship across different age categories of labour trying to overcome one common limitation of previous studies. In our analysis, we introduce the possibility of imperfect substitutability between different categories of workers. In previous empirical works, regardless of the choice of the production function specification in labour and capital (Cobb-Douglas or translog), different workers types were simply summed up under the assumption of perfect substitutability.

3 The model: production function with labour as nested CES

The assumption of perfect substitutability between workers with different characteristics implies that employing one worker while dismissing another one will not lead to any change in the marginal products of either of them as one is perfectly substitutable for another. However, it has been noticed that actually there might arise an interaction of workers within a firm (Lengermann, 2002) . The productivity of a certain employee might be affected by co-worker's characteristics. It might matter whether the employee works together with a colleague with the same level of skills, similar age, etc. In particular, there exists empirical evidence that the human capitals of young and older workers are imperfect substitutes (Kremer and Thomson, 1998) . Hence, labour is not necessarily as easily substitutable as it seems at first glance. This study tries to take into account this possibility by choosing such a form of the production function that would take into account the potential imperfect substitutability within the workforce - between high-skilled and low-skilled workers and between different age categories within each skill group. For this purpose, we estimate the Cobb-Douglas production function specification in capital (K) and labour (L) whereas the labour input itself takes the form of the nested constant-elasticity-of-substitution (CES) function.

Since Arrow and al. (1961) have formulated the CES function, numerous studies have aimed at estimating its parameters, but not so far in the context of the labour productivity analysis. As mentioned before, most of research on age-productivity pattern has been based so far on the estimation of the production function. However, by imposing the additive functional form for different categories of labour inputs, they assumed a perfect substitution between them.

Figure 1: Scheme of the production structure



Our benchmark (see Figure 1) takes into account two skill groups (low-skilled (L_l) and high-skilled (L_h)) and within each skill category - three age groups of workers (young (L_y), middle-aged (L_m), old (L_o)). The labour input is allowed to be heterogeneous across but homogeneous within closely defined groups of workers. Thus, it is assumed that the employees belonging to the same skill-age group (e.g. young low-skilled) are perfectly substitutable.

At the "highest level", our production function takes the Cobb-Douglas form given by:

$$Y = f(K, L) = AK^{\alpha}L^{\beta}$$

where K denotes capital, L stands for labour and A is a Hicks neutral technological progress.

At the “second level” the labour aggregate is defined as a CES function of high-skilled and low-skilled workers:

$$L = \left(\sum_i \delta_i L_i^{\rho_i}\right)^{\frac{1}{\rho_i}} \tag{1}$$

where i indicates the skill category. Finally, each skill group of workers is a CES function by itself:

$$L_i = \left[\sum_j \delta_{ij} L_{ij}^{\rho_{ij}}\right]^{\frac{1}{\rho_{ij}}} \tag{2}$$

where the age category is denoted by j .

Based on this choice of production function, we aim at estimating the distribution parameters: δ_i, δ_{ij} as well as the substitution parameters: ρ_i and ρ_{ij}. The elasticity of substitution is defined as $\sigma = \frac{d\ln(x_1/x_2)}{d\ln\left(\frac{\partial Y}{\partial x_1}/\frac{\partial Y}{\partial x_2}\right)}$ and is a measure of the percentage change in factors demand due to a percentage change in the marginal rate of technical substitution so that the output remains constant. For the case of constant returns to scale it takes the form: $\sigma = \frac{1}{1-\rho}$. The inverse of sigma $\left(\frac{1}{\sigma}\right)$ denotes a change in the marginal rate of technical substitution due to a change in factor proportions so that output remains constant.

Labour productivity contribution

Given the estimate value of the production function parameters, we compute the marginal product of labour for different categories of labour. Since, in our setting, constant returns to scale are assumed at the level of labour input, we can make use of Euler's theorem to specify the function of labour. According to Euler, if a function is homogenous of degree 1, it can be represented as a sum of its inputs multiplied by their marginal products. Thus, our labour function takes the following form:

$$f(L_1, L_2, ..., L_n) = L_1\frac{\partial f}{\partial L_1} + L_2\frac{\partial f}{\partial L_2} + ... + L_n\frac{\partial f}{\partial L_n} \tag{3}$$

Consequently, the labour production can be entirely explained by the sum of the marginal products of different labour categories multiplied by their employment level.

Skill differentiation

The marginal product of a particular labour input is defined as the partial derivative of the production function with respect to the specific category of labour. For the given skill group, it takes the following form:

$$MP_i = \frac{\partial Y}{\partial L}\frac{\partial L}{\partial L_i} \tag{4}$$

$$MP_i = AK^{\alpha}\beta\left(\sum_i \delta_i L_i^{\rho_i}\right)^{\frac{\beta}{\rho_i}-1}\delta_i L_i^{\rho_i-1} \tag{5}$$

The marginal rate of technical substitution depends not only on the factor intensity and the distribution parameter but also on the level of substitution between different labour categories. It shows the rate at which one input may be substituted for another, while maintaining the same level of production. The relative marginal product of labour for workers differentiated by skills is given by:

$$\frac{MP_1}{MP_2} = \frac{\partial L/\partial L_1}{\partial L/\partial L_2} = \lambda = \frac{\delta_1}{\delta_2}\left(\frac{L_1}{L_2}\right)^{\rho_i-1} \tag{6}$$

In order to compare productivity contribution over different skill categories, we compute for each firm ratios of marginal products of workers belonging to certain skill group in relation to the average marginal product of labour. For two categories of skills, the ratios take the following form:

$$\frac{MP_1}{MP_{av}} = \frac{L}{L_1+\lambda^{-1}L_2} \text{ and } \frac{MP_2}{MP_{av}} = \frac{L}{\lambda L_1 + L_2}$$

where MP_{av} is the average marginal product of total labour.

Age differentiation

The marginal product of labour for a given age group in a specified skill category is defined as:

$$MP_{ij} = \frac{\partial Y}{\partial L}\frac{\partial L}{\partial L_i}\frac{\partial L_i}{\partial L_{ij}} \tag{7}$$

$$MP_i = AK^{\alpha}\beta\left(\sum_i \delta_i L_i^{\rho_i}\right)^{\frac{\beta}{\rho_i}-1}\delta_i L_i^{\frac{\rho_i}{\rho_j}-1}\delta_{ij}L_{ij}^{\rho_{ij}-1} \tag{8}$$

The relative marginal product of any two age groups of workers in a given skill group is:

$$\frac{MP_{i1}}{MP_{i2}} = \frac{\delta_{i1}}{\delta_{i2}}\left(\frac{L_{i1}}{L_{i2}}\right)^{\rho_{ij}-1} \tag{9}$$

In our setting, we define relative marginal products as: $\frac{MP_{iY}}{MP_{iM}} = \varphi$, $\frac{MP_{iY}}{MP_{iO}} = \gamma$ and $\frac{MP_{iM}}{MP_{iO}} = \eta$ (where Y - young, M - middle-aged, O - old). The productivity contribution of each age group is given by the ratio of marginal product of respective age group over the average labour marginal productivity of a specific skill group:

$$\frac{MP_{iY}}{MP_{av}} = \frac{L_i}{L_{iY} + \varphi^{-1}L_{iM} + \gamma^{-1}L_{iO}} \tag{10}$$

$$\frac{MP_{iM}}{MP_{av}} = \frac{L_i}{\varphi L_{iY} + L_{iM} + \eta^{-1}L_{iO}} \tag{11}$$

$$\frac{MP_{iO}}{MP_{av}} = \frac{L_i}{\gamma L_{iY} + \eta L_{iM} + L_{iO}} \tag{12}$$

Wage share

Our dataset contains rich information on earnings. Hence, according to the procedure above, we compute the share of a distinct age group in the wage bill of the given skill category. Given the productivity contributions and the analogously constructed wage shares, we can compare an earnings-productivity pattern for different categories of workers.

4 The method

In the econometric estimation of the production function, one of the major problems we are confronted with is the possible endogeneity of inputs, which will result in inconsistency of direct estimators. In fact, the input demand might be correlated with the productivity shocks unobservable by the econometrician but observed or predicted by the firm (Marschak and Andrews, 1944). A profit maximizing (or cost-minimizing) firm facing positive productivity shocks will expand its production and thus increase the inputs level. Conversely, a consequence of the negative shocks will be a decrease in production and lower input usage. A particular concern about the endogeneity bias concerns the analysis of labour disaggregated by age. Aubert and Crépon (2003) draw attention to the fact that enterprise could respond to negative productivity shocks by postponing its hiring decisions. Then, one might observe a decline in production accompanied by ageing workforce. On the other hand, positive shocks could encourage the firm to hire some young workers. As a result, the rise in output would be associated with a relatively younger workforce. The causality problem that appears in such case is whether firms employing relatively older workforce are less productive or if firms employ relatively older workforce because they are less productive.

In order to obtain consistent estimates of the production function parameters, we use the method developed by Levinshon and Petrin (2003). The procedure consists in including in the estimation equation a proxy for the productivity shocks potentially observed by firms while making input decisions[1].

We consider the following value added production function:

$$y_{it} = \beta_0 + \alpha k_{it} + \beta ln\left(\left(\sum_i \delta_i \left(\sum_j \delta_{ij} L_{ij\,it}^{\rho_{ij}}\right)^{\frac{\rho_i}{\rho_{ij}}}\right)^{\frac{1}{\rho_i}}\right) + \omega_{it} + \eta_{it} \tag{13}$$

The error term is composed of 2 elements: ω_{it} denoting productivity shocks likely to be observed by the firm and η_{it} having no impact on the firm's inputs decisions. y_{it} is the natural logarithm of value added and k_{it} denotes the natural logarithm of capital.

Following LP, we assume that firms decide on the level of capital at $t-1$, thus capital is a dynamic input. Labour and the intermediate inputs (materials) m_{it} are chosen at time t . The productivity shock ω_{it} is assumed to follow a first order Markov process:

$$p(\omega_{it}|I_{it-1}) = p(\omega_{it}|\omega_{it-1}) \tag{14}$$

where I is firm's i's information set at t.

The approach adopted in the current work consists in using intermediate input as a proxy for the unobservable productivity shocks. Hence, materials control for the part of the error term correlated with inputs. Given the above timing assumptions, the firm's demand for the intermediate input m_{it} is assumed to depend on the state variables k_{it} and ω_{it}.

$$m_{it} = f_t(k_{it}, \omega_{it}) \tag{15}$$

The assumption that intermediate input is strictly monotonic in the productivity shock allows inversion of materials demand function for ω_{it}

$$\omega_{it} = f_t^{-1}(k_{it}, m_{it}) \tag{16}$$

Substituting it into the production function leads to the following first stage equation:

$$y_{it} = \beta ln\left(\left(\sum_i \delta_i \left(\sum_j \delta_{ij} L_{ij\,it}^{\rho_{ij}}\right)^{\frac{\rho_i}{\rho_{ij}}}\right)^{\frac{1}{\rho_i}}\right) + \phi_{it}(k_{it}, m_{it}) + \eta_{it} \tag{17}$$

[1] An alternative approach toward endogeneity bias has been developed by Arellano and Bond (1991) and consisted in estimating an equation in first-differences with appropriately lagged levels as instruments. However, since lagged variables in levels are often weak instruments for contemporaneous differences, in case of highly persistent data, the method appeared to suffer from finite sample bias and poor precision of the estimates. This problem has been further addressed by Blundell and Bond (1998), Blundell and Bond (2000), Blundell, Bond and Windmeijer (2000) and Windmeijer (2000). Within this framework lagged levels are used as instruments for contemporaneous differences and lagged differences as instruments for contemporaneous levels.

where $\phi_{it}(k_{it}, m_{it}) = \beta_0 + \alpha k_{it} + \omega_{it}(k_{it}, m_{it})$.

By substituting a third-order polynomial in k_{it} and m_{it} to $\phi_{it}(k_{it}, m_{it})$:

$$y_{it} = \delta_0 + \beta ln \left(\left(\sum_i \delta_i \left(\sum_j \delta_{ij} L_{ij\,it}^{\rho_{ij}} \right)^{\frac{\rho_i}{\rho_{ij}}} \right)^{\frac{1}{\rho_i}} \right) + \sum_{j=0}^{3} \sum_{n=0}^{3-j} \delta_{jn} k_{it}^{j} m_{it}^{n} + \eta_{it} \tag{18}$$

we can consistently estimate parameters for labour using the non-linear least squares method.

Identification of the input coefficients according to the method by Levinsohn and Petrin has been questioned recently. Bond and Soderbom (2005) argue that production function parameters are not identified from cross section variation when inputs are perfectly flexible and chosen optimally, and input price are common to all firms. However, their result holds on the property that Cobb Douglas optimal capital and labour inputs demand can be expressed as log linear functions of real input prices. This is not the case in our model where labour input takes a form of a CES production function. Moreover, we do not impose the condition of an optimal input choice implying wage and marginal productivity equality.

On the other hand, Ackerberg, Caves and Frazer (2006) pointed out the potential problem with the identification of labour input coefficients in the first stage of the Levinsohn and Petrin method. The authors raise the question of collinearity issues. They claim that the simultaneous choice of labour input and material induces collinearity between the variables in the first step regression. Considering their statement, the identification of the labour parameters in our model is relying on the non-linearities of the production function[2].

The second stage of the procedure helps us identify the coefficient for capital.[3] It starts with the computation of the estimated value for $\hat{\phi}_{it}$:

[2] Ackerberg, Caves and Frazer (2006) propose an alternative procedure to estimate production functions. In their approach, all input coefficients are estimated in the second stage. Their procedure draws on the Levinsohn and Petrin method. Intermediate input demand is indeed used as a proxy to net out an error term of the production function. The main difference is that the firm's demand for the intermediate input is assumed to be a strictly monotonic function of the productivity shock and all the input variables, i.e. capital and all types of labour. In the more complex model that we propose, the firm's demand for the intermediate input is thus a function of 8 arguments. When substituting the productivity shock with a third order polynomial depending on materials and other inputs, identification became really fragile with our data. We thus privileged the Levinsohn and Petrin method in our context.

[3] Even if we consider only marginal labour productivity in the sequel, we implement the two steps of the estimation procedure.

$$\hat{\phi_{it}} = \hat{y_{it}} - \hat{\beta} ln \left(\left(\sum_i \hat{\delta}_i \left(\sum_j \hat{\delta}_{ij} L_{ij\,it}^{\hat{\rho}_{ij}} \right)^{\frac{\hat{\rho}_i}{\hat{\rho}_{ij}}} \right)^{\frac{1}{\hat{\rho}_i}} \right) = \hat{\delta}_0 + \sum_{j=0}^{3} \sum_{n=0}^{3-j} \hat{\delta}_{jn} k_{it}^j m_{it}^n \tag{19}$$

For any candidate value α^*, we can compute the prediction for ω_{it} for all periods t:

$$\hat{\omega}_{it} = \hat{\phi}_{it} - \beta_0^* - \alpha^* k_{it} \tag{20}$$

Given these values, we regress ω_{it} on its lagged term ω_{it-1}:

$$\hat{\omega}_{it} = \gamma_0 + \gamma_1 \omega_{it-1} + \gamma_2 \omega_{it-1}^2 + \gamma_3 \omega_{it-1}^3 + \epsilon_{it} \tag{21}$$

in order to get the residual ξ_{it} and the conditional expectation $E\left[\omega_{it} \mid \omega_{it-1}\right] = \omega_{it} - \xi_{it}$. Given labour coefficients, a starting value for $\alpha^*$and $E\left[\omega_{it} \mid \omega_{it-1}\right]$, we can find a consistent estimate of a parameter for capital through the minimisation of a squared sum of a residuals of our production function:

$$\underset{\alpha^*}{min} \sum_t (\eta_{it} + \xi_{it})^2 = \underset{\alpha^*}{min} \sum_t \left(y_{it} - \hat{\beta} ln \left(\left(\sum_i \hat{\delta}_i \left(\sum_j \hat{\delta}_{ij} L_{ij\,it}^{\hat{\rho}_{ij}} \right)^{\frac{\hat{\rho}_i}{\hat{\rho}_{ij}}} \right)^{\frac{1}{\hat{\rho}_i}} \right) - \alpha^* k_{it} - E[\omega_{it} | \omega_{it-1}] \right)^2 \tag{22}$$

The asymptotic standard errors for estimated parameters can be constructed using a bootstrap approach.

5 Data and summary statistics

The dataset used in this study covers a short panel of data for years 2003 and 2004 for manufacturing, services and trade sectors in France. It comes from merging two different data sources: *Bénéfices Réels Normaux* (BRN) and *Déclarations Administratives de Données Sociales* (DADS). They both constitute mandatory employers' reports to the Fiscal Office. The BRN consists of firms' balance sheets and provides important information on the employers' output, capital stock and economic sector. The DADS contains rich data on the characteristics of the workforce. The number of hours worked is decomposed by workers' age and occupation. The valuable information on earnings allows measuring the share of a distinct labour category in the total wage bill. However, the dataset is not without imperfections: unfortunately, the DADS does not contain any information on workers' education level and tenure.

In order to distinguish among workers according to the level of skills, we make use of the available decomposition by occupation. The DADS employment data are arranged by occupation according to the French socio-professional classification. This classification is used in collective agreements for wage determination. A higher level of education places a worker directly on the "higher starting point" and experience then allows further wage increases. However, as emphasised by Thesmar and Thoenig (2000), this classification, based on the mix of education and experience, contains no information about the task-assignment. Therefore, the executives ("Cadres") may include high-ranked directors as well as e.g. consultants without any supervision duty. Using this occupation classification, we distinguish two skills categories of workers: high-skilled and low-skilled. The high-skilled correspond to employers, the senior and intermediate personnel. The office and sales employees as well as blue collar workers are included in the low-skilled category. For details, see Table 1.

Table 1: Skill classification

High-skilled labour	Low-skilled labour
Employers	*Non-manual workers*
Craftsmen	Office employees
Traders	Sales workers
Employers (of 50 or more employees)	
	Manual workers
Liberal professions, senior and executive personnel	Skilled industrial manual workers
	Skilled craftsman
Liberal professions	Drivers
Professors and scientific professions	Skilled handling, storage and transport workers
Artistic professions	
Senior administrative personnel	Unskilled industrial workers
Engineers and senior technicians	Unskilled artisans
Intermediate personnel	
Medical and social services	
Intermediate administrative personnel	
Technicians	
Foreman, supervisors	

Regarding labour force composition, three age groups are considered within each skill group. We define young workers as those who are under 30 years old, the middle-aged workers between 30 and 50, and the senior employees as those over 50. The reason why we choose these age groups is twofold. Firstly, since we keep only firms where all age categories are present, the condition for having sufficient number of observations within each age group is verified. Secondly, the data analysis revealed that the employment level is much more heterogenous among the young (up to 30) and among seniors (over 50) compared to the middle-aged (30-50) group. In particular, the lowest employment rates characterise the young under 25 and older persons over 55. Nowadays, many young people decide to prolong their education and, thus, enter relatively late into labour market. Similarly, an earlier exit from the labour market is still quite common among the seniors.

For the purpose of our analysis, the volume of production is represented by value added and the employment level is measured by the number of hours worked. It allows distinguishing between part-time and full-time employees. Outliers have been eliminated. Value added, capital, labour cost and employment are required to take positive values. Only firms employing at least fifty workers have been considered. As a result of these operations, the final dataset contains 15 992 observations.

As regards sectoral division, manufacturing, trade and services are distinguished according to NES16 (Nomenclature économique de synthèse en 16 postes). Agriculture, forestry and fishing as well as the construction sector (due to high ratio of seasonal workers) have been excluded from manufacturing. Administration and financial services have not been taken into account in the services sector.

Summary statistics for the main variables as well as the labour force composition by age and skills are represented in Table 2. We can observe substantial differences with respect to age-employment and age-earnings patterns of workers belonging to different skill groups.

Employment pattern

First of all, we can see that in all three sectors, i.e. manufacturing, trade and services, workers between 30 and 50 years old account for around 60% of total hours worked. The employment share of the young and the seniors is considerably lower. If we look separately at each skill group, we can notice that among the high-skilled, the number of hours worked by older workers exceeds those of the young ones. In particular, discrepancies are the biggest in manufacturing. The opposite pattern characterises the low-skilled employees (with exception of manufacturing sector), where the youngs are more numerous than the seniors.

Earnings pattern

We observe that hourly earnings are increasing with age, for both skill groups and in all sectors. The remuneration of young workers is the lowest and the oldest employees are paid the most. According to economic expectations, high-skilled workers are better paid than the low-skilled. Taking into account the desaggregation by skills, the profile of hourly earnings of the low-skilled looks considerably flatter - the differences between consecutive age groups are on the average of 15% and 4%. Interestingly, the respective mean differentials in salaries between the high-skilled age groups are of 40% and 30%. Consequently, the range of salaries in this skill category is wider.

Table 2: Sample statistics, DADS-BRN, 2004

Variables*	Manufacturing			Services			Trade		
	share	mean	sdv	share	mean	sdv	share	mean	sdv
ln value added		-2.67	1.15		-3.01	1.26		-3.04	1.04
ln capital		-2.57	1.54		-3.32	1.85		-3.28	1.38
ln capital (t-1)		-2.61	1.57		-3.38	1.88		-3.34	1.39
ln materials		-2.81	1.63		-5.54	2.06		-6.81	2.15
ln materials (t-1)		-2.86	1.63		-5.59	2.03		-6.82	2.17
Hours worked by age:									
total	1.00	4.85	22.47	1.00	5.28	44.68	1.00	3.83	22.14
(*Ly*) young (<30)	0.18	0.79	4.04	0.21	1.09	4.99	0.28	1.08	6.74
(*Lm*) middle-aged (30-50)	0.60	2.85	11.68	0.58	3.07	24.65	0.57	2.19	12.90
(*Lo*) old (>50)	0.22	1.21	7.09	0.21	1.11	15.93	0.15	0.56	2.76
Hours worked by skills and age:									
(*Ll*) low-skilled	0.59	2.85	13.07	0.58	3.05	18.63	0.65	2.50	17.41
(*Lly*) young	0.18	0.52	2.61	0.25	0.74	3.57	0.34	0.86	5.88
(*Llm*) middle-aged	0.58	1.65	6.65	0.56	1.72	9.64	0.53	1.32	9.70
(*Llo*) old	0.24	0.68	4.05	0.19	0.59	6.24	0.13	0.32	1.99
(*Lh*) high-skilled	0.41	2.00	10.24	0.42	2.23	26.92	0.35	1.33	5.40
(*Lhy*) young	0.14	0.27	1.52	0.16	0.35	1.71	0.17	0.22	1.04
(*Lhm*) middle-aged	0.59	1.19	5.65	0.61	1.36	15.59	0.65	0.87	3.65
(*Lho*) old	0.27	0.53	3.27	0.23	0.51	9.94	0.18	0.24	0.86

* All variables have been standardised (divided by 100 000 before taking logarithms)

Variables	Manufacturing			Services			Trade		
	share	mean	sdv	share	mean	sdv	share	mean	sdv
Hourly earnings by age:									
total	1.00	15.66	3.93	1.00	14.65	5.84	1.00	14.36	4.42
(*Ly*) young (<30)	0.14	12.00	2.46	0.20	11.74	5.51	0.22	10.70	2.42
(*Lm*) middle-aged (30-50)	0.60	15.70	3.98	0.58	14.90	8.04	0.58	14.87	4.52
(*Lo*) old (>50)	0.26	18.99	6.75	0.22	17.90	7.70	0.20	18.58	7.57
Hourly earnings by skills and age:									
(*Ll*) low-skilled	0.52	12.13	2.44	0.51	11.20	2.12	0.50	10.63	1.92
(*Lly*) young	0.17	10.54	1.97	0.25	10.15	1.72	0.30	9.49	1.72
(*Llm*) middle-aged	0.60	12.35	2.53	0.56	11.46	2.39	0.55	11.03	7.49
(*Llo*) old	0.23	12.92	3.08	0.20	11.95	2.88	0.15	11.54	5.76
(*Lh*) high-skilled	0.48	22.13	4.84	0.49	19.94	6.96	0.50	20.62	5.32
(*Lhy*) young	0.11	15.05	4.52	0.16	14.58	6.44	0.11	14.28	0.36
(*Lhm*) middle-aged	0.59	21.73	4.86	0.58	19.88	7.14	0.62	20.36	8.54
(*Lho*) old	0.30	28.00	10.83	0.26	25.96	18.89	0.27	26.66	7.75
Number of observations		8185			4498			3309	

6 Results

We start our analysis with the estimation of the production function whose structure has been detailed in Part 3. Based on estimated parameters, we will generate and compare the age-productivity and age-earnings pattern for the low-skilled and for the high-skilled workers belonging to different sectors. First, on the basis of median values, we will present the general pattern. Afterwards, we will analyse the density estimations of inter-firm distributions of productivity and earnings. The detailed analysis will be carried out consecutively by skills and then by age within each skill group.

6.1 Econometric results

Our estimation procedure consists in estimating three successive models:

model (1) with labour differentiated by skills:

$$Y = AK^{\alpha}\left(\delta_s L_l^{\rho_s} + (1-\delta_s)L_h^{\rho_s}\right)^{\frac{\beta}{\rho_s}}$$

model (2) with labour differentiated by age:

$$Y = AK^{\alpha}\left(\delta_y L_y^{\rho_a} + \delta_m L_m^{\rho_a} + (1-\delta_y-\delta_m)L_o^{\rho_a}\right)^{\frac{\beta}{\rho_a}}$$

model (3) with labour differentiated simultaneously by age and skills:

$$Y = AK^{\alpha}\Bigg(\gamma\left(\delta_{ly}L_{ly}^{\rho_l} + \delta_{lm}L_{lm}^{\rho_l} + (1-\delta_{ly}-\delta_{lm})L_{lo}^{\rho_l}\right)^{\frac{\rho_s}{\rho_l}} +$$

$$+\ (1-\gamma)\left(\delta_{hy}L_{hy}^{\rho_h} + \delta_{hm}L_{hm}^{\rho_h} + (1-\delta_{hy}-\delta_{hm})\,L_{ho}^{\rho_h}\right)^{\frac{\rho_s}{\rho_h}}\ \Bigg)^{\frac{\beta}{\rho_s}}$$

The estimation results of the production function for each sector are presented respectively in Tables 3 to 6[4]. The first column refers to results obtained according to the nonlinear least squares method. The second column reports the production function estimates based on the two-stages procedure by Levinsohn and Petrin (2003) controlling for the potential endogeneity. Since parameters enter into the function in a nonlinear way, estimators only have asymptotic validity. Standard errors have been constructed according to a bootstrap approach with 200 replications.

Elasticity of substitution

[4]In order to check the validity and robustness of our results, in the appendix we present the results of the estimation of different models with sub-sector dummy variables in manufacturing and services sectors (Tables 17 and 18). The results confirm the robustness of the estimates after introducing the sub-sector controls.

In models with labour differentiated by skills, the inter-skill substitution parameter ρ_s has surprisingly been found to converge to 1 in all the sectors (see Table 3) which means perfect substitutability between workers belonging to different skill groups. We suppose that this result might come from the classification of low-skilled and high-skill workers. It is possible that, in fact, there are not much differences in skill levels for certain socio-professional categories. To try to overcome this problem, we have run the estimation with 3 skill groups, taking apart the intermediate personnel but it did not have much impact on the results. In the sequel, estimations are thus made directly under the constraint $\rho_s = 1$.

Table 3: Production function estimates for each sector, labour differentiated by skills, model (1)

Parameters	Manufacturing		Services		Trade	
	NLLS	LP	NLLS	LP	NLLS	LP
c	-2.197*** (0.015)		-2.243*** (0.022)		-2.421*** (0.027)	
α	0.187*** (0.005)	0.301*** (0.047)	0.234*** (0.006)	0.173*** (0.038)	0.118*** (0.008)	0.164*** (0.038)
β	0.829*** (0.008)	0.729*** (0.009)	0.743*** (0.011)	0.699*** (0.023)	0.903*** (0.012)	0.875*** (0.015)
δ_s	0.231*** (0.011)	0.207*** (0.015)	0.190*** (0.033)	0.163*** (0.043)	0.216*** (0.016)	0.216*** (0.019)
ρ_s	1.257*** (0.078)	1.327*** (0.093)	2.408*** (0.322)	2.649*** (0.512)	1.336*** (0.111)	1.328*** (0.118)
No of obs.	8185	8185	4498	4498	3309	3309

Notes: Bootstrapped standard errors in parentheses. *** Significant at 1%, ** significant at 5%, * significant at 10%.

The elasticity of substitution between workers differentiated only by age appears quite different by sector (see Table 4). The substitution parameter ρ_a is not significantly different from zero in services, between zero and one in manufacturing and higher than unity in the trade sector. These results imply different work organization in each sector. They hold in the model with labour differentiated simultaneously by age and skills (see Table 5). There we observe that the substitution parameter ρ_h in the services sector is not significantly different from 0. In trade, the respective parameter for both skill groups tends to converge to 1.

Table 4: Production function estimates for each sector, labour differentiated by age, model (2)

Parameters	Manufacturing		Services		Trade	
	NLLS	LP	NLLS	LP	NLLS	LP
c	-1.947*** (0.022)		-2.067*** (0.036)		-2.287*** (0.039)	
α	0.200*** (0.005)	0.278*** (0.038)	0.240*** (0.006)	0.186*** (0.037)	0.104*** (0.008)	0.159*** (0.038)
β	0.835*** (0.008)	0.726*** (0.010)	0.735*** (0.011)	0.691*** (0.021)	0.921*** (0.013)	0.879*** (0.016)
δ_y	0.314*** (0.018)	0.281*** (0.020)	0.221*** (0.027)	0.238*** (0.036)	0.070*** (0.019)	0.074*** (0.017)
δ_m	0.443*** (0.026)	0.491*** (0.032)	0.479*** (0.057)	0.540*** (0.084)	0.326*** (0.064)	0.357*** (0.072)
δ_o	0.243	0.228*** (0.017)	0.300	0.222** (0.094)	0.604	0.569*** (0.074)
ρ_a	0.367*** (0.074)	0.246*** (0.094)	1.321*** (0.314)	0.955 (1.482)	1.912*** (0.340)	1.848*** (0.366)
No of obs.	8185	8185	4498	4498	3309	3309

Notes: Bootstrapped standard errors in parentheses. *** Significant at 1%, ** significant at 5%, * significant at 10%.

Table 5: Production function estimates for each sector, labour differentiated by age and skills, model (3)

Parameters	Manufacturing		Services		Trade	
	NLLS	LP	NLLS	LP	NLLS	LP
c	-1.294*** (0.021)		-1.538*** (0.037)		-1.538*** (0.036)	
α	0.186*** (0.004)	0.291*** (0.045)	0.222*** (0.006)	0.166*** (0.040)	0.112*** (0.008)	0.159*** (0.036)
β	0.833*** (0.007)	0.736*** (0.009)	0.759*** (0.011)	0.717*** (0.022)	0.911*** (0.012)	0.879*** (0.015)
γ	0.274*** (0.013)	0.254*** (0.015)	0.367*** (0.027)	0.357*** (0.023)	0.226*** (0.021)	0.215*** (0.026)
$1\text{-}\gamma$	0.725	0.746*** (0.015)	0.633	0.643*** (0.023)	0.774	0.785*** (0.026)
δ_{ly}	0.437*** (0.034)	0.395*** (0.032)	0.275*** (0.036)	0.282*** (0.030)	0.236*** (0.042)	0.263*** (0.046)
δ_{lm}	0.417*** (0.046)	0.458*** (0.052)	0.497*** (0.075)	0.558*** (0.068)	0.608*** (0.099)	0.673*** (0.124)
δ_{lo}	0.145	0.147*** (0.030)	0.227	0.159** (0.065)	0.155	0.063 (0.121)
ρ_l	0.463*** (0.119)	0.372*** (0.120)	0.810*** (0.286)	0.574*** (0.186)	1.341*** (0.427)	1.039** (0.518)
δ_{hy}	0.247*** (0.029)	0.242*** (0.031)	0.117** (0.061)	0.134 (0.099)	0.209*** (0.032)	0.192*** (0.058)
δ_{hm}	0.405*** (0.036)	0.432*** (0.042)	0.372*** (0.103)	0.388** (0.181)	0.421*** (0.049)	0.427*** (0.091)
δ_{ho}	0.347	0.326*** (0.026)	0.511	0.477* (0.268)	0.370	0.380*** (0.054)
ρ_h	0.725*** (0.122)	0.594*** (0.132)	2.302*** (0.820)	2.065 (10.898)	0.789*** (0.169)	0.792** (0.324)
No of obs.	8185	8185	4498	4498	3309	3309

Notes: Bootstrapped standard errors in parentheses. *** Significant at 1%, ** significant at 5%, * significant at 10%.

The results shown in Table 6 include already all these constraints on parameters ρ_l and ρ_h. We observe that low-skilled services workers of different ages are closer substitutes than high-skilled ones. Interestingly, in manufacturing, high-skilled workers have been found more easily substitutable between each other than low-skilled ones.

Table 6: Production function estimates for each sector, labour differentiated by age and skills, constrained model (3)

Parameters	Manufacturing		Services		Trade	
	NLLS	LP	NLLS	LP	NLLS	LP
c	-1.294*** (0.021)		-1.587*** (0.035)		-1.534*** (0.035)	
α	0.186*** (0.004)	0.291*** (0.045)	0.222*** (0.006)	0.165*** (0.040)	0.113*** (0.008)	0.159*** (0.036)
β	0.833*** (0.007)	0.736*** (0.009)	0.751*** (0.011)	0.709*** (0.022)	0.910*** (0.012)	0.878*** (0.015)
γ	0.274*** (0.013)	0.254*** (0.015)	0.405*** (0.027)	0.387*** (0.024)	0.218*** (0.019)	0.213*** (0.022)
$1\text{-}\gamma$	0.725	0.746*** (0.015)	0.595	0.612*** (0.024)	0.782	0.786*** (0.022)
δ_{ly}	0.437*** (0.034)	0.395*** (0.032)	0.252*** (0.038)	0.263** (0.030)	0.240*** (0.037)	0.262*** (0.037)
δ_{lm}	0.417*** (0.046)	0.458*** (0.052)	0.430*** (0.071)	0.505 (0.074)	0.668*** (0.085)	0.680*** (0.085)
δ_{lo}	0.145	0.147*** (0.030)	0.318	0.231** (0.079)	0.092	0.058 (0.091)
ρ_l	0.463*** (0.119)	0.372*** (0.120)	1.062*** (0.324)	0.778*** (0.230)	1	1
δ_{hy}	0.247*** (0.029)	0.242*** (0.031)	0.081** (0.028)	0.110 (0.034)	0.222*** (0.032)	0.201*** (0.056)
δ_{hm}	0.405*** (0.036)	0.432*** (0.042)	0.832*** (0.045)	0.802** (0.052)	0.373*** (0.029)	0.385*** (0.038)
δ_{ho}	0.347	0.326*** (0.026)	0.085	0.087* (0.034)	0.404	0.414*** (0.039)
ρ_h	0.725*** (0.122)	0.594*** (0.132)	0	0	1	1
elasticity of substitution : σ=1/(1-ρ)						
Lly-Llm-Llo		1.59		4.5		→∞
Lhy-Lhm-Lho		3.64		1		→∞
No of obs.	8185	8185	4498	4498	3309	3309

Notes: Bootstrapped standard errors in parentheses. *** Significant at 1%, ** significant at 5%, * significant at 10%.

Control for endogenity bias

The nonlinear least squares estimates appear to suffer from the endogeneity bias implying existing correlation between productivity and input choices. Interestingly, the existing bias has different character in distinct sectors. In manufacturing and in trade, the NLLS method tends to underestimate the capital coefficient (α) and overestimate the labour coefficients (β). Such situation takes place if capital and labour are positively correlated and labour's correlation with the productivity shock is higher than capital's correlation. On the other hand, in services, both NLLS coefficients α and β

tend to be biased upwards. It might be the case when only labour responds to the shock and at the same time capital and labour are positively correlated[5].

We can also observe interesting results regarding the potential endogeneity bias within the labour input. In the model with labour differentiated by skills (model (1) and (3)), we observe that the coefficients of low-skilled workers are slightly biased upwards in all the sectors. Among different age categories, the coefficients of old workers (as well as young workers in manufacturing) are overestimated. Although these biases are not statistically significant, they might however indicate that the correlation of these categories of labour with the productivity shock is higher. It could imply that a greater part of this labour category is hired/made redundant in response to the positive/negative productivity shock.

Within the labour differentiated simultaneously by age and skills, in all the sectors, the NLLS estimates tend to underestimate the middle-aged workers coefficient. For other skill-age categories, the results are more sector-specific. Among the underestimated coefficients suggesting lower correlation with the productivity shock, we find those of young low-skilled workers in services and trade, those of old low-skilled workers in manufacturing, those of young high-skilled workers in services and those of senior high-skilled workers in trade. Nevertheless, these biases remain not very significant.

6.2 Age-productivity and age-earnings profiles: general pattern

According to the methodology presented in part 3, we construct the productivity contributions and wage shares for different categories of workers. Thanks to the information on earnings in the dataset, the share of a distinct age group in the wage bill of a given skill category may be easily computed. Based on the estimated parameters values corrected for the endogeneity bias (right columns of Table 6), we estimate the marginal product of labour. Consequently, we can compare an earnings-productivity pattern for different skill and age categories of workers in three different sectors.

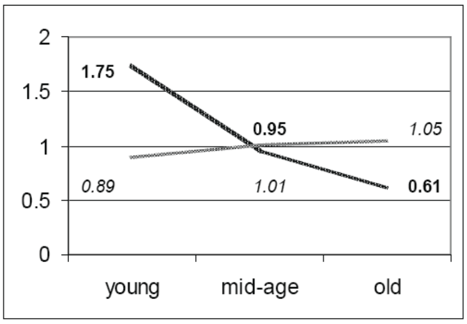
A general tendency regarding productivity contributions and wage shares is presented in Table 7 and is given by the median. In our case, where the data are not necessarily symmetrically distributed, the median is a summary that gives a better idea of a general pattern than given by the mean. In fact, if data are symmetrically distributed, using either the mean or the median gives identical results. In case of skewed distributions however, using the mean could be misleading as means are very sensitive to outliers.

[5]According to Levinsohn and Petrin (2003), these two cases might be the most relevant for short panels because between-firm variation often plays a dominant role in identification and, in this dimension, capital and labour tend to be highly correlated.

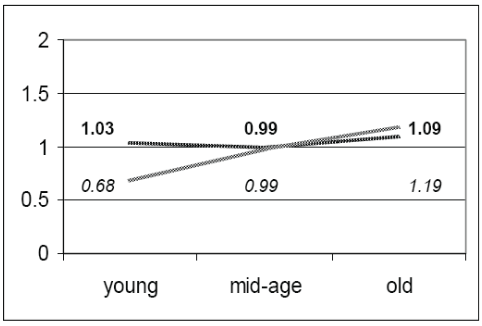
Table 7: Age-productivity and age-earnings pattern

Manufacturing

low-skilled

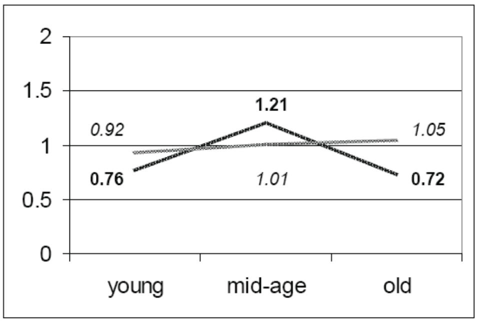


high-skilled

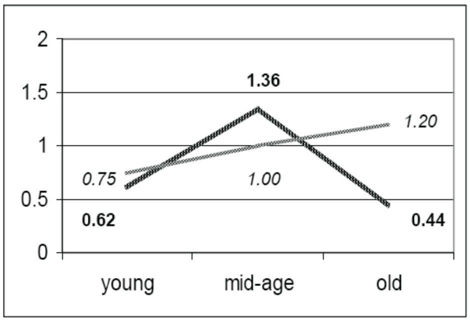


Services

low-skilled

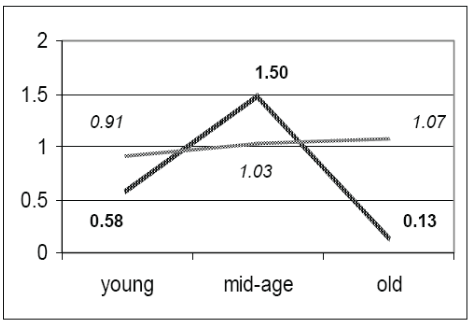


high-skilled

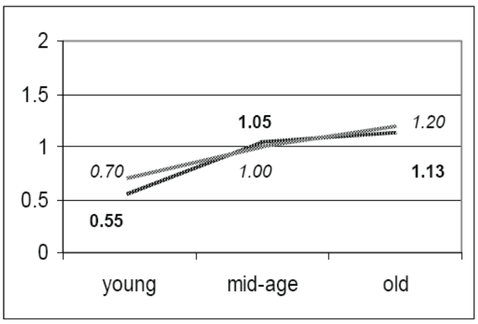


Trade

low-skilled



high-skilled



productivity contribution (median)

wage share (median)

For the median firm, we observe throughout the sectors and for both skill categories of workers that wage shares vary substantially less than workers' productivity. In manufacturing, the age-productivity and age-earnings profiles are compatible with a deferred compensation system. It might indicate that, in this sector, the effort incentive problem has been regulated in practice by many firms by offering at the start of the career wages under the workers' marginal productivity and compensating this difference in the later periods. On the other hand, in services and in trade, we observe the combined relevance of specific human capital and deferred compensation. For young employees, the productivity profile is steeper than the wage profile suggesting that investments in specific human capital are important at the beginning of employees' careers. For older workers, the wage share is higher than the contribution to productivity suggesting rather an incentive-based compensation scheme.

Interestingly, for high-skilled workers in manufacturing, there is not much difference in productivity across age groups. Though, it is the highest for the oldest employees. In trade, the productivity has a clearly increasing slope. Importantly, in both of these sectors, the profile of wage share of middle-aged and senior workers does not diverge much from the profile of productivity contribution.

At the same time, in the low-skilled category, the estimated workers' productivity is clearly the lowest for the oldest workers. However, this result may suffer from a selection bias: experience upgrade low-skilled workers qualification and they may move to high-skilled occupation as time is passing; thus, the senior workers who stay in the low-skilled jobs may be those who are not very productive. A reverse phenomenon could be also observed among the people in high-skilled occupations. Seniors working as highly-skilled experts are not an exception in certain jobs.

6.3 Density estimations

The results for the median firm are interesting but do not reflect all the complexity of variation in wage shares and productivity contributions of different labour categories across the enterprises. Since the estimated parameters (right columns of Table 6) allow calculating the marginal productivities of all labour categories in each firm, we can also analyse the shape of the inter-firm density functions of wage and productivity distributions. For this purpose, we make use of kernel density estimation which is a non-parametric way of estimating the probability density function. It is clearly smoother than some other density estimators such as histogram. The univariate kernel density estimator is computed using:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left[\frac{x - X_i}{h}\right] \tag{23}$$

where K is the Epanechnikov Kernel function and h is a smoothing parameter called the bandwith [Parzen, 1962].

6.3.1 Productivity

The productivity contribution (MP_{ij}/MP_iav) is defined as the ratio of the marginal product of labour of a specific skill and age group (ij) over the average marginal product of labour of a given skill group (i). If the productivity of a certain age group is equal to the sector average, this ratio is equal to 1. In Tables 8 - 10, this case is expressed as a black vertical line. Since the distribution of productivity across the age groups is highly sector-specific, we analyse each sector separately as follows.

Manufacturing

In manufacturing, as shown in Table 8, the productivity of low-skilled workers across firms is characterised by higher variability than the one of high-skilled workers. In general, the median absolute deviations are higher and there are more positive outliers. If we look closer at different age groups, we can notice that in both skill groups a greater variability in productivity is observed among young and older workers. At the same time, the productivity contribution of the middle-aged group does not vary that much between the firms. Its values are well concentrated around the sector average.

Certain particularities can be observed within each skills category. Among the low-skilled, the young workers appear the most productive. Most of them have productivity contributions exceeding the sector average (>1) and we observe many positive outliers. On the other hand, the great majority of older workers have productivity below the average (<1). Though a few positive outliers occur.

The density estimations concerning the high-skilled workers are quite different. The productivity distributions of different age groups have much more symmetric shapes and the median values are quite close to the sector average. In this skill category, the older workers appear the most productive group. More than half of them are more productive than an average high-skilled person.

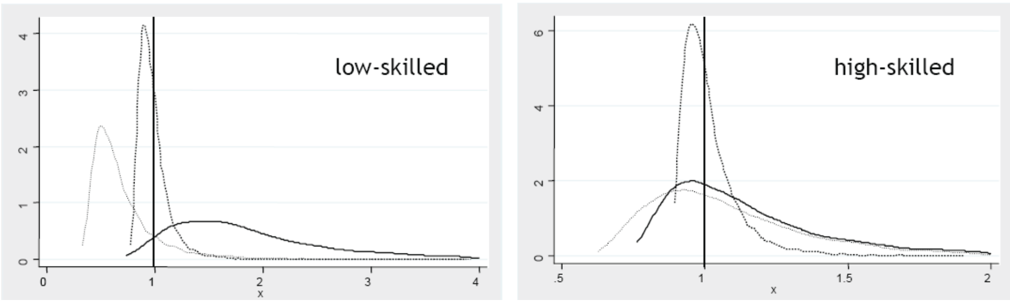
Services

Table 9 shows the pattern of the productivity distribution in the sector of services. In this case, a higher inter-firm variability can be observed for the high-skilled employees. Both, the median absolute deviations as well as positive outliers take much higher values for this type of workers.

Nevertheless, in this case the productivity profile across age groups is similar in both skill groups. The middle-aged workers are found clearly the most productive. Almost all of them have productivities that are higher than the sector average (>1). In contrast, most juniors and seniors are characterised by a productivity below the mean. However, there are some positive outliers, especially among high-skilled workers.

Table 8: Share in average marginal productivity (manufacturing)

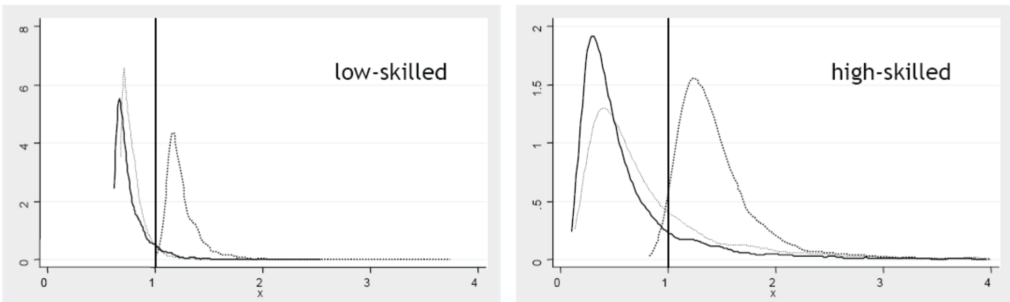
	median	median absolute deviation	min	max
low-skilled				
young	1.75	0.48	0.73	44.44
middle-aged	0.95	0.07	0.77	3.88
old	0.61	0.13	0.33	16.18
high-skilled				
young	1.03	0.17	0.63	7.55
middle-aged	0.99	0.05	0.89	2.77
old	1.09	0.16	0.76	8.96



young ; middle-aged ; old

Table 9: Share in average marginal productivity (services)

	median	median absolute deviation	min	max
low-skilled				
young	0.76	0.05	0.68	1.96
middle-aged	1.21	0.07	0.99	3.74
old	0.72	0.06	0.62	2.53
high-skilled				
young	0.62	0.27	0.13	47.11
middle-aged	1.36	0.19	0.83	27.85
old	0.44	0.18	0.09	119.00



young ; middle-aged ; old

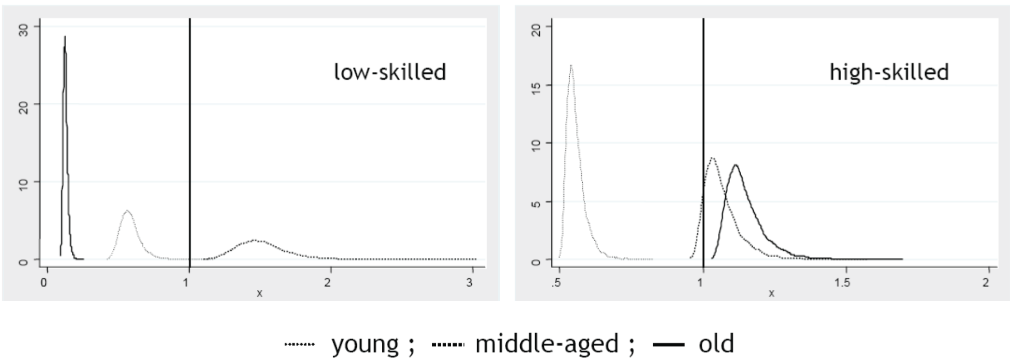
Trade

As can be seen in Table 10, in trade the productivity pattern is very different from those observed in other sectors. First of all, the productivity variability within narrowly defined age groups is much lower. Furthermore, there are large differences between skill groups. Among the low-skilled workers, the productivity distributions almost do not cross each other. The middle-aged workers are the most productive with the productivity contribution over the sector average in all the enterprises. The productivity of young and seniors is considerably lower, well below the sector mean.

A very different pattern is observed within the high-skilled category of workers. Here, seniors are the most productive group followed closely by middle-aged workers with very similar productivity distribution. The young high-skilled employees are significantly less productive.

Table 10: Share in average marginal productivity (trade)

	median	median absolute deviation	min	max
low-skilled				
young	0.58	0.04	0.42	1.17
middle-aged	1.50	0.11	1.10	3.03
old	0.13	0.01	0.09	0.26
high-skilled				
young	0.55	0.02	0.49	0.82
middle-aged	1.05	0.03	0.95	1.57
old	1.13	0.04	1.03	1.69



6.3.2 Earnings

Distributions of wages over age and skill categories are presented in Tables 11, 12 and 13. The wage share (W_{ij}/W_iav) is defined as a ratio of earnings of a specific age group

(j) over the average earnings of a given skill group (i). The ratio equals to 1 (expressed in the tables by a black vertical line) if earnings correspond to the sector average.

In all the sectors we observe a very similar pattern. In general, wage rates vary substantially less than workers' productivity. This result is in line with empirical evidence rejecting the paradigm of wage and marginal productivity equality (Frank (1984), Campbell and Kamlani (1997)).

Looking separately at the two skill classes, we notice that wages of low-skilled workers are less variable than those of the high-skilled. In the low-skilled category, the middle-aged group of workers is characterised by the earnings distribution with the lowest variability, well concentrated around the mean. The earnings variability of young and older workers is very comparable.

Within the high-skilled group we observe more positive outliers, in particular for senior workers, possibly due to better remuneration offered to high-skilled employees with a long tenure. The distribution of earnings for the middle-aged workers is the least variable, followed by young and senior employees.

Table 11: Share in average wage (manufacturing)

	median	median absolute deviation	min	max
low-skilled				
young	0.89	0.05	0.05	2.74
middle-aged	1.01	0.02	0.54	1.49
old	1.05	0.04	0.44	4.79
high-skilled				
young	0.68	0.08	0.01	2.94
middle-aged	0.99	0.05	0.40	2.23
old	1.19	0.13	0.35	8.34

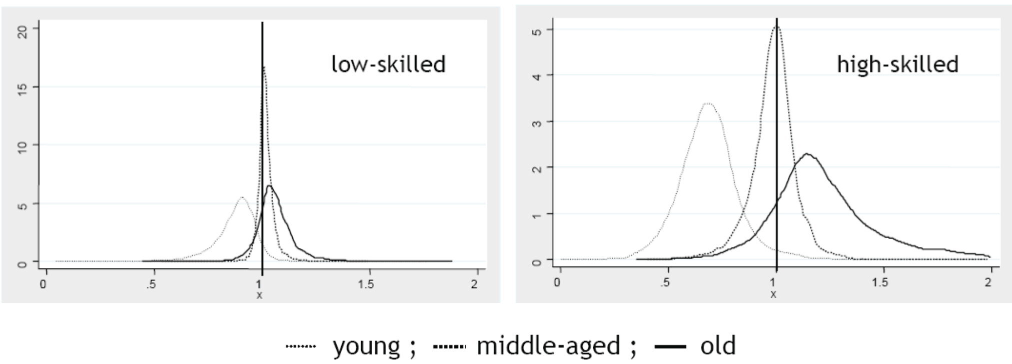


Table 12: Share in average wage (services)

	median	median absolute deviation	min	max
low-skilled				
young	0.92	0.04	0.24	1.43
middle-aged	1.01	0.02	0.78	1.80
old	1.05	0.05	0.34	3.52
high-skilled				
young	0.75	0.08	0.02	5.02
middle-aged	1.00	0.05	0.31	2.23
old	1.20	0.15	0.28	37.84

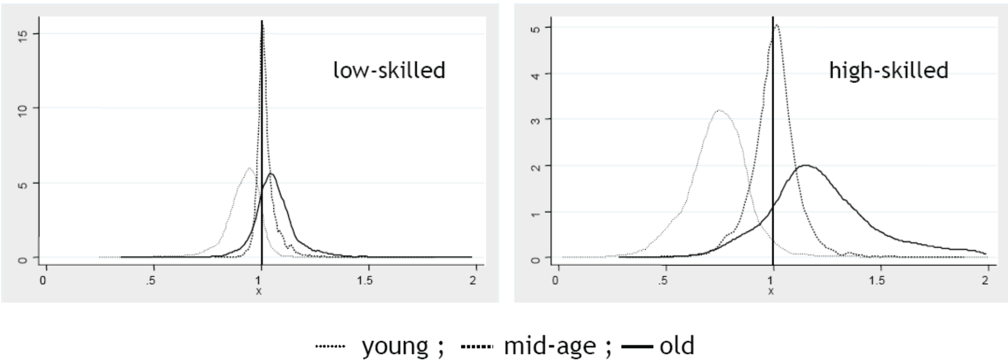
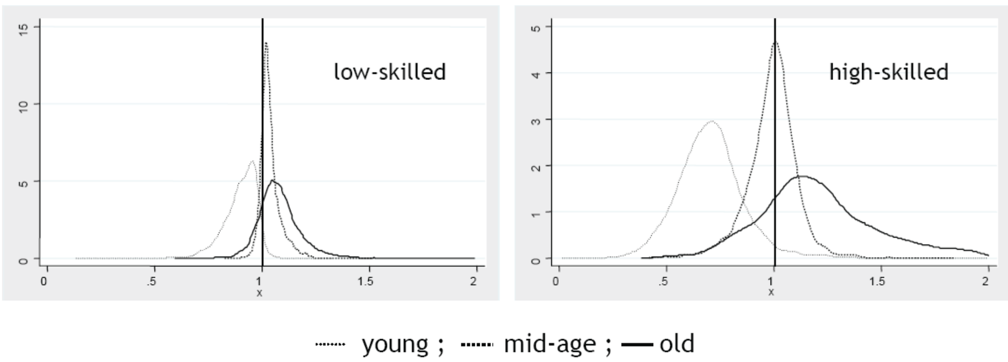


Table 13: Share in average wage (trade)

	median	median absolute deviation	min	max
low-skilled				
young	0.91	0.05	0.13	1.33
middle-aged	1.03	0.02	0.82	1.51
old	1.07	0.05	0.59	2.32
high-skilled				
young	0.70	0.09	0.02	3.75
middle-aged	1.00	0.06	0.42	1.83
old	1.20	0.17	0.38	5.49



6.3.3 Productivity/earnings ratio

The most interesting aspect from the perspective of the employer is the workers' productivity in relation to their cost. It is particularly important as it may create an incentive for employers to exclude some age groups from the labour market and to give preference to the others.

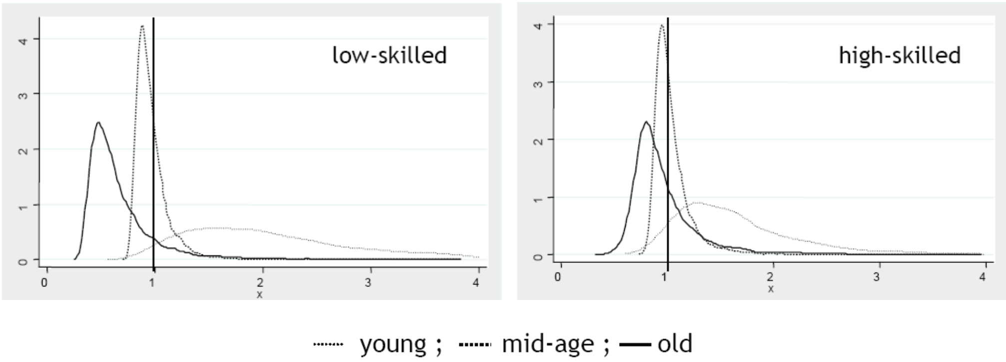
It is possible that some workers having the same productivity are paid differently or that some others are paid equally but have different productivities. Therefore, apart from analysing the productivity distribution separately from wages, we consider also the inter-firm distribution of the productivity/earnings ratio with respect to all age and skill groups (see Tables 14, 15 and 16). We define it as $\frac{MP_{ij}}{MP_iav}/\frac{W_{ij}}{W_iav}$.

It appears that in manufacturing the relative productivity over the wage ratio is the highest for the young, followed by the middle-aged and the old. The possible explanation could be that the young are paid the least and at the same time they are highly motivated to work hard. If there exists incomplete information about the workers' ability at the beginning of their career, the young workers might exert much more effort in order to suggest high ability level and keep their current job (Grund and Westergård-Nielsen, 2008). The high variability in distribution for the young comes from the positive outliers.

This productivity/earnings ratio decreases with age for both skill groups. It means that the attractiveness of an employee for the employer decreases with age. Though, it is not so strong for the high-skilled workers. The distribution of the ratio shows higher variability for the older compared to the middle-aged workers. It has a lower median and a significant majority of observations are below the sector average.

Table 14: Ratio of relative productivity over relative wage (manufacturing)

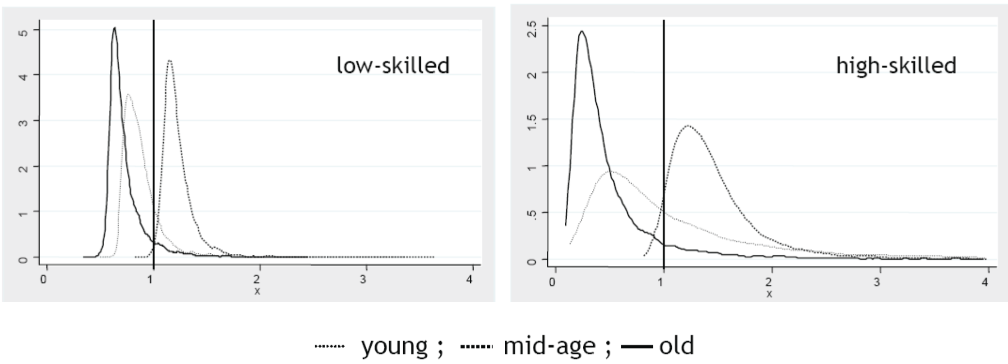
	median	median absolute deviation	min	max
low-skilled				
young	2.02	0.59	0.56	106
middle-aged	0.93	0.07	0.70	4.03
old	0.58	0.13	0.24	17.77
high-skilled				
young	1.53	0.33	0.60	291
middle-aged	1.00	0.07	0.73	3.82
old	0.88	0.13	0.32	10.17



In the services sector, we observe lower variability of the ratio among the low-skilled workers. The high variability for the high-skilled comes, among others, from the positive outliers in this group. In both skill categories, the productivity/earnings ratio is the highest for the middle-aged, followed by young and senior workers. Thus, similarly to the pure productivity profile, the middle-aged workers are the most attractive employees. However, the biggest positive outliers are found among the junior workers.

Table 15: Ratio of relative productivity over relative wage (services)

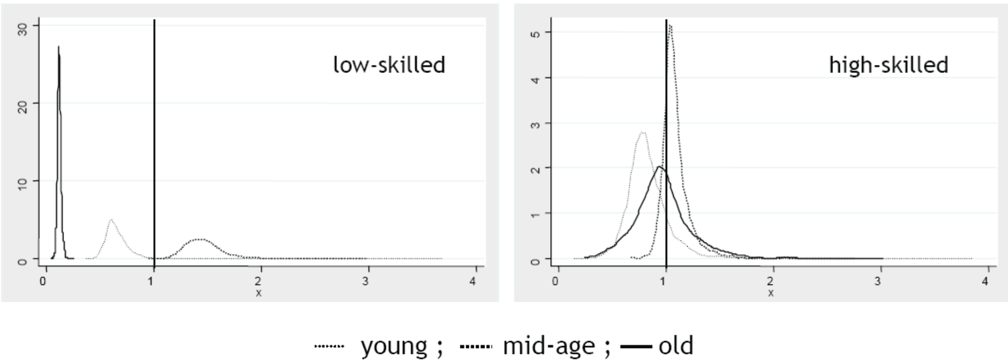
	median	median absolute deviation	min	max
low-skilled				
young	0.84	0.08	0.50	4.08
middle-aged	1.20	0.07	0.83	3.63
old	0.68	0.06	0.34	2.44
high-skilled				
young	0.85	0.40	0.13	223.53
middle-aged	1.36	0.21	0.82	35.50
old	0.36	0.14	0.09	67.48



Contrasting results are found in the trade sector as shown in Table 16. For low-skilled employees, the productivity/earnings ratio is very dispersed across age groups. Likewise in services, middle-aged workers are the most attractive, followed by juniors and seniors. Among the high-skilled, for all age groups the distribution of the ratio converge closely around the mean and does not vary much. Again, the prime-age workers form the group whose majority has the higher ratio of productivity over cost higher. They are followed by the older workers, whose distribution is well symmetric around the mean.

Table 16: Ratio of relative productivity over relative wage (trade)

	median	median absolute deviation	min	max
low-skilled				
young	0.64	0.06	0.37	3.67
middle-aged	1.44	0.10	0.95	2.97
old	0.12	0.01	0.05	0.26
high-skilled				
young	0.80	0.10	0.14	29.40
middle-aged	1.06	0.05	0.67	2.75
old	0.95	0.14	0.23	3.02



7 Conclusions

This paper revisits the question of the actual profiles of marginal productivity across age groups within the given workforce and its potential equality with profiles of earnings. Using French firm-level data, we estimated the parameters of the production function where the labour input, differentiated simultaneously by age and skills, takes a nested CES functional form. We controlled for the endogeneity bias according to the methodology developped by Levinsohn and Petrin (2003).

Among the main findings, workers of different ages appear to be imperfect substitutes in production. The elasticity of substitution for workers of different ages has been found considerably lower than implied by the additive functional form specification. Our results suggest that wages do not necessarily reflect the actual productivity. Consistent with study by Frank (1984) and Campbell and Kamlani (1997), the wage profile has been found less variable than productivity.

As regards labour productivity, its profile across distinct age groups is likely to depend on the skill category. It has been found the lowest for the low-skilled older workers. Senior high-skilled employees in manufacturing and trade are the most productive group. Results for the manufacturing sector show that the age-productivity and age-earnings profiles are compatible with a deferred compensation system. It might indicate that the effort incentive problem has been regulated in practice by many firms by offering at the start of the career wages under the workers' marginal productivity and compensating this difference in the later periods. On the other hand, in services and in trade, we observe the combined relevance of specific human capital and deferred compensation.

Relative productivity over wage ratio, an important aspect for the employer, has been found sector-specific. In manufacturing, it is the highest for the young workers. In services and trade, the ratio is the highest for the middle-aged employees. This discrepancy between productivity and earnings can be a source of employment difficulties particularly for the older low-skilled workers.

References

Arrow, K., Chenery, H., Minhas, B., and Solow, R. (1961). Capital-labor substitution and economic efficiency. *The Review of Economics and Statistics*, 43(3):225–250.

Aubert, P. and Crépon, B. (2003). Âge, salaire et productivité. Documents de travail G2003/06, Institut National de la Statistique et des Études Économiques, Paris.

Becker, G. (1975). *Human Capital.* National Bureau of Economic Research.

Becker, G. and Stigler, G. (1974). Law enforcement, malfeasance and the compensation of enforcers. *Journal of Legal Studies*, 3:1–18.

Bulow, J. and Summers, L. H. (1986). A theory of dual labor markets with application to industrial policy, discrimination and keynesian unemployment. *Journal of Labor Economics*, 4:376–414.

Campbell, C. M. and Kamlani, K. S. (1997). The reasons for wage rigidity: Evidence from a survey of firms. *The Quarterly Journal of Economics*.

Colonia-Willner, R. (1998). Practical intelligence at work: relationship between aging and cognitive efficiency among managers in a bank environment. *Psychology and Aging*, 13 (1):45–57.

Crépon, B., Deniau, N., and Pérez-Duarte, S. (2003). Productivité et salaire des travailleurs âgés. *Revue Française d'Economie*, 18(1):157–185.

Daveri, F. and Maliranta, M. (2007). Age, seniority and labour costs: lessons from the finnish it revolution. *Economic Policy*, 22(49):117–175.

Frank, R. (1984). Are workers paid their marginal products? *The American Economic Review*, 74 (4):549–571.

Grund, G. and Westergård-Nielsen, N. (2008). Age structure of the workforce and firm performance. *International Journal of Manpower*, 29(5):410–422.

Guest, R. and Shacklock, K. (2005). The impending shift to an older mix of workers: Perspectives from the management and economics literatures. *International Journal of Organisational Behaviour*, 10(3):713–728.

Hashimoto, M. (1981). Firm-specific human capital as a shared investment. *The American Economic Review*, 71(3):475–482.

Hellerstein, J. and Neumark, D. (2007). *Hard to measure goods and services: essays in honor of Zvi Griliches*, chapter Production function and wage equation estimation with heterogeneous labor: evidence from a new matched employer-employee data set. University of Chicago Press.

Hellerstein, J. K., Neumark, D., and Troske, K. R. (1999). Wages, productivity, and worker characteristics: Evidence from plant-level production functions and wage equations. *Journal of Labor Economics*, 17(3).

Hutchens, R. (1986). Delayed payment contracts and a firm's propensity to hire older workers. *Journal of Labor Economics*, 4:439 – 457.

Kremer, M. and Thomson, J. (1998). Why isn't convergence instantaneous? young workers, old workers, and gradual adjustment. *Journal of Economic Growth*, 3:5–28.

Lam, D. (1989). Population growth, age structure, and age-specific productivity. does a uniform age distribution minimize lifetime wages? *Journal of Population Economics*, 2:189–210.

Lazear, E. P. (1979). Why is there mandatory retirement? *Journal of Political Economy*, 87(6):1261–1284.

Lazear, E. P. (1981). Agency, earnings profiles, productivity, and hours restrictions. *The American Economic Review*, 71(4):606–620.

Lengermann, P. A. (2002). Is it who you are, where you work, or with whom you work? reassessing the relationship between skill segregation and wage inequality. Technical Paper TP-2002-10, U.S. Census Bureau.

Levinsohn, J. and Petrin, A. (2003). Estimating production functions using inputs to control for unobservables. *Review of Economic Studies*, 70:317–341.

Loewenstein, G. and Sicherman, N. (1991). Do workers prefer increasing wage profiles? *Journal of Labor Economics*, 9 (1):67–84.

Marschak, J. and Andrews, W. H. (1944). Random simultaneous equations and the theory of production. *Econometrica*, 12 (3-4):143–205.

Maurer, T. (2001). Career-relevant learning and development, worker age, and beliefs about self-efficacy for development. *Journal of Management*, 27 (2).

Mincer, J. (1974). *Schooling, Experience and Earnings*. NY: Columbia University Press.

Parzen, E. (1962). On estimation of a probability density function and mode. *The Annals of Mathematical Statistics*, 33(3):1065–1076.

Remery, C., Henkens, K., Schippers, J., and Ekamper, P. (2003). Managing an ageing workforce and a tight labour market. *Population Research and Policy Review*, 22(1):21–40.

Shapiro, C. and Stiglitz, J. (1984). Equilibrium unemployment as a worker discipline device. *American Economics Review*, 74:433–444.

Skirbekk, V. (2004). Age and individual productivity: A literature survey. *Vienna Yearbook of Population Research*, Verlag der Österreichischen Akademie der Wissenschaften.

Thesmar, D. and Thoenig, M. (2000). Creative destruction and firm organization choice. *The Quarterly Journal of Economics*, 115(4):1201–1237.

Yellen, J. (1984). Efficiency wage models of unemployment. *American Economic Review*, 74:200–208.

8 Appendix

Table 17: Production function estimates for each sector, labour differentiated by age, sub-sector controls

Parameters	Manufacturing		Services	
	NLLS	LP	NLLS	LP
α	0.222 (0.005)***	0.284 (0.029)***	0.241 (0.006)***	0.189 (0.039)***
β	0.811 (0.008)***	0.679 (0.010)***	0.710 (0.011)***	0.642 (0.024)***
δ_y	0.325 (0.019)***	0.293 (0.023)***	0.275 (0.026)***	0.299 (0.030)***
δ_m	0.450 (0.027)***	0.508 (0.035)***	0.427 (0.048)***	0.425 (0.051)***
δ_o	0.224	0.198 (0.017)***	0.298	0.276 (0.051)***
ρ_a	0.345 (0.075)***	0.188 (0.099)*	0.935 (0.213)***	0.759 (0.176)***
sub-sector controls	*yes*	*yes*	*yes*	*yes*
No of obs.	8185	8185	4498	4498

Notes: Bootstrapped standard errors in parentheses.
*** Significant at 1%, ** significant at 5%, * significant at 10%.

Table 18: Production function estimates for each sector, labour differentiated by age and skills, sub-sector controls

Parameters	Manufacturing		Services			
	no constraints		*no constraints*		*ρ_h=0*	
	NLLS	LP	NLLS	LP	NLLS	LP
α	0.191 (0.005)***	0.283 (0.041)***	0.218 (0.006)***	0.167 (0.042)***	0.219 (0.006)***	0.167 (0.042)***
β	0.827 (0.008)***	0.714 (0.009)***	0.733 (0.011)***	0.668 (0.025)***	0.724 (0.011)***	0.660 (0.025)***
γ	0.279 (0.013)***	0.269 (0.016)***	0.331 (0.025)***	0.324 (0.023)***	0.364 (0.026)***	0.356 (0.026)***
$1-\gamma$	0.721	0.730 (0.016)***	0.669	0.675 (0.023)***	0.635	0.643 (0.026)***
δ_{ly}	0.441 (0.033)***	0.404 (0.033)***	0.342 (0.038)***	0.371 (0.034)***	0.334 (0.038)***	0.365 (0.034)***
δ_{lm}	0.420 (0.046)***	0.465 (0.052)***	0.435 (0.071)***	0.451 (0.062)***	0.387 (0.066)***	0.409 (0.062)***
δ_{lo}	0.138	0.131 (0.029)***	0.222	0.177 (0.055)***	0.278	0.226 (0.060)***
ρ_l	0.437 (0.117)***	0.323 (0.118)***	0.653 (0.234)***	0.473 (0.160)***	0.814 (0.248)***	0.608 (0.172)***
δ_{hy}	0.248 (0.029)***	0.246 (0.032)***	0.211 (0.041)***	0.214 (0.067)***	0.132 (0.028)***	0.144 (0.035)***
δ_{hm}	0.403 (0.037)***	0.435 (0.044)***	0.391 (0.067)***	0.361 (0.091)***	0.696 (0.045)***	0.664 (0.054)***
δ_{ho}	0.348	0.318 (0.027)***	0.398	0.424 (0.132)***	0.172	0.191 (0.037)***
ρ_h	0.726 (0.124)***	0.581 (0.138)***	1.009 (0.288)***	0.984 (3.484)	0	0
sub-sector controls	*yes*	*yes*	*yes*	*yes*	*yes*	*yes*
No of obs.	8185	8185	4498	4498	3309	3309

Notes: Bootstrapped standard errors in parentheses.
*** Significant at 1%, ** significant at 5%, * significant at 10%.