DISCUSSION PAPER SERIES

No. 4956

INSURANCE POLICIES FOR MONETARY POLICY IN THE EURO AREA

Keith Küster and Volker Wieland

INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP4956.asp

ISSN 0265-8003

INSURANCE POLICIES FOR MONETARY POLICY IN THE EURO AREA

Keith Küster, Johann Wolfgang Goethe-Universität Frankfurt
Volker Wieland, Johann Wolfgang Goethe-Universität Frankfurt and CEPR

Discussion Paper No. 4956
March 2005

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 4956

March 2005

ABSTRACT

Insurance Policies for Monetary Policy in the Euro Area*

In this paper, we examine the cost of insurance against model uncertainty for the euro area considering four alternative reference models, all of which are used for policy analysis at the ECB. We find that maximal insurance across this model range in terms of a Minimax policy comes at moderate costs in terms of lower expected performance. We extract priors that would rationalize the Minimax policy from a Bayesian perspective. These priors indicate that full insurance is strongly oriented towards the model with highest baseline losses. Furthermore, this policy is not as tolerant towards small perturbations of policy parameters as the Bayesian policy rule. We propose to strike a compromise and use preferences for policy design that allow for intermediate degrees of ambiguity-aversion. These preferences allow the specification of priors but also give extra weight to the worst uncertain outcomes in a given context.

JEL Classification: E52, E58 and E61
Keywords: euro area, minimax, model uncertainty, monetary policy rules and robustness

Keith Küster
Johann-Wolfgang-Goethe Universität
Mertonstrasse 17
D-60325 Frankfurt am Main
GERMANY
Tel: (49 69) 798 25283
Email: kkuester@stud.uni-frankfurt.de

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=162191

Volker Wieland
Monetary Theory and Policy
Johann-Wolfgang-Goethe Universität
Mertonstrasse 17
D-60325 Frankfurt am Main
GERMANY
Tel: (49 69) 798 252 88
Fax: (49 69) 798 252 72
Email: wieland@wiwi.uni-frankfurt.de

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=152242

*This version of this paper was first presented in the context of a discussion of ‘The Performance and Robustness of Interest-Rate Rules in Models of the Euro Area’ by R. Adalid, G. Coenen, P. McAdam and S. Siviero at the ECB Conference on ‘Monetary policy and imperfect knowledge’ in Würzburg, October 14-15, 2004.

Submitted 22 February 2005

1 Introduction

What if our perception of the workings of the economy and the effects of monetary policy is seriously biased? Can we take out any insurance against this uncertainty? These are practical questions faced by policy makers that motivate much of the recent literature on policy robustness and model uncertainty. One line of research has applied various forms of worst-case analysis to insure against perturbations of single reference models of the economy.[1] Another line of research has focused on comparing policy rules across a limited set of well-studied reference models.[2] We follow the second approach and consider four models of the Euro area, all of which are being used for monetary policy analysis at the European Central Bank.

Our starting point is an investigation of the performance of simple rules that are optimized for one specific model across the other models as in the preceding literature.[3] Given the range of output and inflation dynamics implied by the models considered in this study it is not likely to find fully robust rules in this manner. Thus, the main purpose of our paper is to identify rules that are effective at insuring against particularly bad outcomes while still delivering fairly good performance compared to rules optimized within each model. Or in other words, we attempt to identify insurance policies for monetary policy that come at an affordable cost in terms of expected performance loss

[1] See for example Sargent (1999), Hansen and Sargent (2002), Giannoni (2001,2002), Onatski and Stock (2002), Onatski and Williams (2003), Tetlow and von zur Muehlen (2001), Tetlow and von zur Muehlen (2004) and Zakovic, Rustem and Wieland (2004). An early contribution is von zur Muehlen (1982).

[2] See for example Levin, Wieland and Williams (1999, 2003), Levin and Williams (2003) and Coenen (2003). Early contributions are Becker, Dwolatzky, Karakitsos and Rustem (1986) and McCallum (1988).

[3] In 2003 a Euro system project was started with the objective of evaluating robustness of policy rules across relevant Euro area models in the vein of Levin *et al.* (1999, 2003) and Coenen (2003). Wieland and Küster were involved in this project in 2003, Wieland as a consultant in two project meetings and Küster as an intern contributing to the preparation for this project of one the models, the Area-Wide model of Fagan *et al.* (2001). The first ECB paper out of this project, Adalid *et al.* (2004), and the present paper were written in parallel and independently for the ECB Conference on 'Monetary policy and imperfect knowledge' in Würzburg, October 14-15, 2004. As a result they present interesting alternative views on the question how to design robust policy for the Euro area.

compared to model-specific rules. To this end, we employ Bayesian as well as Minimax analysis.

We take Sims' (2001) criticism that Minimax policies may imply unreasonable priors seriously and construct the priors that would rationalize our Minimax policies from a Bayesian perspective. We confirm that those priors may seem excessively biased towards one model to most policy makers. As a compromise we propose so-called ambiguity-averse preferences for policy design. These preferences allow the specification of priors but also give extra weight to the worst uncertain outcomes in a given context (see also Brock *et al.*, 2003). We also consider extensions to non-quadratic preferences and carefully analyze the implications of small perturbations in our policies (i.e. the policy maker's "trembling hand") for policy performance.

The remainder of the paper is structured as follows. Section 2 provides a brief overview of the models used in this study. Section 3 discusses the risks of focusing on one model and the insurance properties of Bayesian policy. Section 4 provides further perspectives on robustness, namely a look at full insurance, intermediate ambiguity aversion and differing degrees of risk aversion. The conclusions follow. Further details of the analysis, methods and sensitivity studies are discussed in the appendices.

2 The Models

Each of the models considered in this study exhibits long-run monetary neutrality as well as short-run nominal inertia. As a result, monetary policy has short-run real effects and the central bank enjoys the ability to stabilize inflation and output fluctuations. The central bank also faces substantial uncertainty regarding output and inflation dynamics and policy effectiveness that is reflected in the differences between these models. The models differ, for example, in the degree of forward-looking expectation formation, the extent of optimizing behavior by economic agents, and in terms of magnitude, scope and parameter estimates. The four models, which have all been developed at the ECB, are estimated with (or calibrated to) aggregate quarterly Euro area data.

The Area-Wide model (AW) (see Fagan *et al.*, 2001) is a linearized version of the model used as an element in the ECB's forecasting process. Expectation formation in this model is largely backward-looking. The two models of Coenen and Wieland (2003) are much smaller but incorporate forward-looking expectations. The CW-T variant with Taylor-style staggered wage contracting (see Taylor, 1980) exhibits less nominal rigidity than the CW-F variant with Fuhrer-Moore style contracts (see Fuhrer and Moore, 1995). Finally, the Smets and Wouters (2003) model (SW) most completely embodies recent advances in modelling optimizing behavior of economic agents. For a more detailed discussion of these models we refer the reader to the appendix A.

To provide a common empirical benchmark for monetary policy in these models, we use a rule estimated with euro area data by Gerdesmeier and Roffia (2003). We adjust the monthly estimates to a quarterly frequency:

$$r_t = 0.87^3 r_{t-1} + \left(1 - 0.87^3\right)\left(1.93\pi_t + 0.28y_t\right).$$

Figure 1 reports autocorrelation functions for the four models using the identical interest rate rule. In computing these we assume that the central bank credibly commits to following the Gerdesmeier-Roffia rule.

As a basis for comparison we show the autocorrelation function implied by the actual data (solid line), which indicates that inflation and the output gap exhibit substantial persistence. The four models imply quite different dynamics thereby spanning a significant range of model uncertainty. In the CW-T and SW models inflation dynamics die out within 6 quarters, while the CW-F model generates somewhat longer-lasting dynamics. The AW model stands out with the highest degree of inflation persistence. Output is strongly serially correlated in the AW model and the CW-T model, but less so in the SW and CW-F models. In spite of these differences, the four models fall within the +/- 2 standard deviation bands we computed (not shown).[4]

Figure 1: Autocorrelation Functions

Annual Inflation

Output Gap

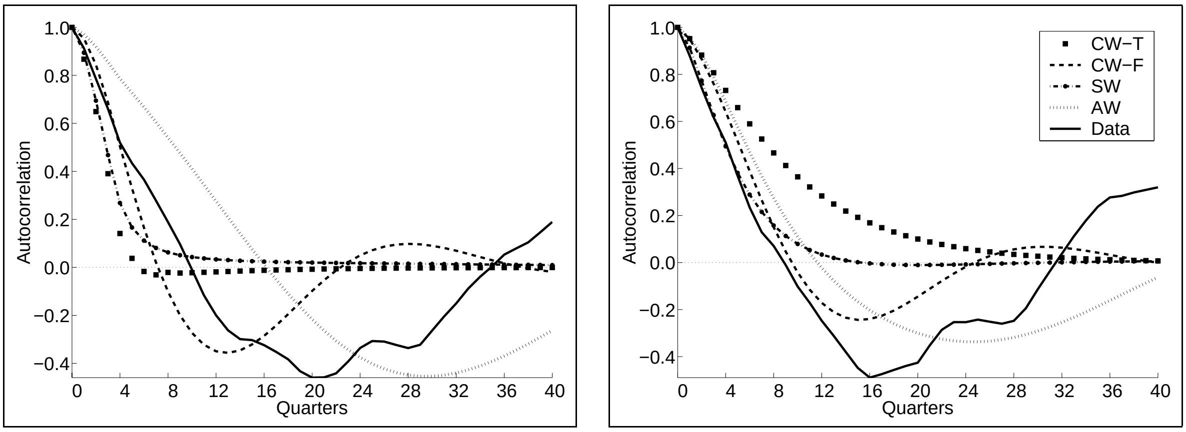

Figure 1: Autocorrelation functions. Shown are the autocorrelations of inflation and the output gap under the rule $r_t = 0.87^3 r_{t-1} + (1 - 0.87^3)\ (1.98\pi_t^a + 0.28y_t)$ estimated by Gerdesmeier and Roffia (2003). Also shown is the autocorrelation in the data. The data are from the AWM data set and range from 1985:1 to 2002:4. The inflation series has been linearly detrended. Model autocorrelations lie within ± 2 standard deviation bounds (not shown) of the data. The standard deviation for $\widehat{\rho_j}$ is computed on the assumption that the data generating process is an MA(j-1).

To illustrate the extent of uncertainty about policy effectiveness, we report impulse response functions for inflation and the output gap in Figure 2. We simulate a 100 basis point interest rate shock assuming that monetary policy subsequently follows the Gerdesmeier-Roffia rule. Again, the AW model generates the highest degree of endogenous persistence with inflation reaching its trough more than five years after the shock. Similarly, output returns only very gradually to baseline in this model. The SW model again marks the smallest degree of output and inflation persistence. The CW-F and CW-T models lie in between in terms of persistence of the dynamics in response to a policy shock. As expected Taylor-contracts generate less inflation persistence than Fuhrer-Moore contracts but output is more persistent in the CW-T model.

Table 1 compares the unconditional second moments of inflation, the output gap and

[4] Of course, these significance bands heavily depend on assumptions made regarding the data-generating process.

Figure 2: Impulse Responses

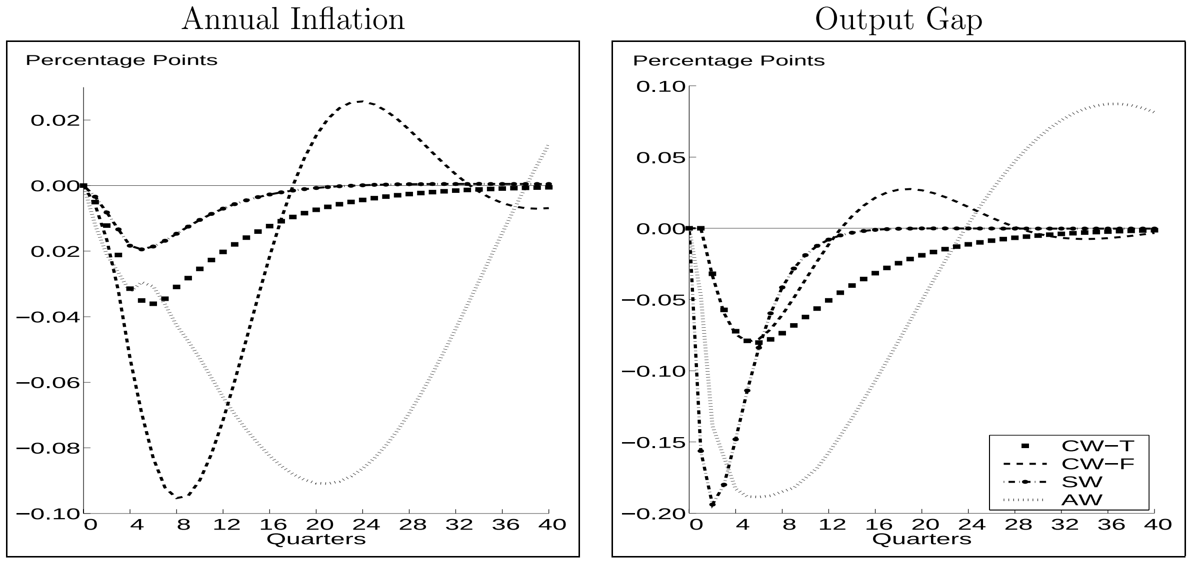

Figure 2: Impulse-responses to a 100bps temporary unanticipated rise in the nominal short term rate. The monetary policy rule is the Gerdesmeier and Roffia (2003) rule, $\rho = 0.87^3$, $\alpha = (1 - 0.87^3)1.93$, $\beta = (1 - 0.87^3)0.28$, with annual inflation. All variables are measured in percentage points.

the change in the nominal interest, π, y and Δr, in the above models with those of the data. Under the Gerdesmeier-Roffia rule the CW-F model induces the highest variance for inflation–more than twice as the SW model and consequently rather high variability of interest rate increments. All models generate a standard deviation of the output gap that is almost twice the value observed in the data (2.5 for the AW model).

3 Policy Performance Across Models

In this study we focus on interest rate reaction functions that belong to the class of simple outcome-based Taylor-style rules

$$r_t = \rho r_{t-1} + \alpha \pi_t + \beta y_t, \tag{1}$$

Table 1: Standard Deviations

	y	π	Δr
Data	1.0	0.8	0.5
CW-F	2.0	2.2	0.9
CW-T	1.9	1.1	0.5
SW	2.1	0.8	0.4
AW	2.5	1.3	0.4

Unconditional standard deviations of the target variables in the four models when the policy maker commits himself to the Gerdesmeier and Roffia (2003) rule (with year on year inflation rates). Also shown are the standard deviations in the data. The sample and the data source are as in Figure 1.

with three response parameters (ρ, α, β) corresponding to the lagged interested rate r_{t-1}, the year-on-year inflation rate π_t and the output gap y_t, all in deviations from steady state.[5]

To measure policy performance and determine the cost of insurance against model uncertainty we use the weighted sum of unconditional variances of inflation, the output gap and the change in the interest rate, which is consistent with a standard quadratic loss function (see Levin *et al.*, 1999, 2003, Coenen, 2003, and others in this literature),[6]

$$\mathcal{L}_m = \mathrm{Var}(\pi) + \lambda_y \mathrm{Var}(y) + \lambda_{\Delta r} \mathrm{Var}(\Delta r), \; m \in \mathcal{M}. \tag{2}$$

Var($\cdot$) denotes the unconditional variance, while $\mathcal{M} = \{$CW-F, CW-T, SW, AW$\}$ comprises the model space. The parameter $\lambda_y \geq 0$ determines the policy maker's preference for reducing output variability around potential relative to curbing inflation variability. The weight $\lambda_{\Delta r} > 0$ introduces a preference for restraining the variability of changes to nominal interest rates. For the remainder of this paper we use values of $\lambda_y \in \{0, 0.5, 1\}$

[5] Simple outcome-based rules of this form tend to be more robust than rules with more states (see Levin *et al.*, 1999) and forecast-based rules with longer horizons (see Levin *et al.*, 2003). Further improvements in stabilization performance that can be achieved with forecast based rules, are hampered by the need for a forecasting model that may be incorrect and introduce additional uncertainty (see Levin *et al.*, 2003, and Coenen, 2003).

[6] An interesting alternative measure would be the welfare of the representative consumer in the Smets and Wouters (2003) model.

that cover the range from strict to flexible inflation targeting and values of $\lambda_{\Delta r} \in \{0.5, 1\}$.[7]

3.1 Rules Across Models

For comparing rules across models and measuring costs of insurance we use the outcome under commitment to the best simple rule of the type specified by equation (1) in each model. The best simple rule is obtained by choosing the response parameters, (ρ, α, β), in (1) so as to minimize the loss defined by equation (2) for each model.[8] Table 2 summarizes the parameters of the simple rules optimized for each model for the three different values of the weight on output variability in the loss function, λ_y. $\lambda_{\Delta r}$ is set to 0.5.

Table 2: Optimized Simple Rules

	$\lambda_y = 0$			$\lambda_y = 0.5$			$\lambda_y = 1$		
Model	ρ	α	β	ρ	α	β	ρ	α	β
CW-F	0.9	0.8	0.4	0.8	0.7	0.6	0.8	0.7	0.8
CW-T	1.0	0.3	0.1	0.8	0.2	0.6	0.8	0.2	0.8
SW	1.0	0.4	0.0	1.0	0.2	0.8	1.0	0.2	1.3
AW	0.6	0.5	0.5	0.4	0.6	1.3	0.4	0.5	1.7

Parameters of optimal simple rules for each model. Shown is the case $\lambda_{\Delta r} = 0.5$ for loss function (2).

The best rule in the CW-F model features moderate to strong interest rate smoothing (ρ between 0.8 and 0.9) and strong feedback to inflation ($\alpha = 0.8$). The best CW-T rule implies a bit more interest smoothing (greater ρ) but smaller response coefficients for inflation due to the lower degree of inflation persistence. The best SW policy consistently

[7] These values for $\lambda_{\Delta r}$ ensure that interest rate variability does not stray very far from what is observed empirically for the Euro area. We have conducted further sensitivity studies that are not reported in this version of the paper.

[8] An alternative would be to use the overall first-best policy under commitment as a benchmark. We have not done so because we are interested in measuring the cost of insurance implied by a simple rule (chosen due to a preference for robustness concerning model uncertainty), rather than being interested in the benefits that may be possible from first-best policy compared to simple rules in any specific model. For this purpose the model-optimized simple rule is the proper benchmark. As to the potential benefits of first-best policies compared to optimized three-parameter rules, those are moderate for models with rational expectations but can be substantial for models with primarily backward-looking expectations (see Dieppe *et al.*, 2004).

uses the highest weight on the lagged interest rate ($\rho = 1$) essentially implementing a first-difference rule. It is well-known from earlier research that the prominence of rational expectations and forward-looking behavior in models with optimizing agents renders such a policy strategy optimal. In sharp contrast the rule optimized in the AW model exhibits much lower values of ρ ranging between 0.4 and 0.6. Optimal AW policy also features a strong feedback to the output gap as a proxy for future inflation (see also Dieppe *et al.*, 2004).

The performance of simple rules optimized for one model in the other three models is sufficiently diverse to deem it risky to use a single reference model. Table 3 reports the percentage increase in loss when using a rule optimized for Model X in Model Y relative to the rule that is optimal for Y. We confirm earlier results in the literature. Rules with a high degree of interest-rate smoothing such as those optimized in the SW model generate substantial losses and even explosiveness in models with a significant backward-looking component such as the AW model. On the other hand, policy designed for models with strong intrinsic persistence such as AW may not be active enough to anchor expectations if agents indeed were more forward-looking. For example, for $\lambda_y = 1.0$, the best AW policy would imply indeterminacy in CW-F and CW-T. Otherwise percentage losses are largest in AW and CW-F, reaching up to 360%.

The percentage losses reported in Table 3, which have been commonly used in the recent literature, may overemphasize the extent of model uncertainty in particular when the baseline loss is rather low or when the loss function heavily penalizes small deteriorations[9] in economic outcomes. Similarly, substantial changes in economic outcomes may be de-emphasized when the baseline loss is already large.

As a remedy to this drawback of relative percentage losses we introduce the implied premium on inflation variability or "Implied Inflation Premium", the IIP. This premium measures the increase in the standard deviation of inflation relative to the outcome under the best simple rule that is necessary to match the loss under the alternative policy. In

[9] A case in point are the non-quadratic preferences discussed in Section 4.3.

Table 3: Robustness of Rules Optimized for a Specific Model: Relative Losses

	CW-F rule evaluated in			CW-T rule evaluated in		
λ_y	CW-T	SW	AWM	CW-F	SW	AW
0.0	31	53	126	161	9	∞
0.5	12	17	82	103	15	29
1.0	12	27	71	145	18	36

	SW rule evaluated in			AW rule evaluated in		
λ_y	CW-F	CW-T	AW	CW-F	CW-T	SW
0.0	360	8	∞	78	24	78
0.5	66	19	324	167	15	49
1.0	101	26	220	ME	ME	54

Percentage losses relative to first-best simple rule for each model (in %). The notation "∞" indicates that the implemented rule results in instability; the notation "ME" indicates that the implemented rule results in multiple equilibria. Shown is the case $\lambda_{\Delta r} = 0.5$ for loss function (2).

other words, we attribute the deterioration in loss that results from a lack of robustness entirely to inflation variability keeping the standard deviation of output and interest rates at the benchmark level. The benefit of this premium is that units are intuitive (percentage points of the inflation rate) and interpretable on an economic scale. The corresponding premia are reported in Table 4.

The implied inflation variability premia shed new light on the comparison of rules across models. With no weight on the output gap the SW model exhibits low baseline losses. Using the CW-F policy in the SW model implies an increase in loss of 53% (cp. Table 3), which seems sizeabe. The IIP of 0.16 percentage points on annual inflation (an increase from 0.82 to 0.98 percentage points, say), however, is rather small. Similarly, using the CW-F policy in AW generates an increase in losses of 126%, which appears prohibitive. The implied increase of 0.56 percentage points in inflation variability is sizeable but does not appear extreme. As these examples show, relative losses may be

Table 4: Robustness of Model-Specific Rules: Implied Inflation Premia

	CW-F rule evaluated in			CW-T rule evaluated in		
λ_y	CW-T	SW	AWM	CW-F	SW	AW
0.0	.15	.16	.56	1.13	.03	∞
0.5	.10	.11	.79	1.03	.09	.34
1.0	.13	.19	.93	1.57	.13	.55

	SW rule evaluated in			AW rule evaluated in		
λ_y	CW-F	CW-T	AW	CW-F	CW-T	SW
0.0	2.06	.04	∞	.63	.12	.23
0.5	.72	.16	2.06	1.50	.12	.28
1.0	1.18	.27	2.01	ME	ME	.36

Implied Inflation (Variability) Premium relative to first-best simple rule for each model (in percentage points). The notation "∞" indicates that the implemented rule results in instability; the notation "ME" indicates that the implemented rule results in multiple equilibria. Shown is the case $\lambda_{\Delta r} = 0.5$ for loss function (2).

misleading. We will therefore report the IIP where appropriate in the remainder of the paper.

3.2 The Bayesian Perspective

A natural first step in the search for a rule that performs more consistently across models than the rules discussed in the preceding sub-section is to take a Bayesian perspective as recommended by Levin *et al.* (2003). A more robust rule could thus be found by minimizing a weighted loss function. The Bayesian loss is

$$\mathcal{L}^B = \min_{(\rho,\alpha,\beta)} E_{\mathcal{M}}\{\mathcal{L}_m\} = \min_{(\rho,\alpha,\beta)} \sum_{m\in\mathcal{M}} p_m \mathcal{L}_m, \tag{3}$$

where p_m are the policy maker's priors as to model m. As a start, we consider flat priors $p_m = 1/|\mathcal{M}|$. The performance of the Bayesian policy in terms of implied inflation variability premia is reported in Table 5. Considering all models in policy design in this

Table 5: Flat Bayesian Priors versus Model-Specific Simple Rules

	Optimal Rule			Inflation (Variability) Premium			
λ_y	ρ	α	β	CW-F	CW-T	SW	AW
0.0	0.8	0.6	0.4	.07	.10	.16	.12
0.5	0.7	0.7	0.8	.09	.08	.14	.26
1.0	0.7	0.8	1.1	.11	.12	.21	.32

Optimal policy rule parameters and Implied Inflation (Variability) Premium relative to the simple rules optimized for each model (in percentage points). Shown is the case $\lambda_{\Delta r} = 0.5$ for loss function (2).

manner already avoids the extreme outcomes such as indeterminacy and mulitple equilibria in forward-looking models or explosiveness in backward-looking models. Furthermore the increase in inflation variability that would match the increase in loss relative to the optimized model-specific rules is contained at a maximum of 0.32 percentage points on the standard deviation of annual inflation. Since CW-F features by far the highest benchmark losses of all models (cp. Table 1), it implicitly receives most of the weight in the Bayesian optimization. The Bayesian rule with flat priors looks much like the best CW-F policy albeit with less interest rate smoothing. The reduction in ρ serves to help performance in the AW model.

3.3 Fault Tolerance of Bayesian Policy

Levin and Williams (2003) have proposed so-called fault tolerance as a diagnostic tool in assessing robustness. They suggest to plot variations in the policy rule parameters against percentage increases in losses for each model. The procedure implies varying one parameter at a time while keeping the others fixed at the values implied by the best simple rule for each model. A *model* is deemed fault tolerant if deviations from the best rule for that model do not trigger a steep increase in losses. A drawback of this procedure is that comparisons across models are not ceteris paribus in terms of the policy parameters. For example, when varying the smoothing parameter, ρ, policy rules used in the different

Figure 3: Fault Tolerance for Flat Bayesian Priors

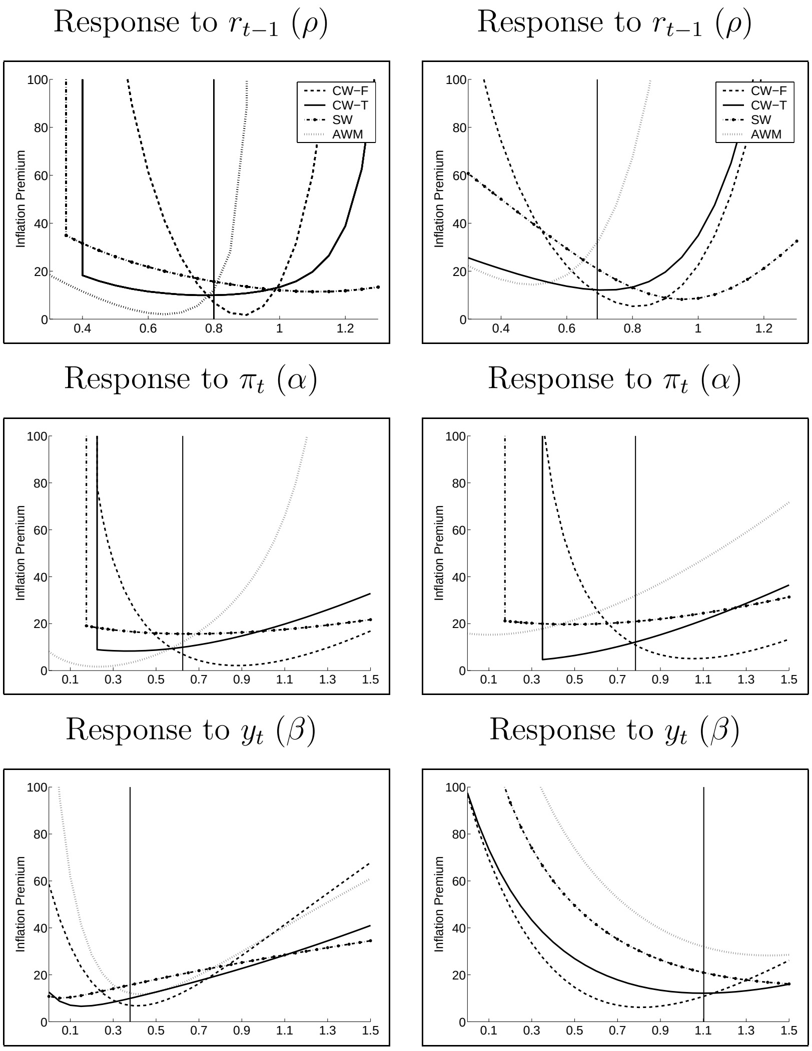

Figure 3: For each of the models, a line traces out the Implied Inflation (Variability) Premium (in basis points) under the flat Bayesian policy as the specified parameter of the policy rule is varied, holding the other two parameters fixed at their respective values under flat Bayesian priors. The left (right) panel pertains to the preferences for $\lambda_y = 0(1)$ and $\lambda_{\Delta r} = 0.5$. A vertical line marks the optimal Bayesian parameter values.

models also have different responses to inflation and the output gap, α and β. Thus, we decided to use the concept of fault tolerance somewhat differently. We ask whether *policy* is fault tolerant, meaning in particular whether minor perturbations in policy parameters have only a minor impact on performance. Our findings for the Bayesian flat prior policy are displayed in Figure 3. Each column of three panels considers implied inflation inflation premia across the four models when varying one parameter of the Bayesian rule at a time, while keeping the others fixed. The respective columns are associated with values for the preference parameter λ_y of zero and one, corresponding to a strict and flexible inflation targeting central bank respectively. Policy is fairly tolerant to deviations in α and β. IIP's do not rise fast in the neighbourhood around the optimal parameter values. The admissible range of variations is smaller for ρ but still permits a safety margin for a policy maker with "trembling hands". We conclude that Bayesian policy under flat priors is fault tolerant.

4 Perspectives on Robustness

4.1 Full Insurance

Bayesian optimization with flat priors is one possible approach when policymakers are fairly agnostic which among a set of models is most appropriate as tool for policy design. Alternatively, such an agnostic policy maker could ask how to best insure herself against adverse model risk, or in other words, how to avoid worst case outcomes. This question can be answered by Minimax analysis. It corresponds to a game between a policy maker who attempts to minimize loss and nature, which chooses from the model space so as to maximize loss,

$$\mathcal{L}^M = \min_{(\rho,\alpha,\beta)} \max_{m \in \mathcal{M}} \mathcal{L}_m. \tag{4}$$

Appendix B describes the algorithm we use to solve the minimax problem in more detail. The policy rule coefficients and premiums implied by such a full insurance policy are summarized in Table 6. The minimax or full insurance policy is close to the best rule for

Table 6: Minimax Policy Relative to Model-Specific Rules

	Optimal Rule			Inflation (Variability) Premium				Loss Premium	
λ_y	ρ	α	β	CW-F	CW-T	SW	AW	$\Delta\mathcal{L}^{worst}$	$\Delta\mathcal{L}^{expect}$
0.0	0.9	0.8	0.5	0	.15	.16	.57	7	17
0.5	0.8	0.7	0.6	0	.10	.11	.73	7	9
1.0	0.8	0.7	0.8	0	.13	.20	.77	7	6

Optimal policy rule parameters and Implied Inflation (Variability) Premium relative to first-best simple rule for each model (in percentage points). Shown is the case $\lambda_{\Delta r} = 0.5$ for loss function (2). $\Delta\mathcal{L}^{worst}$: percentage reduction of worst-case loss relative to worst outcome under flat Bayesian priors. $\Delta\mathcal{L}^{expect}$: percentage increase in expected loss relative to Bayesian policy (flat priors).

the CW-F model, that is the model with the highest baseline loss. The Implied Inflation Premia are relatively modest for the CW-T and SW models but increase with the AW model, adding up to 0.77 percentage points to the standard deviation of the annual inflation rate. The deterioration in stabilization performance compared to model-specific rules in the AW model is noticeably stronger than under the Bayesian policy with flat priors (cp. Table 5).

Overall, the costs for insuring against model risk seem moderate given the considerable range of output and inflation dynamics in the model space. Expected loss relative to flat Bayesian priors increases by 6 to 17%. Similarly the gains from insurance relative to Bayesian policy in the worst case scenario are moderate. Since the CW-F model already receives much weight under a flat Bayesian policy, worst-case losses are only reduced by seven percent. Finally, we note that insurance premiums, i.e. the cost of an increase in expected loss for a given reduction of worst-case loss, decrease as the policy maker puts more weight on the output gap.

Inspired by Sims (2001), who has criticized Minimax for often implying economically unreasonable priors, we proceed to extract the priors that would rationalize the Minimax policy from a Bayesian perspective. The procedure involved is discussed in appendix C.

The implied priors are shown in Table 7. These priors indicate again that model risk in

Table 7: Minimax Implied Priors

λ_y	CW-F	CW-T	SW	AW
0.0	1.000	0.0	0.0	0.000
0.5	0.989	0.0	0.0	0.011
1.0	0.961	0.0	0.0	0.039

Bayesian priors backing the flat minimax solution. Shown is the case $\lambda_{\Delta r} = 0.5$ for loss function (2).

the space we consider lies mainly with the CW-F model. Recall that under the specifically optimized CW-F rule inflation in that model is substantially more volatile than in the AW model (0.9 to 1.6 percentage points), while the output gap is somewhat more volatile in AW (2.0 relative to 1.4 percentage points). Thus, as the preference parameter on the output gap, λ_y, increases this volatility difference gains more importance in the Minimax calculus and the AW model receives some (albeit small) weight in the priors implied by the minimax policy. Neither the CW-T nor the SW model ever appear among the worst case losses.

Interestingly the minimax policy looses the fault tolerance property enjoyed by the Bayesian rule with flat priors. Figure 4 displays fault tolerance plots for the parameter on the lagged interest rate, ρ.[10] Minor changes in policy can lead to a strong increase in the Implied Inflation Premia for the AW model, which does not respond well to a high degree of interest-rate smoothing (see also Dieppe *et al.*, 2004). Minimax monetary policy only just avoids the latter problem in the AW model but not with a wide enough security margin. Thus, "trembling hands" in the conduct of policy could have a strong negative impact when implementing a policy that attempts to guard against worst-case model uncertainty.

[10] The Minimax policy is similarly fault intolerant with respect to the other parameters of the rule. Plots are available upon request.

Figure 4: Fault Intolerance for Full Insurance Policy

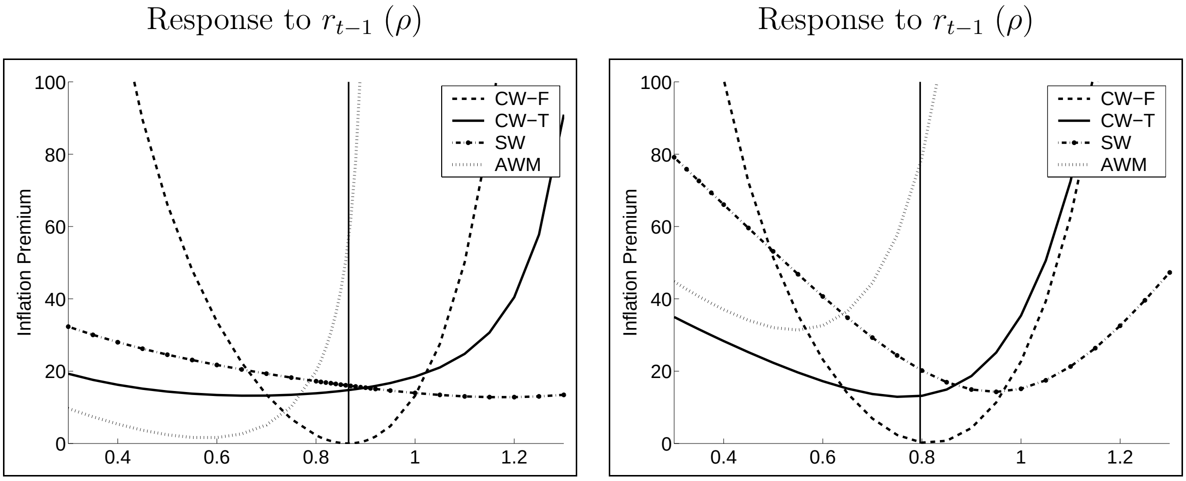

Figure 4: For each of the models, a line traces out the Implied Inflation (Variability) Premium (in basis points) under the full insurance policy as the persistence parameter of the policy rule is varied, holding the other two parameters fixed at their respective values under the full insurance policy. The left (right) panel pertains to the preferences for $\lambda_y = 0\,(1)$ and $\lambda_{\Delta r} = 0.5$. A vertical line marks the optimal Minimax parameter value.

4.2 A Compromise

Implied priors of the Minimax policy provide the policy maker with useful information regarding the risks associated with alternative models and how much weight each of the models receives in the worst-case analysis. In the previous section one model, CW-F, dominated all others in influencing the Minimax outcome or what we called the full insurance policy. Of course, policy makers that prefer the CW-F model over all others will find it easy to accept the policy conclusions from the Minimax approach. Also, if policymakers are not willing to define any priors over the model space, the case of Knightian uncertainty, then the Minimax solution will be appealing even if it focusses almost exclusively on one model. However, the critique of Sims (2001) that the priors needed to justify worst-case policies from a Bayesian perspective may be too extreme will ring true with policy makers in light of the case shown here.

As a solution to this quandary we propose to take some prior information (or preferences across models) into account and balance this prior (or preference) against the worst uncertain outcome. Following Epstein and Wang (1994) such preferences may be formalized as follows

$$\mathcal{L}^A = \min_{(\rho,\alpha,\beta)} \left\{ (1-e) \sum_{m \in \mathcal{M}} p_m \mathcal{L}_m + e \max_{m \in \mathcal{M}} \mathcal{L}_m \right\}, \tag{5}$$

where $e \in [0,1]$ indexes the degree of desired insurance against worst case outcomes. These preferences are also called ambiguity averse (see Brock *et al.*, 2003) because the actor places extra weight on the worst uncertain outcome.[11] At the one extreme, $e = 0$, these preferences amount to minimizing expected losses from Bayesian perspective. At the other extreme, $e = 1$, this coincides with the Minimax policy displayed in Table 6. The desired level of insurance, e, and the model priors, p_m, may be chosen by the policy maker. As an example, we consider a policy maker with profound priors or preferences for the CW-T model (i.e. $p_{CW-T} \approx 1$, and $p_m \approx 0$ for all other models). He is sufficiently open-minded, however, to consider worst cases across the full model space. To reflect this open-mindedness we set $e = 0.5$. Table 8 reports the policy rules and Implied Inflation (Variability) Premia for this case.

Table 8: Ambiguity Averse CW-T Policy Across Models

	Optimal Rule			Inflation (Variability) Premium			
λ_y	ρ	α	β	CW-F	CW-T	SW	AW
0.0	0.9	0.7	0.4	.01	.11	.14	.51
0.5	0.8	0.6	0.6	.01	.07	.10	.74
1.0	0.8	0.6	0.8	.01	.09	.18	.78

Optimal policy rule parameters and Implied Inflation (Variability) Premium relative to first-best simple rule for each model (in percentage points). Shown is the case $\lambda_{\Delta r} = 0.5$ for loss function (2).

The performance results reflect the compromise made by the policy maker. Perfor-

[11] The preferences are a special case of Epstein and Wang's (1994) "ϵ-contamination" preferences. Strictly speaking, also the minimax preferences reflect ambiguity aversion, of course.

mance in the more forward-looking CW-T and SW models improves relative to Minimax outcomes. Policy exhibits more interest-rate smoothing and reacts less to inflation than under Minimax. Thus, it has incorporated some of the policy features that were optimal under the CW-T model (cp. Table 2). From Table 9 it can be seen that the CW-F model still has a strong influence and the AW only a small influence as measured by implied priors. But now the implied priors also respect the CW-T model. With the implied priors

Table 9: Ambiguity Averse CW-T Implied Priors

λ_y	CW-F	CW-T	SW	AW
0.0	0.500	0.5	0.0	0.000
0.5	0.499	0.5	0.0	0.001
1.0	0.476	0.5	0.0	0.024

Bayesian priors backing the ambiguity averse solution. Shown is the case $\lambda_{\Delta r} = 0.5$ for loss function (2). Parameters for the CW-T ambiguity averse solution are $e = 0.5$, $p_{CW-T} \approx 1$ and $p_m \approx 0$ for all other models.

and the IIPs at hand the policy maker has extracted new information that will be helpful in implementing a suitable insurance policy.

4.3 Non-Quadratic Preferences

So far, the policy robustness literature has focussed primarily on linear-quadratic models. As a final point of this paper, we examine the robustness of above policy conclusions to the assumption of quadratic losses. To this end, we consider a generalized loss function that allows alternative degrees of risk aversion[12]:

$$\mathcal{L}_m = \mathrm{E}|\pi|^\xi + \lambda_y \,\mathrm{E}|y|^\xi + \lambda_{\Delta r} \,\mathrm{E}|\Delta r|^\xi, \ \xi > 0. \tag{6}$$

[12] With regard to non-quadratic central bank objectices see Orphanides and Wieland (2000) and recent work by Kilian and Manganelli (2003) on the central bank as risk manager.

The degree to which a policy maker dislikes deviations from target depends on ξ, a measure of risk aversion to upside and downside risks. With respect to deviations from target, ξ close to zero implies a mild degree of risk-aversion. Standard quadratic loss is nested by the case $\xi = 2$. Appendix D motivates these preferences more thoroughly. We consider the following choices for the new preference parameter, $\xi = 1, 2, 5$ and 10 and report the results for Minimax policy under these preferences focusing on a flexible inflation targeter ($\lambda_y = 1$) in Table 10.

Table 10: Minimax Policy – Non-Quadratic Preferences

	Optimal Rule			Inflation (Variability) Premium			
ξ	ρ	α	β	CW-F	CW-T	SW	AW
1	0.8	0.6	0.7	0	.11	.22	.69
2	0.8	0.7	0.8	0	.13	.20	.77
5	0.7	1.1	1.0	.03	.17	.19	.79
10	0.6	1.4	1.2	.05	.20	.19	.77

Optimal policy rule parameters and Implied Inflation (Variability) Premium relative to first-best simple rule for each model (in percentage points). Shown is the case $\lambda_{\Delta r} = 0.5,\ \lambda_y = 1$ for loss function (6).

We find that Minimax policy results in less interest-rate smoothing (i.e. ρ falls) and greater responsiveness to current output and inflation (α and β rise) as risk aversion increases. Implied Inflation Premia remain similar for alternative preferences (cp. Table 10). Thus, the cost of insuring against worst case outcomes does not seem to change much with the degree of risk aversion.

The implied priors in Table 11 reveal that the AW model receives more and more weight with greater risk aversion, while the weight on the CW-F model decreases. The reason was mentioned earlier: while CW-F exhibits greater inflation variability than AW, AW exhibits greater output variability than CW-F. Recall the comparison under the best simple rule for CW-F which generated standard deviations of inflation of 1.6 percentage points in CW-F and 0.9 percentage points in AW, but standard deviations of output of 1.4 and 2.0 percentage points in CW-F and AW, respectively. Under quadratic preferences

Table 11: Non-Quadratic Preferences - Implied Priors

ξ	CW-F	CW-T	SW	AW
1	1.000	0.0	0.0	0.000
2	0.961	0.0	0.0	0.039
5	0.870	0.0	0.0	0.130
10	0.782	0.0	0.0	0.218

Bayesian priors backing the flat minimax solution for varying degrees of risk-aversion. Shown is the case $\lambda_{\Delta r} = 0.5$, $\lambda_y = 1$ for loss function (6).

these differences roughly balance so full insurance policy is close to the rule optimized for CW-F. As preferences turn more risk averse, however, the variability of the output gap in the AW that is larger than inflation variability in CW-F receives relatively more weight. Thus, Minimax policy is driven more towards the features that are optimal for the AW model.

5 Conclusions

Considering four alternative models of the Euro area, all of which are used for policy-analysis at the ECB, we have found that maximal insurance against model uncertainty via Minimax policy comes at moderate costs in terms of lower expected performance. However, the implied priors that would rationalize the Minimax policy from a Bayesian perspective indicate that Minimax-type insurance is strongly oriented towards the model with highest baseline losses. Furthermore, this policy is not as tolerant towards small perturbations of policy parameters as the Bayesian policy rule. Consequently, we propose to strike a compromise and use so-called ambiguity-averse preferences for policy design. These preferences allow the specification of priors but also give extra weight to the worst uncertain outcomes in a given context. The implied priors and the Implied Inflation Variability Premia that we have computed are intended to help policy makers in refining their views on alternative reference models and in reconsidering their desired degree of insurance. For practical application, an iterative procedure using implied priors and calculating costs of alternative degrees of insurance should be a useful tool for policy design.

The uncertainty we have considered by means of the linearized benchmark models concerned the dynamics around a given steady state. For future research, it would be interesting to also consider uncertainty about the steady state itself. Most dominantly this will mean including uncertainty with regard to such variables as the natural rates of interest and unemployment, and the level of potential output.

References

Adalid, R., Coenen, G., McAdam, P., and Siviero, S. (2004). The performance and robustness of interest-rate rules in models of the euro area. mimeo.

Angeloni, I., Coenen, G., and Smets, F. (2003). Persistence, the transmission mechanism and robust monetary policy. *Scottish Journal of Political Economy*, **50**(5), 527–549.

Becker, R., Dwolatzky, B., Karakitsos, E., and Rustem, B. (1986). The simultaneous use of rival models in policy optimisation. *The Economic Journal*, **96**, 89–101.

Brayton, R. K., Director, S. W., Hachtel, G. D., and Vidigal, L. M. (1979). A new algorithm for statistical circuit design based on quasi-newton methods and function splitting. *IEEE Transactions on Circuits and Systems*, **26**(9), 784–794.

Brock, W. A., Durlauf, S. N., and West, K. D. (2003). Policy evaluation in uncertain economic environments. NBER Working Paper 10025, New York: NBER.

Calvo, G. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, **12**, 383–398.

Coenen, G. (2003). Inflation persistence and robust monetary policy design. ECB Working Paper Series 290, Frankfurt: European Central Bank.

Coenen, G. and Wieland, V. (2003). A small estimated euro area model with rational expectations and nominal rigidities. *European Economic Review*. Forthcoming.

Dieppe, A. and Henry, J. (2003). The euro area viewed as a single economy: How does it respond to shocks? *Economic Modelling, special edition*. Forthcoming.

Dieppe, A., Küster, K., and McAdam, P. (2004). Optimal Monetary Policy Rules for the Euro Area: An Analysis Using the Area Wide Model. ECB Working Paper Series 360, Frankfurt: European Central Bank.

Epstein, L. and Wang, T. (1994). Intertemporal asset pricing under Knightian uncertainty. *Econometrica*, **62**(2), 283–322.

Fagan, G., Henry, J., and Mestre, R. (2001). An area-wide model (AWM) for the euro area. ECB Working Paper Series 42, Frankfurt: European Central Bank.

Fuhrer, J. and Moore, G. (1995). Inflation persistence. *Quarterly Journal of Economics*, **110**(1), 127–159.

Gerdesmeier, D. and Roffia, B. (2003). Empirical estimates of reaction functions for the euro area. ECB Working Paper Series 206, Frankfurt: European Central Bank.

Giannoni, M. P. (2001). Robust optimal monetary policy in a forward-looking model with parameter and shock uncertainty. mimeo.

Giannoni, M. P. (2002). Does model uncertainty justify caution? Robust optimal monetary policy in a forward-looking model. *Macroeconomic Dynamics*, **6**(1), 111–144.

Hansen, L. P. and Sargent, T. J. (2002). Robust control and model uncertainty in economics. mimeo.

Kilian, L. and Manganelli, S. (2003). The central banker as a risk manager: Quantifying and forecasting inflation risks. ECB Working Paper Series 226, Frankfurt: European Central Bank.

Levin, A. and Williams, J. (2003). Robust monetary policy with competing reference models. *Journal of Monetary Economics*, **50**(5).

Levin, A., Wieland, V., and Williams, J. (1999). Robustness of simple monetary policy rules under model uncertainty. In J. Taylor, editor, *Monetary Policy Rules*, pages 263–299. Univ. of Chicago Press, Chicago.

Levin, A., Wieland, V., and Williams, J. (2003). The performance of forecast-based monetary policy rules under model uncertainty. *American Economic Review*, **93**(3), 622–645.

McCallum, B. T. (1988). Robustness properties of a rule for monetary policy. *Carnegie-Rochester Conference Series on Public Policy*, **29**(autumn), 173–203.

Onatski, A. and Stock, J. H. (2002). Robust monetary policy under model uncertainty in a small model of the U.S. economy. *Macroeconomic Dynamics*, **6**(1), 85–110.

Onatski, A. and Williams, N. (2003). Modeling model uncertainty. *Journal of the European Economic Association*, **1**(5), 1087–1122.

Orphanides, A. and Wieland, V. (2000). Inflation zone targeting. *European Economic Review*, **44**(7), 964–985.

Sargent, T. (1999). Discussion of "Policy rules for open economies". In J. B. Taylor, editor, *Monetary Policy Rules*, pages 145–154. University of Chicago Press, Chicago.

Sims, C. A. (2001). Pitfalls of a minimax approach to model uncertainty. *American Economic Review*, **91**(2), 51–54.

Smets, F. and Wouters, R. (2003). An estimated stochastic dynamic general equilibrium model of the euro area. *Journal of European Economic Association*, **1**(5), 1123–1175.

Taylor, J. B. (1980). Aggregate dynamics and staggered contracts. *Journal of Political Economy*, **88**(1), 1–23.

Tetlow, R. and von zur Muehlen, P. (2001). Robust monetary policy with misspecified models: Does model uncertainty always call for attenuated policy? *Journal of Economic Dynamics and Control*, **25**(6-7), 911–949.

Tetlow, R. and von zur Muehlen, P. (2004). Avoiding nash inflation: Bayesian and robust responses to model uncertainty. *Review of Economic Dynamics*, **7**(4), 869–899.

von zur Muehlen, P. (1982). Activist versus nonactivist monetary policy: optimal rules under extreme uncertainty. Manuscript now available as Finance and Economics Discussion Series paper no. 2001-02 (January), Federal Reserve System.

Zakovic, S., Rustem, B., and Wieland, V. (2004). Stochastic optimisation and worst case analysis in monetary policy design. Presented at Society of Computational Economics, July 8-10, 2004, Amsterdam.

A Detailed Model Description

A.1 The Coenen and Wieland Models

Coenen and Wieland (2003) develop a small-scale monetary macro-model for various staggered pricing schemes. In this study, we utilize the two variants of the model which fit the data best. One variant employs the nominal contract specification of Taylor (1980) (CW-T) while the other employs the relative real wage contract specification of Fuhrer and Moore (1995) (CW-F). These two contract wage equations imply different degrees of inflation persistence – real wage contracts give more weight to past inflation. The model has been estimated on data from the Area Wide Model data set from 1974:1-1998:4. The contract wage specifications have been estimated by a limited information indirect inference technique while the IS equation has been estimated by means of the Generalized Method of Moments. Applications of the models include a robustness analysis by Coenen (2003), which highlights that under model uncertainty policy makers should overemphasize inflation persistence.

A.2 The Smets and Wouters Model

The Smets and Wouters (2003) model (SW) is a micro-founded dynamic general equilibrium model. It models both sticky prices and sticky wages by Calvo (1983) price setting combined with partial indexation of prices and wages. The model has been estimated on synthetic euro area wide data from the AW data set from 1980:2 to 1999:4 by Bayesian estimation techniques. Apart from private consumption as a demand component, the SW model also features investment demand. Applications of the model include a study concerning uncertainty about the values of the various persistence parameters of the model (Angeloni *et al.*, 2003). The main finding is that varying the degree of persistence in output does not matter as much as does variation in the degree of inflation persistence. A policy maker is well advised to overemphasize inflation persistence rather than to understate it. This finding therefore squares with Coenen (2003).

A.3 The Area Wide Model

The SW and CW models treat the Euro area as a closed economy. In contrast, the area wide model by Fagan *et al.* (2001) is a small open economy model of Euro land. Its structure is traditional, having a long-run classical equilibrium with a vertical Phillips curve but with some non-microfounded short-run frictions in price and wage setting as well as factor demands. Consequently, activity is demand-determined in the short run but supply determined in the longer run with employment having converged to a level consistent with the exogenously given level of equilibrium unemployment. Stock-flow adjustments are accounted for by, *e.g.*, the inclusion of a wealth term in consumption. At present, the treatment of expectations in the model is limited; with the exception of the exchange rate (modelled by forward-looking uncovered interest parity) and the (12-year bond) term structure, the model embodies backward-looking expectations. Full model listing and simulation evidence can be found in Fagan *et al.* (2001) and Dieppe and Henry (2003) as well as Dieppe *et al.* (2004). On the balanced growth path, exogenous forces as there are population growth, trend-factor productivity growth and foreign GDP growth as well as government consumption drive the endogenous real variables. Aggregate demand is determined by private consumption, government consumption, investment, variation of inventories, exports and imports. On the nominal side, the AW models the wage rate and six deflators (GDP at factor cost deflator, GDP at market price deflator, private consumption deflator, import and export deflators as well as an investment deflator). The model has been estimated equation by equation.

Table 12 summarizes the previous discussion.

Table 12: Basic Characteristics of the Four Macro-models

	CW-T	CW-F	SW	AW
IS Components	1	1	2	6
Price Variables	2[1]	2[1]	3	6
Labor Variables	0	0	1	1
Estimation Period	1974:1-1998:4	1974:1-1998:4	1980:2-1999:4	1970:1-1997:4[2]
Residual Filtering Period	1979:2-1998:4	1979:2-1998:4		1992:3 to 2000:4

[1] This includes the aggregate price level and the nominal contract wage. [2]The behavioral equations of the AW are estimated on a single equation basis. Depending on the equation, the estimation period may start later (but not later than 1980:1).

B The Minimax Algorithm

We employ the discrete minimax algorithm as implemented in `Matlab 6.1` as `fminimax`. The algorithm is based on Brayton *et al.* (1979). We briefly introduce the idea behind the algorithm.

Quasi-Newton methods solve constraint optimization problems stepwise, finding the optimal step $s\boldsymbol{\Delta}_k$ starting from a point $\mathbf{x}_k$. To illustrate this, consider the equality constrained problem

$$\min_{\mathbf{x}} f(\mathbf{x}), \text{ s.t. } \mathbf{g}(\mathbf{x}) = 0. \tag{7}$$

Let $L(\mathbf{x}, \boldsymbol{\lambda})$ be the corresponding Lagrangian. The Kuhn-Tucker conditions state that there exists a non-negative vector $\boldsymbol{\lambda}^*$ such that at the optimum point $(\mathbf{x}^*, \boldsymbol{\lambda}^*)$ the constraints are met and

$$\mathbf{h}(\mathbf{x}^*, \boldsymbol{\lambda}^*) := \frac{\partial f}{\partial \mathbf{x}}|_{\mathbf{x}^*} + \frac{\partial \mathbf{g}'}{\partial \mathbf{x}}|_{\mathbf{x}^*} \boldsymbol{\lambda}^* = \mathbf{0}.$$

Expanding around $\mathbf{x}_k := \mathbf{x}^* - \boldsymbol{\Delta}_k$, where $\boldsymbol{\Delta}_k$ is 'small',

$$\begin{cases} \mathbf{0} = \mathbf{h}(\mathbf{x}^*, \boldsymbol{\lambda}^*) & \simeq \mathbf{h}(\mathbf{x}_k, \boldsymbol{\lambda}^*) + f_{\mathbf{xx}}|_{\mathbf{x}_k} \boldsymbol{\Delta}_k + \sum \lambda_i^* g_{i\mathbf{xx}}|_{\mathbf{x}_k} \boldsymbol{\Delta}_k \\ \mathbf{0} = \mathbf{g}(\mathbf{x}^*) & \simeq \mathbf{g}(\mathbf{x}_k) + \frac{\partial \mathbf{g}}{\partial \mathbf{x}'}|_{\mathbf{x}_k} \boldsymbol{\Delta}_k. \end{cases} \tag{8}$$

System (8) is the same system of equations as the first order conditions derived by solving the following quadratic subproblem (QSP), provided $\boldsymbol{\Delta}_k$ is small enough so $\boldsymbol{\lambda}_k^{qsp} \simeq \boldsymbol{\lambda}^*$, where $\boldsymbol{\lambda}^{qsp}$ are the multipliers of the QSP

$$\begin{array}{cc} \min_{\boldsymbol{\Delta}_k} & \frac{1}{2}\boldsymbol{\Delta}_k' \mathbf{H} \boldsymbol{\Delta}_k + \frac{\partial f}{\partial \mathbf{x}'}|_{\mathbf{x}_k} \boldsymbol{\Delta}_k \\ \text{s.t.} & \mathbf{g}(\mathbf{x}_k) + \frac{\partial \mathbf{g}}{\partial \mathbf{x}'}|_{\mathbf{x}_k} \boldsymbol{\Delta}_k = \mathbf{0}, \\ \text{where} & \mathbf{H} := L_{\mathbf{xx}}|_{\mathbf{x}_k, \boldsymbol{\lambda}^*}. \end{array} \tag{9}$$

An optimum is found by a sequence of updates $\mathbf{x}_{k+1} = \mathbf{x}_k + s\boldsymbol{\Delta}_k$, and $\boldsymbol{\lambda}_{k+1} = (1-s)\boldsymbol{\lambda}_k + s\boldsymbol{\lambda}_k^{qsp}$. Stepsize s is determined on the basis of a problem-specific merit function.

The minimax problem

$$\min_{\mathbf{x}} \max_{j} f_j(\mathbf{x}), \text{ s.t. } \mathbf{g}(\mathbf{x}) \leq \mathbf{0}, \tag{10}$$

where some of the constraints in $\mathbf{g}(\cdot)$ may be strict equality constraints, can be restated as a non-linear optimization problem

$$\min_{\mathbf{x},\gamma} \gamma, \text{ s.t. } \mathbf{g}(\mathbf{x}) \leq \mathbf{0}, \text{ and } f_j(\mathbf{x}) \leq \gamma \ \forall j. \tag{11}$$

Let $\mathbf{y} := (\gamma, \mathbf{x}')'$, $f(\mathbf{y}) := \gamma$ and stack the constraints $f_j(\mathbf{x}) \leq \gamma$ as $\mathbf{b}(\mathbf{y}) \leq \mathbf{0}$. The Lagrangian for (11) than reads as $L(\mathbf{y}, \boldsymbol{\lambda}) = f(\mathbf{y}) + \boldsymbol{\lambda}_1'\mathbf{g}(\mathbf{y}) + \boldsymbol{\lambda}_2'\mathbf{b}(\mathbf{y})$. Analogous to (9), step $\boldsymbol{\Delta}_k^{(y)}$ is found by solving the quadratic subproblem using active set methods (let $\tilde{\mathbf{b}}$ mark the binding constraints).

$$\begin{cases} \min_{\boldsymbol{\Delta}_k^{(\mathbf{y})}} & \boldsymbol{\Delta}_k^{(\gamma)} + \frac{1}{2}\, \Delta_k^{(\mathbf{x})\,\prime}\mathbf{H}_{\mathbf{x}}\boldsymbol{\Delta}_k^{(\mathbf{x})} \\ \quad\text{s.t.} & \mathbf{g}(\mathbf{y}_k) + \frac{\partial \mathbf{g}}{\partial \mathbf{y}'}|_{\mathbf{y}_k}\boldsymbol{\Delta}_k^{(\mathbf{y})} = \mathbf{0} \\ & \tilde{\mathbf{b}}(\mathbf{y}_k) + \frac{\partial \tilde{\mathbf{b}}}{\partial \mathbf{y}'}|_{\mathbf{y}_k}\boldsymbol{\Delta}_k^{(\mathbf{y})} = \mathbf{0}, \end{cases} \tag{12}$$

where $H_{\mathbf{x}} := L_{\mathbf{x}x}|_{\mathbf{x}_k,\lambda_k}$. For the minimax problem any second (cross) derivative of L involving γ is zero. In `fminimax`, the step-size s in updating $\mathbf{y}_{k+1} := \mathbf{y}_k + s\boldsymbol{\Delta}_k^{(\mathbf{y})}$ has to lead to an improvement in either of two merit functions. Let $\mathcal{E}$ be the set of strict equality constraints in $\mathbf{g}$. The merit functions evaluated at a guess $\mathbf{y}(s) = \mathbf{y}_k + s\boldsymbol{\Delta}_k^{(\mathbf{y})}$ are of the form

$$P(\mathbf{y}, \boldsymbol{\mu}) = Loss(\mathbf{y}) + \sum_{i\in\mathcal{E}} \mu_{1,i}|g_i(\mathbf{x})| + \sum_{i\notin\mathcal{E}} \mu_{1,i} \max(g_i(\mathbf{x}), 0) + \sum_j \mu_{2,j} \max(b_j(\mathbf{y}), 0),$$

where we suppress subscript k for convenience. Above, $\boldsymbol{\mu}_k = \max\left[\boldsymbol{\lambda}_k^{qsp}, \frac{1}{2}(\boldsymbol{\lambda}_k^{qsp} + \boldsymbol{\mu}_{k-1})\right]$. The first merit function, exact for the constrained problem (11), sets $Loss(\mathbf{y}) = \gamma$, while the second merit function sets $Loss(\mathbf{y}) = \max_j f_j(\mathbf{x})$, which makes use of to the minimax problem structure (Brayton *et al.*, 1979). If a step length is found such that either of the penalty functions signals improvement, $\mathbf{y}$ and $\boldsymbol{\lambda}$ are updated accordingly.

As with all Quasi-Newton optimization methods, convergence may occur only to a local minimum. We entertain several starting values to safeguard the results against this feature.

C Bayesian Priors From Unconstrained Minimax

The aim of this section is to explain how priors $\tilde{p}_i$ can be found such that

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \max_i f_i(\mathbf{x}) = \arg\min_{\mathbf{x}} \sum_i \tilde{p}_i f_i(\mathbf{x}).$$

In the form of (11), the first-order conditions to the unconstrained ($\mathbf{g} \equiv \mathbf{0}$) minimax problem are $D_{\mathbf{y}}L := \frac{\partial L}{\partial \mathbf{y}} = \begin{pmatrix} 1 - \sum_i \lambda_i \\ \frac{\partial \sum_i f_i(\mathbf{x})\lambda_i}{\partial \mathbf{x}} \end{pmatrix} = \mathbf{0}$. Define

$$D^2_{\mathbf{y}}L := \frac{\partial L}{\partial \mathbf{y} \partial \mathbf{y}'} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\partial \sum_i f_i(\mathbf{x})\lambda_i}{\partial \mathbf{x} \partial \mathbf{x}'} \end{pmatrix}.$$

So for any local minimum of the unconstrained minimax, the first and second-order conditions for a local minimum of the unconstrained Bayesian optimization are also met, provided one takes $\boldsymbol{\lambda}^*$ (from the final step of the QSP for the minimax) as the Bayesian priors. Priors $\tilde{p}_i = \lambda_i^*$ support the minimax allocation, x^*. For the case of intermediate ambiguity aversion, the problem is $\min_{\mathbf{x}} \max_i \bar{f}_i(\mathbf{x})$, where $\bar{f}_i(\mathbf{x}) = (1-e)\sum_i p_i f_i(\mathbf{x}) + e f_i(\mathbf{x})$. Substituting for $\bar{f}_i(\cdot)$, the first order conditions are

$$\sum_i \bar{\lambda}_i^* = 1 \text{ and } \sum_i \left\{(1-e)p_i + e\bar{\lambda}_i^*\right\} \left.\frac{\partial f_i(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}^*} = 0 \quad \forall i.$$

Priors $\tilde{p}_i := (1-e)p_i + e\bar{\lambda}_i^*$ support the ambiguity averse allocation.

D Non-quadratic Preferences

This appendix serves to motivate the use of the non-quadratic preferences (6) and discusses the effects of those preferences on the relative weighting of models in expected loss minization. Following Kilian and Manganelli (2003) define downward inflation risk, *i.e.* risk of inflation falling short of a lower bound of the inflation target range, $\underline{\pi}$, as

$$DR_{\xi_D}(\underline{\pi}) = -\int_{-\infty}^{\underline{\pi}} (\underline{\pi} - \pi)^{\xi_D} dF(\pi), \ \xi_D > 0. \tag{13}$$

Note that $dF(\pi)$ in our setup is the unconditional (stationary) marginal distribution function of inflation, which we will treat as Gaussian in our study. Similarly upward inflation risk is defined by,

$$UR_{\xi_U}(\overline{\pi}) = \int_{\overline{\pi}}^{\infty} (\pi - \overline{\pi})^{\xi_U} dF(\pi),\ \beta > 0. \tag{14}$$

In general, the target range need not be symmetric about the mean inflation rate nor do the parameters ξ_D and ξ_U governing the inflation risk aversion of the policy maker need to be equal. If the probability of inflation falling within the target range is zero, and hence especially if $\overline{\pi} = \underline{\pi}$, a value of $\xi = 1$ implies risk neutrality.[13] Risk-seeking behaviour on each risk branch is implied by $\xi < 1$, whereas risk averse behavior follows from $\xi > 1$. The academic monetary economics literature conventionally uses $\xi = 2$. Suppose that a central banker focuses his decisions only on the risk-measures DR and UR such that a distribution F is preferred to G iff

$$U(DR_{\xi_D}(\underline{\pi}, F), UR_{\xi_U}(\overline{\pi}, F)) > U(DR_{\xi_D}(\underline{\pi}, G), UR_{\xi_U}(\overline{\pi}, G)).$$

In general, such a central banker's preferences need not satisfy the von Neumann-Morgenstern axioms for expected utility. There is a special case, however, where they do. If the central banker's preferences can be expressed as

$$u(\pi) = \begin{cases} -(1-a)(\underline{\pi} - \pi)^{\xi_D} & if\ \pi < \underline{\pi} \\ 0 & if\ \underline{\pi} \leq \pi \leq \overline{\pi} \\ -a(\pi - \overline{\pi})^{\xi_U} & if \pi > \overline{\pi} \end{cases}$$

then $E(u) = (1-a)DR_{\xi_D} - aUR_{\xi_U}$. For the case in which the central bank is only concerned with inflation and $\mathcal{L}(\pi) = \frac{1}{2}I(\pi < -1)(-1-\pi)^{\xi_D} + \frac{1}{2}I(\pi > 1)(\pi - 1)^{\xi_U}$, Figure 5 illustrates the behaviour of the loss function.

For the sake of expository simplicity, we focus on the case of the same aversion to downside and upside risk, $\xi_U = \xi_D = \xi$, as we have done throughout the paper. We also

[13] For upside- and downside risk considered for themselves.

Figure 5: Loss for Symmetric Zone for Varying ξ

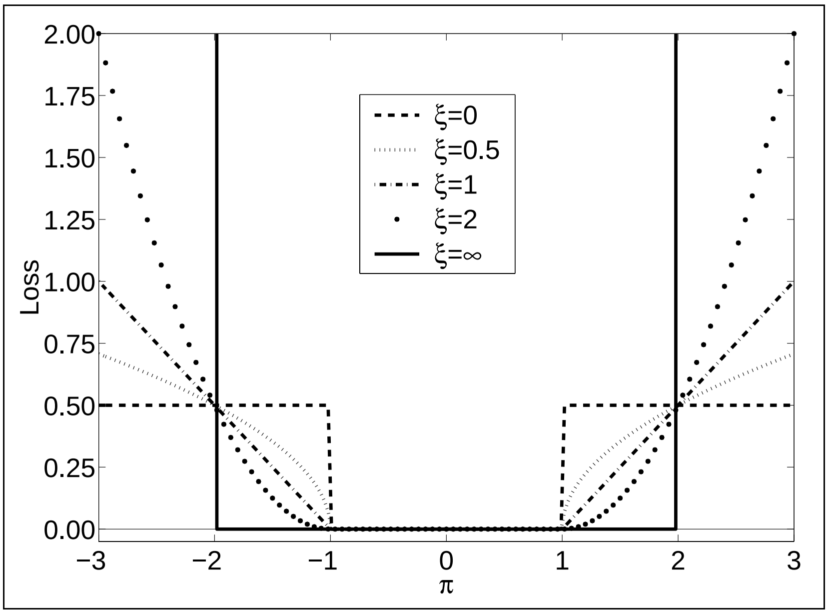

Figure 5: For varying values of ξ the figure plots the loss function $\mathcal{L}(\pi) = \frac{1}{2}I(\pi > 1)(\pi - 1)^{\xi} + \frac{1}{2}I(\pi < -1)(-1 - \pi)^{\xi}$.

assume a symmetric target range about the steady state of zero. As the figure illustrates, when $\xi = 0$, the central-bank exclusively focuses on whether inflation leaves the target range or not. No further information as, say, with regard to the severity of the violation of the range is taken into account. For any $\xi < 1$ the central banker's main concern is that he may fail to attain the target. As ξ increases towards unity, the utility function turns piecewise linear. Abstracting from the target range, the agent would be risk-neutral with respect to upside and downside inflation risks.[14] As ξ increases further, the loss function outside the target range turns more and more convex, leading the policy maker to evermore risk averse policy making. In the limit, as $\xi \to \infty$, the policy maker accepts all realizations of inflation within a range of ± 1 around the target range, but is reluctant to accept any larger deviations from target.[15]

[14] With respect to the entire utility function which is concerned with overall risk, even in the case $\xi = 1$, $\underline{\pi} = \overline{\pi}$, in which risk neutrality on each branch prevails, the agent is risk averse to the overall risk of deviations from target.

[15] Observe, that there may not exist a policy such that the expected loss remains bounded as $\xi \to \infty$.

In the absence of model-uncertainty, as long as the mean of the inflation process falls within the target range, due to the symmetry of the normal distribution about the mean, it is apparent that the best a risk averse policy maker can do if he only implements a linear policy rule is to curb inflation variation (the standard deviation) as much as possible. There is no trade-off between upside and downside inflation risk.

Model uncertainty and multiple policy targets provide for an interesting departure. Policy conclusions will depend on the underlying preferences of the central banker. For expository purposes, Figure 6 illustrates the relative weights attached in the loss function to two different Gaussian inflation models both with mean zero. We assume a flat prior for the two models. Model 1 has a standard deviation of 1 for inflation while model 2's standard deviation is twice as large. The panel illustrates the relative importance attached to the more dispersed second model in terms of influence on the Bayesian loss function, as $\xi_D = \xi_U$ increases.

Figure 6: Share of Loss for Symmetric Zone for Varying ξ

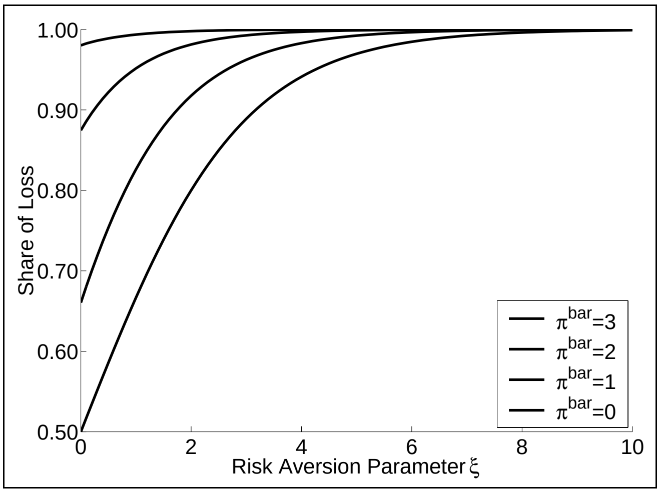

Figure 6: Relative share of losses in a flat Bayesian loss function $\mathcal{L}_B = \frac{1}{2} UR_\xi^{\text{Model 1}}(\overline{\pi}) + \frac{1}{2} UR_\xi^{\text{Model 2}}(\overline{\pi})$ given to model 2. Model 1 has a smaller standard deviation of inflation than model 2, $\sigma_1 = 1$, $\sigma_2 = 2$.

Even under flat Bayesian priors the risk-measure parameters have an important bearing on the composition of the loss function. For the case that a policy maker is not very risk-averse, for a tight target range, he is concerned only with the probability that inflation in each of the models will exceed its steady state value, $\bar{\pi} = 0$. By symmetry of the normal distribution about its mean this probability is equal to $\frac{1}{2}$ in both models. Now, as the weight ξ on deviations rises, the relative weight of model two in the loss function steadily increases. In the extreme, if $\xi > 8$ the policy maker bases his decisions virtually exclusively on the performance of model 2, since this contributes almost the entire losses in relative terms.

E Standard Deviations and Losses

E.1 Single model policy

Table 13: Standard Deviations and Levels of Losses for Single Reference Models

			CW-F				CW-T				SW				AW			
$\lambda_{\Delta r}$	λ_y	ξ	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr
Optimal Policy for CW-F																		
0.5	0.0	1	1.6	1.7	1.5	1.1	1.1	1.4	1.0	0.7	0.8	1.2	0.7	0.5	1.4	3.0	1.1	1.2
		2	2.9	1.7	1.5	1.2	1.3	1.4	1.0	0.8	0.7	1.2	0.7	0.6	2.4	3.4	1.2	1.4
		5	49.3	1.8	1.4	1.3	7.1	1.4	1.0	0.9	1.4	1.2	0.7	0.6	63.6	3.8	1.3	1.6
		10	3.8e4	1.8	1.4	1.3	891.5	1.4	1.0	0.9	36.1	1.2	0.7	0.6	6.6e5	3.9	1.3	1.7
0.5	0.5	1	2.3	1.5	1.6	1.0	1.6	1.2	1.0	0.7	1.2	1.0	0.7	0.6	2.1	2.3	0.9	1.1
		2	4.2	1.5	1.6	1.1	2.1	1.2	1.0	0.8	1.2	1.0	0.7	0.6	4.3	2.3	0.9	1.3
		5	85.3	1.5	1.5	1.2	17.9	1.2	1.0	1.0	4.5	1.0	0.7	0.7	202.5	2.2	0.9	1.5
		10	9.0e4	1.5	1.5	1.3	5.1e3	1.2	1.0	1.1	318.4	0.9	0.7	0.7	1.0e6	2.1	0.9	1.6
0.5	1.0	1	2.8	1.4	1.7	1.0	2.0	1.2	1.0	0.7	1.6	0.9	0.7	0.6	2.8	2.0	0.9	1.2
		2	5.2	1.4	1.6	1.1	2.7	1.2	1.0	0.9	1.6	0.9	0.7	0.7	5.8	2.0	0.9	1.4
		5	108.2	1.4	1.6	1.3	23.9	1.2	1.0	1.0	5.8	0.9	0.7	0.7	238.8	2.0	0.9	1.5
		10	1.1e5	1.5	1.5	1.4	7.0e3	1.2	1.0	1.1	389.2	0.9	0.7	0.8	1.0e6	2.0	0.9	1.6
1.0	0.0	1	2.0	1.6	1.7	0.8	1.2	1.3	1.0	0.5	0.9	1.2	0.7	0.4	1.5	2.5	1.0	0.8
		2	3.4	1.7	1.6	1.0	1.4	1.4	1.0	0.6	0.8	1.2	0.7	0.5	2.3	2.8	1.1	1.0
		5	58.2	1.7	1.5	1.2	7.5	1.4	1.0	0.8	1.5	1.2	0.7	0.6	49.5	3.3	1.2	1.4
		10	4.5e4	1.8	1.4	1.3	932.9	1.4	1.0	0.9	38.1	1.2	0.7	0.6	4.3e5	3.7	1.3	1.6
1.0	0.5	1	2.6	1.5	1.8	0.8	1.7	1.3	1.0	0.5	1.4	1.1	0.7	0.5	2.4	2.3	1.0	0.9
		2	4.7	1.5	1.6	0.9	2.2	1.3	1.0	0.7	1.4	1.0	0.7	0.5	4.8	2.3	1.0	1.1
		5	92.9	1.5	1.5	1.1	18.7	1.2	1.0	0.9	5.0	1.0	0.7	0.6	233.7	2.3	0.9	1.3
		10	9.6e4	1.5	1.5	1.3	5.3e3	1.2	1.0	1.0	372.7	1.0	0.7	0.7	1.2e6	2.2	0.9	1.5
1.0	1.0	1	3.2	1.4	1.8	0.8	2.2	1.2	1.0	0.6	1.8	1.0	0.8	0.5	3.2	2.1	0.9	1.0
		2	5.7	1.4	1.7	0.9	2.9	1.2	1.0	0.7	1.8	1.0	0.7	0.6	6.6	2.1	0.9	1.2
		5	116.0	1.4	1.6	1.1	25.3	1.2	1.0	0.9	6.6	0.9	0.7	0.7	287.0	2.1	0.9	1.4
		10	1.2e5	1.5	1.5	1.3	7.4e3	1.2	1.0	1.0	463.0	0.9	0.7	0.7	1.3e6	2.1	0.9	1.5

Single model standard deviations and losses.

Table 14: Standard Deviations and Levels of Losses for Single Reference Models

			CW-F				CW-T				SW				AW			
$\lambda_{\Delta r}$	λ_y	ξ	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr
Optimal Policy for CW-T																		
0.5	0.0	1	8.4	6.9	9.8	1.5	0.9	1.9	1.0	0.2	0.6	1.9	0.7	0.1	1.9	3.8	2.2	0.3
		2	7.4	2.7	2.6	1.0	1.0	2.0	0.9	0.3	0.5	1.9	0.7	0.2	∞	∞	∞	∞
		5	167.4	2.5	1.9	1.2	4.6	2.0	0.9	0.6	0.9	1.9	0.6	0.4	∞	∞	∞	∞
		10	6.7e4	2.0	1.5	1.2	636.3	1.6	0.9	0.7	52.5	1.4	0.7	0.5	8.2e8	7.1	2.5	1.9
0.5	0.5	1	2.9	1.5	2.6	0.5	1.5	1.3	1.0	0.3	1.2	1.2	0.8	0.3	1.9	2.2	1.0	0.6
		2	8.4	1.2	2.7	0.7	1.9	1.1	1.0	0.5	1.2	1.0	0.8	0.5	3.1	1.9	0.9	1.0
		5	594.1	1.1	2.5	0.9	12.7	1.0	1.1	0.8	3.3	0.8	0.8	0.6	62.0	1.7	0.9	1.3
		10	2.9e6	1.1	2.2	1.0	2.1e3	1.0	1.0	0.9	120.6	0.8	0.8	0.7	8.9e4	1.7	0.8	1.4
0.5	1.0	1	3.8	1.1	3.2	0.7	1.9	1.1	1.1	0.6	1.5	0.9	0.8	0.5	2.7	2.0	0.9	1.1
		2	12.8	1.1	3.4	0.9	2.4	1.0	1.1	0.7	1.5	0.8	0.8	0.6	4.6	1.7	0.9	1.2
		5	2.9e3	1.0	3.4	1.0	16.1	1.0	1.1	0.8	4.0	0.8	0.8	0.7	91.2	1.6	0.9	1.4
		10	1.6e7	1.0	2.7	1.0	2.6e3	1.0	1.1	0.9	143.0	0.7	0.8	0.7	1.2e5	1.6	0.8	1.5
1.0	0.0	1	∞	∞	∞	∞	0.9	2.0	1.0	0.1	0.7	2.0	0.7	0.1	1.6	3.2	1.9	0.2
		2	12.4	3.1	3.4	0.9	1.0	1.9	1.0	0.2	0.5	1.9	0.7	0.2	∞	∞	∞	∞
		5	224.0	2.5	2.0	1.1	4.7	2.0	0.9	0.5	0.9	1.8	0.7	0.3	∞	∞	∞	∞
		10	8.2e4	2.0	1.6	1.1	647.2	1.6	0.9	0.7	54.6	1.4	0.7	0.5	1.8e8	6.5	2.3	1.7
1.0	0.5	1	4.0	2.2	3.4	0.6	1.6	1.5	1.0	0.2	1.4	1.4	0.8	0.2	2.1	2.4	1.1	0.4
		2	8.7	1.3	2.7	0.6	2.0	1.2	1.0	0.4	1.4	1.1	0.8	0.4	3.5	2.0	0.9	0.8
		5	746.8	1.1	2.6	0.8	13.3	1.1	1.1	0.7	3.8	0.9	0.8	0.6	75.2	1.8	0.9	1.2
		10	4.0e6	1.1	2.3	0.9	2.2e3	1.0	1.0	0.8	136.5	0.8	0.8	0.7	1.0e5	1.7	0.9	1.4
1.0	1.0	1	3.4	1.4	2.3	0.6	2.1	1.2	1.0	0.4	1.8	1.1	0.8	0.4	3.0	2.1	1.0	0.7
		2	11.1	1.2	3.0	0.7	2.6	1.1	1.1	0.6	1.7	0.9	0.8	0.5	5.4	1.9	0.9	1.0
		5	3.1e3	1.0	3.4	0.9	17.1	1.0	1.1	0.8	4.7	0.8	0.8	0.6	118.0	1.7	0.9	1.3
		10	2.6e7	1.0	2.8	1.0	2.7e3	1.0	1.1	0.9	165.2	0.8	0.8	0.7	1.5e5	1.6	0.9	1.4

Single model standard deviations and losses. The notation "∞" indicates that the implemented rule results in instability.

Table 15: Standard Deviations and Levels of Losses for Single Reference Models

			CW-F				CW-T				SW				AW			
$\lambda_{\Delta r}$	λ_y	ξ	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr
Optimal Policy for SW																		
0.5	0.0	1	∞	∞	∞	∞	0.9	2.4	1.1	0.2	0.6	2.5	0.7	0.1	∞	∞	∞	∞
		2	13.1	3.7	3.5	1.4	1.0	2.4	1.0	0.4	0.4	2.5	0.6	0.2	∞	∞	∞	∞
		5	214.5	2.7	2.0	1.5	4.9	2.2	0.9	0.7	0.8	1.8	0.6	0.4	∞	∞	∞	∞
		10	6.7e4	2.0	1.5	1.4	976.4	1.6	1.0	0.9	30.5	1.2	0.7	0.5	∞	∞	∞	∞
0.5	0.5	1	2.8	1.3	2.4	0.9	1.6	1.2	1.1	0.8	1.1	0.8	0.8	0.5	4.5	5.2	1.5	3.2
		2	6.9	1.2	2.4	1.1	2.2	1.1	1.1	0.9	1.0	0.7	0.8	0.6	10.1	3.3	1.1	2.7
		5	320.0	1.2	2.2	1.2	17.9	1.1	1.1	1.0	2.6	0.7	0.8	0.7	1.5e3	3.1	1.0	2.8
		10	9.4e5	1.3	2.0	1.3	4.0e3	1.1	1.1	1.1	83.4	0.7	0.8	0.7	5.5e7	3.1	1.0	2.9
0.5	1.0	1	4.1	1.0	3.2	1.8	2.3	1.0	1.1	1.7	1.4	0.5	0.8	0.9	3.5	1.9	0.9	3.3
		2	10.5	1.1	2.9	1.4	3.1	1.0	1.1	1.3	1.3	0.6	0.8	0.8	10.7	2.4	0.9	2.9
		5	627.4	1.1	2.5	1.3	24.4	1.1	1.1	1.1	3.0	0.7	0.8	0.7	1.4e3	2.7	0.9	2.8
		10	2.0e6	1.2	2.2	1.3	5.4e3	1.1	1.1	1.1	95.3	0.7	0.8	0.7	5.0e7	2.8	0.9	2.9
1.0	0.0	1	∞	∞	∞	∞	1.0	2.6	1.2	0.1	0.6	2.7	0.7	0.0	∞	∞	∞	∞
		2	28.3	4.6	5.1	1.4	1.1	2.4	1.0	0.3	0.5	2.4	0.6	0.2	∞	∞	∞	∞
		5	310.4	2.8	2.1	1.4	5.0	2.2	0.9	0.6	0.9	1.9	0.6	0.4	∞	∞	∞	∞
		10	8.3e4	2.0	1.5	1.4	977.2	1.6	1.0	0.9	31.4	1.2	0.7	0.5	∞	∞	∞	∞
1.0	0.5	1	3.5	2.0	2.7	0.7	1.7	1.5	1.1	0.4	1.3	1.2	0.8	0.3	5.8	6.5	2.5	1.6
		2	7.6	1.4	2.4	0.9	2.4	1.2	1.1	0.7	1.2	0.9	0.8	0.5	117.9	11.7	3.1	6.3
		5	426.6	1.3	2.3	1.1	19.3	1.1	1.1	0.9	2.9	0.7	0.8	0.6	2.7e3	3.5	1.1	2.8
		10	1.4e6	1.3	2.1	1.2	4.2e3	1.1	1.1	1.0	90.9	0.7	0.8	0.6	8.4e7	3.2	1.0	2.8
1.0	1.0	1	4.2	1.3	2.9	1.0	2.4	1.2	1.1	0.8	1.7	0.8	0.8	0.5	6.2	3.9	1.2	2.6
		2	11.2	1.2	2.9	1.1	3.3	1.1	1.1	0.9	1.5	0.7	0.8	0.6	16.8	3.0	1.0	2.5
		5	810.2	1.2	2.6	1.2	26.6	1.1	1.1	1.0	3.5	0.7	0.8	0.6	2.0e3	2.9	1.0	2.6
		10	3.2e6	1.2	2.3	1.2	5.8e3	1.1	1.1	1.1	105.3	0.7	0.8	0.7	6.9e7	2.9	1.0	2.8

Single model standard deviations and losses. The notation "∞" indicates that the implemented rule results in instability.

Table 16: Standard Deviations and Levels of Losses for Single Reference Models

			CW-F				CW-T				SW				AW			
$\lambda_{\Delta r}$	λ_y	ξ	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr
Optimal Policy for AW																		
0.5	0.0	1	2.2	1.6	2.4	0.7	1.0	1.4	1.0	0.4	0.8	1.4	0.8	0.4	1.0	2.0	1.0	0.5
		2	5.1	1.5	2.2	0.7	1.2	1.3	1.0	0.5	0.8	1.3	0.8	0.5	1.1	1.9	0.9	0.6
		5	263.5	1.4	2.1	0.8	7.6	1.3	1.0	0.7	2.3	1.2	0.8	0.6	4.5	1.7	0.9	0.8
		10	7.3e6	1.3	2.4	0.8	1.3e3	1.2	1.0	0.7	120.5	1.2	0.8	0.7	475.6	1.6	0.9	0.8
0.5	0.5	1	3.0	1.3	2.7	0.9	1.6	1.2	1.0	0.8	1.4	1.1	0.8	0.8	1.7	1.6	0.9	0.9
		2	11.1	1.2	3.1	1.0	2.1	1.2	1.0	0.9	1.6	1.1	0.8	0.8	2.4	1.5	0.9	1.1
		5	ME	ME	ME	ME	ME	ME	ME	ME	7.1	1.0	0.8	0.9	27.9	1.4	0.8	1.2
		10	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	1.5e4	1.4	0.8	1.3
0.5	1.0	1	4.1	1.1	3.5	1.3	2.2	1.0	1.1	1.2	1.7	0.8	0.8	1.0	2.3	1.3	0.8	1.5
		2	ME	ME	ME	ME	ME	ME	ME	ME	1.9	0.9	0.8	1.0	3.4	1.3	0.8	1.4
		5	ME	ME	ME	ME	ME	ME	ME	ME	9.0	0.9	0.8	1.0	42.7	1.3	0.8	1.3
		10	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	2.4e4	1.3	0.8	1.3
1.0	0.0	1	2.9	2.0	3.0	0.6	1.0	1.6	1.0	0.3	0.9	1.6	0.8	0.3	1.2	2.2	1.1	0.3
		2	6.2	1.6	2.4	0.7	1.2	1.4	1.0	0.4	0.8	1.4	0.8	0.4	1.2	2.0	1.0	0.5
		5	317.1	1.4	2.2	0.8	7.6	1.3	1.0	0.6	2.3	1.3	0.8	0.6	5.1	1.8	0.9	0.7
		10	1.0e7	1.4	2.5	0.8	1.3e3	1.3	1.0	0.7	136.6	1.2	0.8	0.7	547.6	1.7	0.9	0.8
1.0	0.5	1	3.7	1.6	3.1	0.7	1.8	1.4	1.0	0.5	1.6	1.4	0.8	0.5	2.0	1.9	1.0	0.5
		2	12.6	1.4	3.3	0.8	2.3	1.3	1.0	0.7	1.9	1.2	0.8	0.7	2.8	1.7	0.9	0.8
		5	ME	ME	ME	ME	ME	ME	ME	ME	9.1	1.1	0.8	0.8	34.1	1.5	0.9	1.1
		10	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	1.9e4	1.4	0.9	1.2
1.0	1.0	1	ME	ME	ME	ME	ME	ME	ME	ME	2.2	1.2	0.8	0.7	2.7	1.7	0.9	0.8
		2	ME	ME	ME	ME	ME	ME	ME	ME	2.5	1.1	0.8	0.8	4.0	1.5	0.9	1.0
		5	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	52.9	1.4	0.9	1.2
		10	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	ME	3.1e4	1.4	0.9	1.3

Single model standard deviations and losses. The notation "ME" indicates that the implemented rule results in multiple equilibria.

E.2 Flat Bayesian Priors

Table 17: Standard Deviations and Levels of Losses

			CW-F				CW-T				SW				AW				
$\lambda_{\Delta r}$	λ_y	ξ	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_B$
Flat Bayesian priors																			
0.5	0.0	1	1.7	1.7	1.8	0.8	1.0	1.4	1.0	0.5	0.7	1.3	0.7	0.4	1.1	2.3	1.0	0.6	1.1
		2	3.1	1.7	1.6	0.9	1.2	1.4	1.0	0.6	0.7	1.3	0.7	0.5	1.3	2.3	1.0	0.8	1.5
		5	53.7	1.7	1.5	1.2	7.0	1.4	1.0	0.9	1.5	1.2	0.7	0.6	10.7	2.5	1.0	1.0	18.2
		10	4.6e4	1.7	1.5	1.3	1.1e3	1.4	1.0	1.0	49.0	1.2	0.7	0.7	7.3e3	2.3	1.0	1.2	1.3e4
0.5	0.5	1	2.3	1.4	1.8	0.8	1.5	1.2	1.0	0.6	1.2	1.1	0.8	0.5	1.8	1.9	0.9	0.9	1.7
		2	4.5	1.4	1.7	1.0	2.0	1.2	1.0	0.8	1.3	1.0	0.8	0.7	2.9	1.8	0.9	1.1	2.7
		5	102.8	1.4	1.6	1.3	18.7	1.2	1.0	1.1	5.2	1.0	0.7	0.8	52.5	1.7	0.8	1.3	44.8
		10	1.3e5	1.4	1.6	1.4	8.1e3	1.2	1.0	1.3	491.4	0.9	0.7	0.9	6.7e4	1.6	0.8	1.4	5.2e4
0.5	1.0	1	2.9	1.2	2.0	1.0	2.0	1.1	1.0	0.9	1.6	0.9	0.8	0.7	2.5	1.6	0.8	1.3	2.2
		2	5.6	1.2	1.8	1.1	2.7	1.1	1.0	1.0	1.6	0.9	0.8	0.8	4.0	1.6	0.8	1.3	3.5
		5	129.0	1.3	1.7	1.3	25.6	1.1	1.0	1.2	6.9	0.9	0.8	0.9	73.0	1.5	0.8	1.4	58.6
		10	1.6e5	1.4	1.6	1.5	1.1e4	1.1	1.0	1.3	692.0	0.9	0.7	0.9	8.9e4	1.5	0.8	1.4	6.7e4
1.0	0.0	1	2.1	1.8	2.0	0.7	1.1	1.4	1.0	0.4	0.9	1.4	0.7	0.3	1.2	2.3	1.1	0.5	1.3
		2	3.6	1.7	1.7	0.8	1.2	1.4	1.0	0.5	0.7	1.3	0.7	0.4	1.5	2.3	1.0	0.7	1.8
		5	62.9	1.7	1.5	1.1	7.2	1.4	1.0	0.8	1.6	1.2	0.7	0.6	11.7	2.4	1.0	0.9	20.8
		10	5.3e4	1.7	1.5	1.3	1.1e3	1.4	1.0	0.9	51.9	1.2	0.7	0.6	8.0e3	2.4	1.0	1.1	1.5e4
1.0	0.5	1	2.7	1.5	2.0	0.6	1.7	1.3	1.0	0.4	1.4	1.2	0.8	0.4	2.1	2.1	1.0	0.6	2.0
		2	4.9	1.4	1.8	0.8	2.2	1.2	1.0	0.6	1.5	1.1	0.8	0.5	3.4	1.9	0.9	0.9	3.0
		5	112.7	1.4	1.7	1.2	19.8	1.2	1.0	1.0	6.2	1.0	0.8	0.8	61.8	1.7	0.8	1.2	50.1
		10	1.4e5	1.4	1.6	1.4	8.6e3	1.2	1.0	1.2	604.8	1.0	0.7	0.9	7.8e4	1.6	0.8	1.3	5.8e4
1.0	1.0	1	3.3	1.3	2.0	0.7	2.2	1.2	1.0	0.5	1.9	1.1	0.8	0.5	2.9	1.9	0.9	0.8	2.6
		2	6.1	1.3	1.9	0.9	2.9	1.1	1.0	0.7	1.9	1.0	0.8	0.6	4.8	1.7	0.9	1.1	3.9
		5	140.0	1.3	1.7	1.2	27.3	1.1	1.0	1.0	8.4	0.9	0.8	0.8	87.9	1.6	0.8	1.3	65.9
		10	1.8e5	1.4	1.6	1.4	1.2e4	1.2	1.0	1.2	866.6	0.9	0.8	0.9	1.0e5	1.6	0.8	1.4	7.5e4

Standard deviations and losses for flat Bayesian priors. Final column: expected loss.

E.3 Minimax and Ambiguity Averse

Table 18: Standard Deviations and Levels of Losses

			CW-F				CW-T				SW				AW			
$\lambda_{\Delta r}$	λ_y	ξ	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr
									Minimax									
0.5	0.0	1	1.6	1.7	1.5	1.1	1.1	1.4	1.0	0.7	0.8	1.2	0.7	0.5	1.4	3.0	1.1	1.2
		2	2.9	1.7	1.5	1.2	1.3	1.4	1.0	0.8	0.7	1.2	0.7	0.6	2.4	3.4	1.2	1.4
		5	49.3	1.8	1.4	1.3	7.1	1.4	1.0	0.9	1.4	1.2	0.7	0.6	49.3	3.6	1.2	1.5
		10	4.0e4	1.7	1.4	1.3	1.0e3	1.4	1.0	1.0	40.9	1.2	0.7	0.6	4.0e4	2.8	1.1	1.3
0.5	0.5	1	2.3	1.5	1.6	1.0	1.6	1.2	1.0	0.7	1.2	1.0	0.7	0.6	2.1	2.3	0.9	1.1
		2	4.2	1.5	1.6	1.1	2.1	1.2	1.0	0.8	1.2	1.0	0.7	0.6	4.2	2.2	0.9	1.3
		5	89.0	1.5	1.5	1.3	18.6	1.2	1.0	1.0	5.0	1.0	0.7	0.8	89.0	1.9	0.9	1.3
		10	1.1e5	1.4	1.6	1.4	7.1e3	1.2	1.0	1.2	438.8	1.0	0.7	0.9	1.1e5	1.7	0.8	1.4
0.5	1.0	1	2.8	1.4	1.7	1.0	2.0	1.2	1.0	0.7	1.6	0.9	0.7	0.6	2.8	2.0	0.9	1.2
		2	5.2	1.4	1.6	1.1	2.7	1.2	1.0	0.9	1.6	0.9	0.7	0.7	5.2	1.9	0.9	1.3
		5	113.0	1.4	1.6	1.3	25.3	1.2	1.0	1.1	6.6	0.9	0.7	0.8	113.0	1.7	0.8	1.4
		10	1.4e5	1.4	1.6	1.5	1.0e4	1.2	1.0	1.3	604.2	0.9	0.7	0.9	1.4e5	1.6	0.8	1.4
1.0	0.0	1	2.0	1.6	1.7	0.8	1.2	1.3	1.0	0.5	0.9	1.2	0.7	0.4	1.5	2.5	1.0	0.8
		2	3.4	1.7	1.6	1.0	1.4	1.4	1.0	0.6	0.8	1.2	0.7	0.5	2.3	2.8	1.1	1.0
		5	58.2	1.7	1.5	1.2	7.5	1.4	1.0	0.8	1.5	1.2	0.7	0.6	49.5	3.3	1.2	1.4
		10	4.6e4	1.7	1.4	1.3	1.0e3	1.4	1.0	0.9	42.6	1.2	0.7	0.6	4.6e4	2.8	1.1	1.3
1.0	0.5	1	2.6	1.5	1.8	0.8	1.7	1.3	1.0	0.5	1.4	1.1	0.7	0.5	2.4	2.3	1.0	0.9
		2	4.7	1.5	1.6	0.9	2.2	1.3	1.0	0.7	1.4	1.0	0.7	0.5	4.7	2.3	0.9	1.1
		5	97.9	1.5	1.6	1.2	19.6	1.2	1.0	0.9	5.8	1.0	0.7	0.7	97.9	1.9	0.9	1.2
		10	1.2e5	1.4	1.6	1.4	7.6e3	1.2	1.0	1.2	533.9	1.0	0.7	0.8	1.2e5	1.7	0.8	1.3
1.0	1.0	1	3.2	1.4	1.8	0.8	2.2	1.2	1.0	0.6	1.8	1.0	0.8	0.5	3.2	2.1	0.9	1.0
		2	5.7	1.4	1.7	0.9	2.9	1.2	1.0	0.7	1.9	1.0	0.7	0.6	5.7	1.9	0.9	1.1
		5	123.6	1.4	1.6	1.2	27.3	1.2	1.0	1.0	8.0	0.9	0.7	0.8	123.6	1.7	0.8	1.3
		10	1.5e5	1.4	1.6	1.4	1.1e4	1.2	1.0	1.2	759.9	0.9	0.7	0.9	1.5e5	1.6	0.8	1.4

Standard deviations and losses for minimax policy.

Table 19: Standard Deviations and Levels of Losses

			CW-F				CW-T				SW				AW			
$\lambda_{\Delta r}$	λ_y	ξ	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr	$\mathcal{L}_m$	y	π	Δr
Ambiguity Averse CW-T																		
0.5	0.0	1	1.7	1.7	1.6	0.9	1.0	1.4	1.0	0.5	0.7	1.2	0.7	0.4	1.3	2.9	1.2	0.9
		2	2.9	1.8	1.5	1.1	1.2	1.4	1.0	0.7	0.6	1.2	0.7	0.5	2.3	3.4	1.2	1.2
		5	49.5	1.8	1.4	1.3	6.8	1.4	1.0	0.9	1.3	1.2	0.7	0.6	49.5	3.7	1.3	1.5
		10	4.0e4	1.7	1.4	1.3	989.4	1.4	1.0	1.0	40.8	1.2	0.7	0.6	4.0e4	2.8	1.1	1.3
0.5	0.5	1	2.3	1.5	1.7	0.8	1.5	1.2	1.0	0.6	1.2	1.1	0.7	0.5	2.1	2.3	1.0	1.0
		2	4.2	1.5	1.6	1.0	2.0	1.2	1.0	0.7	1.2	1.0	0.7	0.6	4.2	2.3	0.9	1.2
		5	89.5	1.5	1.6	1.2	17.4	1.2	1.0	1.0	4.7	1.0	0.7	0.7	89.5	1.9	0.9	1.3
		10	1.1e5	1.4	1.6	1.4	6.4e3	1.2	1.0	1.2	411.3	0.9	0.7	0.8	1.1e5	1.7	0.8	1.4
0.5	1.0	1	2.9	1.3	1.8	0.9	2.0	1.1	1.0	0.7	1.6	0.9	0.8	0.6	2.8	2.0	0.9	1.2
		2	5.3	1.3	1.7	1.0	2.6	1.1	1.0	0.8	1.6	0.9	0.7	0.6	5.3	1.9	0.9	1.3
		5	113.9	1.4	1.6	1.3	23.4	1.1	1.0	1.1	6.1	0.9	0.7	0.8	113.9	1.7	0.8	1.4
		10	1.4e5	1.4	1.6	1.4	9.0e3	1.2	1.0	1.2	558.9	0.9	0.7	0.9	1.4e5	1.6	0.8	1.4

Standard deviations and losses for ambiguity averse CW-T policy. Parameters: $e = 0.5$, $p_{CW-T} \approx 1$, $p_m \approx 0$ for all other models.

F Relative Performance

F.1 Single Models

Table 20: Performance Relative to First-best Simple

			Rule			Relative Loss				Inflation Premium			
$\lambda_{\Delta r}$	λ_y	ξ	ρ	α	β	CW-F	CW-T	SW	AW	CW-F	CW-T	SW	AW
						Optimal CW-F policy							
0.5	0.0	1	0.9	0.7	0.4	0	23	35	39	0	.25	.25	.49
		2	0.9	0.8	0.4	0	31	53	126	0	.15	.16	.56
		5	0.9	0.9	0.5	0	54	61	1321	0	.09	.08	.69
		10	0.9	0.9	0.5	0	40	18	1.4e5	0	.04	.01	1.05
0.5	0.5	1	0.8	0.6	0.5	0	7	6	25	0	.13	.09	.54
		2	0.8	0.7	0.6	0	12	17	82	0	.10	.11	.79
		5	0.8	0.9	0.7	0	41	73	627	0	.11	.13	1.10
		10	0.8	1.0	0.8	0	138	282	6392	0	.12	.13	1.16
0.5	1.0	1	0.8	0.5	0.7	0	4	13	25	0	.11	.22	.70
		2	0.8	0.7	0.8	0	12	27	71	0	.13	.19	.93
		5	0.8	0.9	0.9	0	49	90	459	0	.15	.16	1.15
		10	0.8	1.0	0.9	0	167	309	4020	0	.15	.14	1.16
1.0	0.0	1	0.9	0.4	0.4	0	32	54	30	0	.37	.40	.43
		2	0.9	0.6	0.4	0	37	67	87	0	.17	.20	.44
		5	0.9	0.8	0.4	0	58	72	875	0	.10	.09	.59
		10	0.9	0.9	0.5	0	44	22	7.9e4	0	.04	.02	.96
1.0	0.5	1	0.9	0.4	0.4	0	10	7	22	0	.20	.12	.53
		2	0.8	0.5	0.5	0	12	15	69	0	.11	.11	.76
		5	0.8	0.8	0.7	0	40	74	586	0	.11	.14	1.14
		10	0.8	0.9	0.8	0	142	310	6480	0	.13	.14	1.20
1.0	1.0	1	0.8	0.4	0.5	0	5	8	20	0	.12	.17	.67
		2	0.8	0.5	0.6	0	11	22	63	0	.13	.19	.95
		5	0.8	0.8	0.8	0	48	91	443	0	.15	.18	1.21
		10	0.8	0.9	0.9	0	170	340	4106	0	.15	.16	1.21

Percentage losses relative to first-best simple rule in %. Implied Inflation (Variability) Premium in percentage points.

Table 21: Performance Relative to First-best Simple

$\lambda_{\Delta r}$	λ_y	ξ	Rule ρ	α	β	Relative Loss CW-F	CW-T	SW	AW	Inflation Premium CW-F	CW-T	SW	AW
Optimal CW-T policy													
0.5	0.0	1	1.0	0.1	0.0	414	0	7	91	8.51	0	.05	1.13
		2	1.0	0.3	0.1	161	0	9	∞	1.13	0	.03	∞
		5	0.9	0.6	0.1	240	0	4	∞	.47	0	.01	∞
		10	0.9	0.7	0.3	75	0	72	1.7e8	.10	0	.05	3.38
0.5	0.5	1	0.9	0.1	0.3	28	0	8	11	.79	0	.12	.24
		2	0.8	0.2	0.6	103	0	15	29	1.03	0	.09	.34
		5	0.8	0.3	0.8	596	0	27	122	.94	0	.06	.57
		10	0.8	0.4	1.0	3131	0	45	470	.74	0	.04	.70
0.5	1.0	1	0.9	0.1	0.6	33	0	10	21	1.19	0	.18	.58
		2	0.8	0.2	0.8	145	0	18	36	1.57	0	.13	.55
		5	0.8	0.2	1.0	2627	0	32	114	1.85	0	.07	.68
		10	0.8	0.3	1.0	1.4e4	0	50	389	1.13	0	.04	.75
1.0	0.0	1	1.0	0.1	0.0	∞	0	10	41	∞	0	.07	.60
		2	1.0	0.2	0.1	262	0	10	∞	1.81	0	.04	∞
		5	0.9	0.5	0.1	285	0	5	∞	.54	0	.01	∞
		10	0.9	0.7	0.3	83	0	74	3.3e7	.11	0	.05	2.53
1.0	0.5	1	0.9	0.1	0.2	54	0	6	9	1.76	0	.09	.23
		2	0.8	0.2	0.4	87	0	15	24	.96	0	.11	.32
		5	0.8	0.2	0.7	704	0	30	121	1.03	0	.07	.61
		10	0.8	0.3	0.9	4038	0	50	451	.80	0	.04	.72
1.0	1.0	1	0.9	0.2	0.4	7	0	10	12	.29	0	.21	.39
		2	0.8	0.2	0.6	95	0	17	34	1.18	0	.15	.59
		5	0.8	0.2	0.8	2590	0	35	123	1.86	0	.08	.75
		10	0.8	0.3	1.0	2.1e4	0	57	382	1.26	0	.05	.77

Percentage losses relative to first-best simple rule in %. Implied Inflation (Variability) Premium in percentage points. The notation "∞" indicates that the implemented rule results in instability.

Table 22: Performance Relative to First-best Simple

			Rule			Relative Loss				Inflation Premium			
$\lambda_{\Delta r}$	λ_y	ξ	ρ	α	β	CW-F	CW-T	SW	AW	CW-F	CW-T	SW	AW
Optimal SW policy													
0.5	0.0	1	1.0	0.1	0.0	∞	6	0	∞	∞	.06	0	∞
		2	1.0	0.4	0.0	360	8	0	∞	2.06	.04	0	∞
		5	1.0	0.6	0.1	335	6	0	∞	.57	.01	0	∞
		10	1.0	0.8	0.5	74	53	0	∞	.10	.05	0	∞
0.5	0.5	1	1.0	0.2	0.7	22	11	0	168	.64	.21	0	3.56
		2	1.0	0.2	0.8	66	19	0	324	.72	.16	0	2.06
		5	1.0	0.3	0.9	275	40	0	5439	.63	.11	0	2.14
		10	1.0	0.4	1.0	947	87	0	3.5e5	.50	.09	0	2.15
0.5	1.0	1	1.0	0.2	1.7	44	19	0	57	1.58	.46	0	1.61
		2	1.0	0.2	1.3	101	26	0	220	1.18	.27	0	2.01
		5	1.0	0.2	1.1	480	52	0	3223	.91	.15	0	2.09
		10	1.0	0.4	1.0	1666	108	0	2.0e5	.63	.11	0	2.13
1.0	0.0	1	1.0	0.1	0.0	∞	10	0	∞	∞	.11	0	∞
		2	1.0	0.3	0.0	728	8	0	∞	3.65	.04	0	∞
		5	1.0	0.6	0.1	433	6	0	∞	.68	.01	0	∞
		10	1.0	0.7	0.4	83	51	0	∞	.11	.05	0	∞
1.0	0.5	1	1.0	0.1	0.2	34	10	0	196	1.11	.20	0	4.84
		2	1.0	0.2	0.6	63	20	0	4085	.73	.18	0	9.87
		5	1.0	0.2	0.8	359	45	0	8011	.74	.12	0	2.50
		10	1.0	0.4	0.9	1351	90	0	4.3e5	.57	.09	0	2.29
1.0	1.0	1	1.0	0.1	0.7	30	15	0	130	1.21	.40	0	4.37
		2	1.0	0.1	0.8	95	27	0	314	1.19	.30	0	2.81
		5	1.0	0.2	0.9	599	56	0	3760	1.02	.17	0	2.30
		10	1.0	0.3	1.0	2524	112	0	2.2e5	.72	.12	0	2.23

Percentage losses relative to first-best simple rule in %. Implied Inflation (Variability) Premium in percentage points. The notation "∞" indicates that the implemented rule results in instability.

Table 23: Performance Relative to First-best Simple

$\lambda_{\Delta r}$	λ_y	ξ	Rule ρ	α	β	Relative Loss CW-F	CW-T	SW	AW	Inflation Premium CW-F	CW-T	SW	AW
Optimal AW policy													
0.5	0.0	1	0.7	0.4	0.4	31	15	40	0	.65	.16	.29	0
		2	0.6	0.5	0.5	78	24	78	0	.63	.12	.23	0
		5	0.6	0.7	0.7	434	66	171	0	.66	.11	.16	0
		10	0.5	0.6	0.9	1.8e4	111	295	0	1.04	.09	.12	0
0.5	0.5	1	0.5	0.6	1.0	32	10	22	0	.91	.19	.32	0
		2	0.4	0.6	1.3	167	15	49	0	1.50	.12	.28	0
		5	0.4	0.4	1.5	ME	ME	173	0	ME	ME	.23	0
		10	0.4	0.3	1.6	ME	ME	ME	0	ME	ME	ME	0
0.5	1.0	1	0.4	0.6	1.9	46	11	24	0	1.63	.28	.42	0
		2	0.4	0.5	1.7	ME	ME	54	0	ME	ME	.36	0
		5	0.4	0.4	1.7	ME	ME	197	0	ME	ME	.27	0
		10	0.4	0.3	1.7	ME	ME	ME	0	ME	ME	ME	0
1.0	0.0	1	0.8	0.2	0.2	45	15	45	0	1.14	.17	.34	0
		2	0.7	0.4	0.4	80	22	81	0	.71	.11	.24	0
		5	0.6	0.6	0.6	445	60	171	0	.69	.10	.17	0
		10	0.5	0.6	0.8	2.2e4	111	335	0	1.09	.09	.13	0
1.0	0.5	1	0.6	0.4	0.5	41	12	25	0	1.33	.23	.41	0
		2	0.5	0.5	0.9	170	17	59	0	1.62	.15	.37	0
		5	0.4	0.5	1.3	ME	ME	215	0	ME	ME	.28	0
		10	0.5	0.3	1.4	ME	ME	ME	0	ME	ME	ME	0
1.0	1.0	1	0.5	0.5	0.8	ME	ME	30	0	ME	ME	.64	0
		2	0.4	0.5	1.2	ME	ME	67	0	ME	ME	.49	0
		5	0.4	0.4	1.5	ME	ME	ME	0	ME	ME	ME	0
		10	0.4	0.3	1.6	ME	ME	ME	0	ME	ME	ME	0

Percentage losses relative to first-best simple rule in %. Implied Inflation (Variability) Premium in percentage points. The notation "ME" indicates that the implemented rule results in multiple equilibria.

F.2 Flat Bayesian Priors

Table 24: Performance Relative to First-best Simple

			Rule			Relative Loss					Inflation Premium				
$\lambda_{\Delta r}$	λ_y	ξ	ρ	α	β	CW-F	CW-T	SW	AW	$\Delta\mathcal{L}^{worst}$	CW-F	CW-T	SW	AW	$\Delta\sigma_\pi$
						Flat Bayesian priors									
0.5	0.0	1	0.8	0.4	0.3	6	13	29	7	∞	.12	.14	.21	.09	∞
		2	0.8	0.6	0.4	7	21	51	22	∞	.07	.10	.16	.12	∞
		5	0.8	0.9	0.5	9	52	77	140	∞	.03	.09	.09	.20	∞
		10	0.8	1.1	0.6	21	83	61	1445	∞	.03	.07	.04	.35	∞
0.5	0.5	1	0.8	0.5	0.5	3	4	8	9	94	.08	.07	.12	.18	3.38
		2	0.7	0.7	0.8	7	9	22	21	149	.09	.08	.14	.26	1.80
		5	0.6	1.2	1.0	20	47	102	89	∞	.09	.12	.16	.49	∞
		10	0.6	1.4	1.2	48	278	489	327	∞	.09	.18	.17	.64	∞
0.5	1.0	1	0.7	0.6	1.0	3	4	13	9	41	.11	.10	.23	.25	1.38
		2	0.7	0.8	1.1	7	11	29	19	∞	.11	.12	.21	.32	∞
		5	0.6	1.2	1.2	19	59	128	71	∞	.10	.17	.20	.55	∞
		10	0.6	1.4	1.3	44	340	626	261	∞	.09	.22	.20	.68	∞
1.0	0.0	1	0.9	0.3	0.2	7	18	42	8	∞	.16	.20	.31	.11	∞
		2	0.8	0.5	0.3	7	23	59	21	∞	.07	.11	.18	.12	∞
		5	0.8	0.8	0.4	8	53	85	131	∞	.03	.09	.10	.20	∞
		10	0.8	1.0	0.5	18	81	65	1375	∞	.03	.07	.04	.35	∞
1.0	0.5	1	0.8	0.3	0.3	3	5	8	8	115	.11	.10	.14	.19	4.65
		2	0.8	0.5	0.5	6	9	23	20	2.2e3	.08	.08	.16	.28	9.59
		5	0.7	1.0	0.9	21	49	115	82	∞	.10	.13	.19	.51	∞
		10	0.6	1.3	1.1	53	285	566	299	∞	.10	.19	.19	.66	∞
1.0	1.0	1	0.8	0.3	0.5	2	3	11	9	∞	.09	.08	.24	.29	∞
		2	0.7	0.6	0.8	6	10	30	19	∞	.10	.12	.25	.36	∞
		5	0.6	1.0	1.1	21	60	142	66	∞	.11	.18	.24	.58	∞
		10	0.6	1.3	1.2	49	348	723	239	∞	.10	.22	.22	.70	∞

Percentage losses relative to first-best simple rule in %. $\Delta\mathcal{L}^{worst}$: percentage reduction of worst-case loss relative to worst single model policy. Implied Inflation (Variability) Premium in percentage points. Final column: reduction of worst Implied Inflation (Variability) Premium relative to worst single model policy.

F.3 Minimax and Ambiguity Averse

Table 25: Performance Relative to First-best Simple

			Rule			Relative Loss					Inflation Premium				Premium
$\lambda_{\Delta r}$	λ_y	ξ	ρ	α	β	CW-F	CW-T	SW	AW	$\Delta\mathcal{L}^{worst}$	CW-F	CW-T	SW	AW	$\Delta\mathcal{L}^{expect}$
								Minimax							
0.5	0.0	1	0.9	0.7	0.4	0	23	35	39	6	0	.25	.25	.48	7
		2	0.9	0.8	0.5	0	32	53	129	7	0	.15	.16	.57	17
		5	0.9	0.9	0.5	0	55	62	1003	9	0	.09	.08	.61	47
		10	0.8	1.0	0.5	5	59	34	8390	16	.01	.05	.03	.58	48
0.5	0.5	1	0.8	0.6	0.5	0	7	6	25	3	0	.13	.09	.54	3
		2	0.8	0.7	0.6	0	12	18	74	7	0	.10	.11	.73	9
		5	0.7	1.1	0.9	4	46	92	219	16	.02	.12	.15	.74	12
		10	0.7	1.3	1.1	23	233	426	611	20	.05	.17	.16	.74	10
0.5	1.0	1	0.8	0.6	0.7	0	4	13	25	3	0	.11	.22	.69	3
		2	0.8	0.7	0.8	0	12	29	56	7	0	.13	.20	.77	6
		5	0.7	1.1	1.0	4	57	116	165	14	.03	.17	.19	.79	10
		10	0.6	1.4	1.2	22	292	534	464	19	.05	.20	.19	.77	9
1.0	0.0	1	0.9	0.4	0.4	0	32	53	30	7	0	.36	.40	.43	6
		2	0.9	0.6	0.4	0	37	67	86	7	0	.18	.20	.44	11
		5	0.9	0.8	0.4	0	58	72	876	8	0	.10	.09	.59	40
		10	0.8	0.9	0.5	3	58	36	8435	15	.01	.05	.03	.59	50
1.0	0.5	1	0.9	0.4	0.4	0	10	7	22	3	0	.20	.12	.53	3
		2	0.8	0.6	0.5	0	12	15	66	6	0	.11	.11	.73	8
		5	0.7	0.9	0.8	5	47	100	187	15	.03	.13	.17	.74	10
		10	0.7	1.2	1.0	27	241	488	531	20	.06	.17	.18	.75	9
1.0	1.0	1	0.8	0.4	0.5	0	5	8	19	2	0	.12	.17	.64	2
		2	0.8	0.6	0.6	0	12	26	42	6	.01	.14	.22	.70	4
		5	0.7	1.0	0.9	7	59	129	134	13	.04	.18	.22	.78	7
		10	0.6	1.3	1.1	27	306	622	398	18	.06	.21	.21	.78	7

Relative loss: percentage losses relative to first-best simple rule in %. $\Delta\mathcal{L}^{worst}$: percentage reduction of worst-case loss relative to worst outcome under flat Bayesian priors. Implied Inflation (Variability) Premium in percentage points. Premium: percentage increase in expected loss relative to Bayesian policy (flat priors).

Table 26: Performance Relative to First-best Simple

			Rule			Relative Loss				Inflation Premium			
$\lambda_{\Delta r}$	λ_y	ξ	ρ	α	β	CW-F	CW-T	SW	AW	CW-F	CW-T	SW	AW
Ambiguity Averse CW-T													
0.5	0.0	1	0.9	0.5	0.3	2	16	28	28	.03	.17	.21	.35
		2	0.9	0.7	0.4	1	23	45	114	.01	.11	.14	.51
		5	0.9	0.9	0.4	0	48	57	1006	0	.08	.07	.61
		10	0.8	1.0	0.5	5	55	34	8392	.01	.05	.02	.58
0.5	0.5	1	0.9	0.4	0.5	1	4	5	22	.02	.07	.08	.47
		2	0.8	0.6	0.6	1	8	15	75	.01	.07	.10	.74
		5	0.8	1.0	0.8	5	36	80	221	.03	.10	.13	.74
		10	0.7	1.3	1.1	24	197	393	613	.05	.15	.16	.74
0.5	1.0	1	0.8	0.4	0.6	0	3	12	24	.02	.07	.21	.69
		2	0.8	0.6	0.8	1	8	25	57	.01	.09	.18	.78
		5	0.7	1.0	1.0	5	45	99	167	.03	.14	.17	.79
		10	0.7	1.3	1.2	22	245	487	466	.05	.18	.18	.78

Relative loss: percentage losses relative to first-best simple rule in %. Implied Inflation (Variability) Premium in percentage points.

F.4 Minimax and CW-T Ambiguity Averse Implied Priors

Table 27: Minimax Implied Priors

$\lambda_{\Delta r}$	λ_y	ξ	CW-F	CW-T	SW	AW
			Minimax			
0.5	0.0	1	1.000	0.0	0.0	0.000
		2	1.000	0.0	0.0	0.000
		5	0.996	0.0	0.0	0.004
		10	0.960	0.0	0.0	0.040
0.5	0.5	1	1.000	0.0	0.0	0.000
		2	0.989	0.0	0.0	0.011
		5	0.883	0.0	0.0	0.117
		10	0.789	0.0	0.0	0.211
0.5	1.0	1	1.000	0.0	0.0	0.000
		2	0.961	0.0	0.0	0.039
		5	0.870	0.0	0.0	0.130
		10	0.782	0.0	0.0	0.218
1.0	0.0	1	1.000	0.0	0.0	0.000
		2	1.000	0.0	0.0	0.000
		5	1.000	0.0	0.0	0.000
		10	0.967	0.0	0.0	0.033
1.0	0.5	1	1.000	0.0	0.0	0.000
		2	0.994	0.0	0.0	0.006
		5	0.862	0.0	0.0	0.138
		10	0.770	0.0	0.0	0.230
1.0	1.0	1	0.991	0.0	0.0	0.009
		2	0.923	0.0	0.0	0.077
		5	0.831	0.0	0.0	0.169
		10	0.755	0.0	0.0	0.245

Bayesian priors backing the minimax solution.

Table 28: Ambiguity Averse CW-T Implied Priors

$\lambda_{\Delta r}$	λ_y	ξ	CW-F	CW-T	SW	AW
Ambiguity Averse CW-T						
0.5	0.0	1	0.500	0.5	0.0	0.000
		2	0.500	0.5	0.0	0.000
		5	0.497	0.5	0.0	0.003
		10	0.479	0.5	0.0	0.021
0.5	0.5	1	0.500	0.5	0.0	0.000
		2	0.499	0.5	0.0	0.001
		5	0.435	0.5	0.0	0.065
		10	0.386	0.5	0.0	0.114
0.5	1.0	1	0.500	0.5	0.0	0.000
		2	0.476	0.5	0.0	0.024
		5	0.423	0.5	0.0	0.077
		10	0.380	0.5	0.0	0.120

Bayesian priors backing the CW-T ambiguity averse solution. Parameters: $e = 0.5$, $p_{CW-T} \approx 1$ and $p_m \approx 0$ for all other models.

F.5 Insurance Premium: Plots of Minimax Losses

Figure 7: Minimax Losses vs. Bayesian Losses

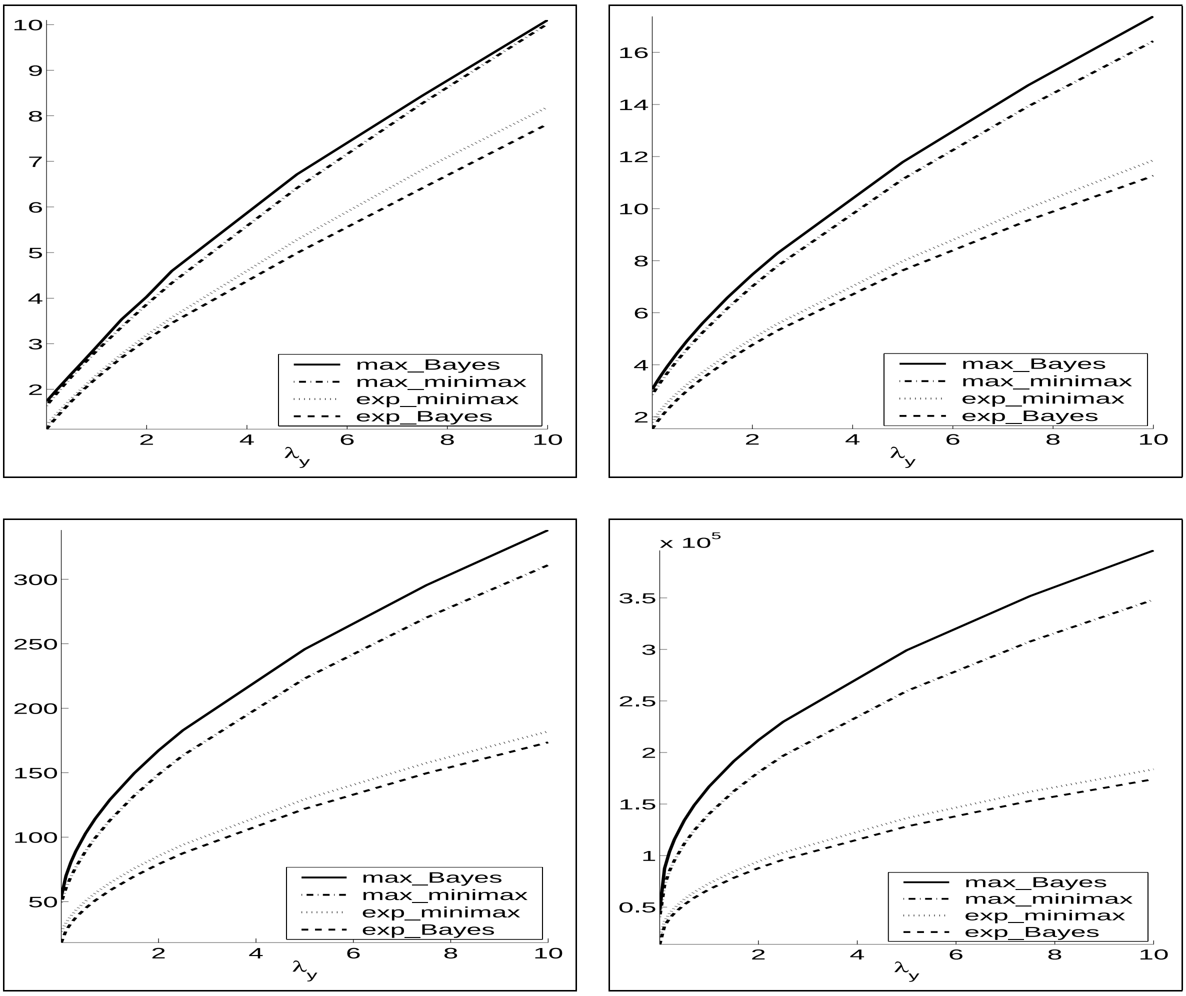

Figure 7: For $\lambda_{\Delta r} = 0.5$, ξ=1 (top left), ξ=2 (top right), $\xi = 5$ (bottom left), $\xi = 10$ (bottom right), flat priors and varying λ_y the panel plots expected losses and maximum losses for the Bayesian and minimax strategy.