

# Country Solidarity, Private Sector Involvement and the Contagion of Sovereign Crises\*

Jean Tirole<sup>†</sup>

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## Abstract

To analyze a sovereign's foreign liabilities and bailouts, this paper develops a new framework and distinguishes between "spontaneous solidarity", an ex-post avoidance of collateral damages inflicted by a distressed country's default, and "contractual solidarity".

Joint liability increases borrowing capability and the risk of contagion. When countries differ substantially in their probability of distress, the optimal pact involves a debt brake but no joint liability. In a more symmetrical context, joint liability is optimal provided that country shocks are sufficiently independent and spillovers costs sufficiently large. Spillovers, when endogenized, are larger under mutual insurance than under one-way insurance.

*Keywords:* Sovereign debt, joint liability, debt brakes, bailouts, contagion, private sector involvement.

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<sup>†</sup>Toulouse School of Economics (TSE) and Institute for Advanced Study in Toulouse (IAST); jean.tirole@tse-fr.eu.

# 1 Introduction

The ongoing Eurozone crisis has reignited the old debate on burden sharing between the official and private sectors when restoring troubled countries' access to the capital market. A further twist in the Eurozone case is the vivid controversy on country solidarity: Should Eurozone countries informally stand by to secure their peers' access to borrowing, as was widely anticipated by markets until Greece's debt was restructured in July 2011? Should Europeans more formally issue Eurobonds, with full joint-and-several liability, and/or create a banking union, and combine such solidarity with a market mechanism for complementary financing? Variants of Eurobonds have been advocated by most leading European politicians, multi-lateral organizations (e.g., the IMF), the media (e.g., *The Economist*), and in several economists' proposals that have attracted wide attention in policy circles.<sup>1</sup> And a banking union is in the making, although its contours are still fuzzy. Assessing the relative merits of such policies requires addressing two key issues:

*Solidarity area:* The policy debate, negotiations and current bailout policies all take it for granted that, just as it fell to the US to rescue Mexico in 1995, Eurozone countries are the natural providers of insurance to each other; even non-Eurozone European countries are exempted from contributing to bailouts.<sup>2</sup> This assumption is at first sight puzzling. After all, insurance economics points at the desirability of spreading risk broadly, rather than allocating it to a small group of countries, which moreover may well face correlated risk.<sup>3</sup> Indeed, alternative cross-insurance mechanisms, such as the IMF's Flexible Credit Line, the Chiang Mai Initiative, or credit lines offered to countries by consortia of banks, already exist, that do not involve insurance among countries within a monetary zone.

*Market vs. official sector borrowing:* While the solidarity area conundrum refers to the allocation of risk within the official sector, another topic for investigation relates to

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<sup>1</sup>See in particular Delpla-von Weizsacker (2010), Euro-nomics group (2011), and Hellwig-Philippon (2011). Related proposals include the European Commission's green paper on "stability bonds" (2011), the Tremonti-Juncker proposal (2010), and the German Council of Economic Experts' "European Redemption Pact" (2011). See Claessens et al (2012) for an extensive overview and discussion of the various proposals.

Most of these proposals advocate coupling Eurobonds with a debt brake mechanism. For example, Olivier Blanchard, IMF's chief economist, argues in the *Financial Times Deutschland* (April 23, 2012) that: "When there was no fiscal treaty nor budgetary discipline instruments, the Germans had good reason to reject bearing the brunt of irresponsible policies by other states. But now we have a fiscal treaty. The Germans should accept that the Eurozone is going by way of Eurobonds." The European Financial Stability Facility created in 2010 already can issue bonds backed by guarantees given by the Euro area member states.

<sup>2</sup>To be certain, the IMF has large programs in the Eurozone; the brunt of the risk however is borne by Eurozone countries.

<sup>3</sup>Philippon (2012) argues that shocks have recently been more asymmetric within the United States than in the Eurozone.

the relative weight of official vs. market borrowing.<sup>4</sup> This allocation of risk between the official and private sectors is particularly relevant for countries that are perceived as risky; a recent case in point is Spain, whose private and public sectors' access to market financing has dwindled and which must rely on the ECB and the European Financial Stability Fund for refinancing.

These observations suggest that an analysis of solidarity must account for both the limited solidarity area (or tax base) and the endogenous allocation of risk between markets and governments. Economic studies of sovereign borrowing link a country's incentive to repay, and therefore its ability to borrow abroad, to either its desire to keep its reputation and access to international financial markets or to the threat of sanctions. While these theories have proved very valuable to study the allocation of risk between a country and its foreign debtors, they make no predictions as to the allocation of risk between private creditors and the official sector; they also do not distinguish among official sector creditors. The purpose of this paper is to start opening the black box of the composition of a sovereign's foreign liabilities.

The idea is to apply the same economic analysis to both the debtor country and its potential guarantors: The latter show solidarity only if it is in their interest to do so. We distinguish between two forms of solidarity: *ex post* (spontaneous and thus involuntary) and *ex ante* (contractual). *Ex post*, the guarantors may stand by the troubled country because they want to avoid the externality or collateral damage inflicted by the latter's default. *Ex ante*, the guarantors may agree on a joint-and-several liability. Spontaneous and contractual bailouts, which correspond roughly to the European approach to date and to the various Eurobonds proposals, are not equivalent. Borrowing capabilities are larger under joint-and-several liability, since a failure to stand by the failing country implies a cost of own default on top of the collateral damage incurred when the failing country defaults. However, joint-and-several liability has redistributive implications; and it further creates a risk of domino effects and increases default costs.

The benchmark model (section 2) involves joint lending by the market and the official sector to a country. We follow the literature by assuming that the country's income realization (or alternatively its liquidity needs realization) is random, and so there are states of nature in which the country cannot or does not want to repay. The twist here is that the official sector is willing to forgive some of its own claim on the country and to reduce private sector involvement (PSI) in order to avoid the collateral damage. Thus, in this "soft budget constraint model", the narrowness of the tax base is rationalized by

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<sup>4</sup>The official sector comprises governments and their agencies, central banks, government controlled institutions and international institutions. For the purpose of an economic analysis, banks that are likely to be bailed out by their government can also be considered as part of the official sector.

the heterogeneity in countries' willingness to stand by the failing country: Countries that have a larger stake in avoiding a country's default are more likely to bail out that country. Consequently, a borrowing country's collateral is provided by the collateral damage its default creates onto peer countries, in short by its nuisance power.

The collateral damage cost admits both economic and political considerations. Economic spillovers include reduced trade, banking exposures and the fear of a run on other countries. The end of the European construction would involve a sizeable political cost; non-Eurozone political costs are evidenced by various countries' access to cash through their nuisance power (collapse of USSR and fear of nuclear weapons proliferation, current assistance to North Korea, US support to Saudi Arabia, Pakistan or Israel) or conversely bailouts motivated by the desire to gain geopolitical influence.<sup>5</sup> Yet another non-economic motivation for bailing out another country is empathy, be it driven by ethnic, religious, vicinity or other considerations.

We first study what happens when the debtor country borrows only from foreign private creditors. Unlike the official sector, individual market investors do not internalize any default cost beyond the monetary loss on their own claim on the country. Unregulated borrowing generates two types of inefficiency, depending on the circumstances: likelihood of default or limited access to the capital market. When the country is particularly eager to borrow or when the probability of a bad income realization is low, the country over-borrows in the private market. The size of private debt may discourage the official sector from coming to the rescue of a distressed country. By contrast, when it is less eager to borrow or when its income is quite risky, the country chooses a low-enough debt level so as to always fall under the umbrella of the official sector's implicit guarantee; it thereby does not maximize its debt capacity.

In general, a Pareto improvement can be obtained through a contract between the country and the official sector. The optimal contract then specifies a cap on private sector borrowing so as to protect the seniority of the official sector's claim on the country.<sup>6</sup> Furthermore, and a central result of our analysis, the optimal contract involves no joint-and-several liability. The intuition is that joint liability allows the debtor country to borrow more by making it more credible that it will be bailed out in case of hardship. But it has no ability to compensate the guarantor for the extra involvement.

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<sup>5</sup>As Roubini (2004) notes, *"Even before the September 11 events, but more so afterwards, the U.S. tendency to support financial aid to countries that are considered as friends, allies or otherwise strategically or systemically important (Turkey, Pakistan, Indonesia, and possibly Brazil) has clearly emerged, more strongly than during the previous administration. Even in the case of Argentina, where IMF support was eventually cut off leading to the sovereign default of this country, political considerations have been dominant: the August 2001 augmented package was pushed for political rather than economic reasons."*

<sup>6</sup>This conclusion is in line with standard models of sovereign borrowing, which predict that countries will spontaneously cap their borrowing so as to make their repayment credible; the borrowing cap is here justified by the co-existence of private and official creditors.

Thus, “asymmetric situations” in which the guarantor is unlikely to enter distress independently of the insuree lead to an implicit form of solidarity, ex post bailouts, but no explicit solidarity.

By contrast, in the “symmetrical environment” studied in section 3, debtor countries have a currency with which to pay for the formal insurance they receive through joint-and-several liability: they can reciprocate by offering guarantors some insurance in a situation in which the fortunes are reversed. We show that joint-and-several liability (cum joint monitoring of countries’ indebtedness) then may emerge as part of the optimal arrangement. More precisely, joint liability (in contrast with currency areas) is optimal provided that country shocks are sufficiently independent and spillovers costs sufficiently large relative to default costs. This contractual solidarity up to a level boosts access to the private capital market, but also enhances the overall instability by generating contagion.

While trade and political disruptions are by and large unavoidable, counterparty risk is in part determined by domestic prudential supervision as well as other mechanisms (such as the ECB’s recent LTRO facility that led to some “running for home”). Section 4 accordingly endogenizes spillovers. Under one-way insurance, the principal generally, although not always, chooses to minimize his exposure to the risky country. By contrast, mutual insurance often leads countries to contractually maximize their cross-exposures.

Section 5 concludes with some alleys for future research.

*Relationship to the literature:* The literature on sovereign defaultable debt<sup>7</sup> has two (complementary) strands. One (e.g., Sachs 1983; Krugman 1985; Eaton et al 1986, Bulow-Rogoff 1989b; Fernandez-Rosenthal 1990) stresses the deterring effect of exogenous default costs, such as trade embargoes, seizure of assets or military interventions. An increase in the cost of default makes the country more prone to repay, but raises the cost of default when the latter occurs due to particularly low resources. Dellas-Niepel (2012) assume that the cost of defaulting is higher when defaulting on the official sector, as the latter can avail itself of sanctions. They thereby obtain an optimal mix of private and official sector borrowing, that delivers the optimal sanction. On the empirical front, Rose (2005) shows that debt renegotiations imply a substantial and long-lasting decline in trade.<sup>8</sup>

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<sup>7</sup>See e.g., chapter 6 of Obstfeld-Rogoff (1996) and Sturzenegger-Zettelmeyer (2007) for reviews of this literature. The following obviously does not do justice to this very rich literature. For example, it leaves aside the large literature on liquidity crises initiated by Calvo (1988).

<sup>8</sup>In Fernandez-Rosenthal (1990), the debtor, when repaying in full, receives a “bonus”, not paid by the creditor, and interpreted as an improved access to international markets. They show that creditors forgive enough of the debt so as to incentivize the debtor to eventually repay in full. Mitchener-Weidenmier (2010) study “supersanctions” (gunboat diplomacy, seizure of railway assets, foreign administration to collect customs and taxes...) during the gold exchange standard period (1870-1913) and find that such

Another line, starting with Eaton-Gersovitz (1981) emphasizes that default tarnishes the country's reputation and limits its future access to international financial markets. On the theory side, Bulow-Rogoff (1989a) argued that reputational concerns may not create access to international finance: a country cannot borrow if it can still save at going rates of interest after default. Some of the subsequent literature revisited Bulow and Rogoff's provocative analysis. Hellwig-Lorenzoni (2009) showed that borrowing is feasible under maintained access to savings if the Bulow-Rogoff assumption that the rate of interest exceeds the rate of growth is relaxed. Cole-P. Kehoe (1995), Eaton (1996) and Kletzer-Wright (2000) stress that commitment is two-sided, as lenders may not comply with the punishment required to maintain discipline. Wright (2002) formalizes banks' tacit collusion to punish a country in default. Cole-P. Kehoe (1998) argue that opportunistic behavior in the financial market may tarnish the sovereign's overall reputation and create a collateral loss in the relationship with third parties (e.g. domestic constituencies). Cole-T. Kehoe (2000) study a country's dynamic debt management in a DSGE reputation model. On the empirical front, Aguiar-Gopinath (2006) show how the presence of trend shocks improves the ability of Eaton-Gersovitz style models to account for actual rate of defaults and other empirical facts for emerging markets. Second, while a number of scholars have documented that defaulting countries recoup unexpectedly quickly access to international capital markets, Cruces-Trebesch (2011) show that large haircuts are associated with high subsequent bond yield spreads and long periods of capital market exclusion.

These papers focus on the allocation of risk between the country and foreign creditors. So does the work of Gennaioli et al (2012) and Mengus (2012), which stresses the role of domestic banking exposures in the sovereign's decision to default.<sup>9</sup> Arteta-Hale (2008), Borensztein-Panizza (2009) and Gennaioli et al (2012) provide empirical evidence on the internal cost of default. Jeske (2006) and Wright (2006) analyze the impact of the allocation of country liabilities between private and public *borrowing*. The innovation in these papers is the introduction of resident default on international borrowing (associated with a lack of enforcement of foreign claims on domestic residents by domestic enforcement institutions), on top of standard default on public debt.

By contrast, this paper takes a shot at analyzing the equilibrium allocation of claims on the sovereign between the private and official sectors as well as the split within the official sector; to this purpose it introduces two features that are traditionally absent in the literature: collateral damage costs and the possibility of cross-insurance among

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sanctions were very effective in resuscitating access to capital markets after default.

<sup>9</sup>This holds even if the sovereign can engage in bailouts of domestic banks, provided that it has incomplete information on the quality of balance sheets: see Mengus (2012). Models of moral hazard (e.g., Tirole 2003) often stress the benefits of a home bias in savings on the government's incentive to behave.

countries.<sup>10</sup>

Corsetti et al (2006) develop a model of mixed private-public financing, in which international institutions serve as a lender of last resort and prevent self-fulfilling liquidity runs. They emphasize the role of the precision of the international institution's information, and show that official lending may not increase moral hazard. Persson-Tabellini (1996) studies cross-country fiscal externalities when political institutions are not integrated but (a varying degree of) fiscal integration is in place. Bolton-Jeanne (2011) shows how monetary integration may create a premium on a healthy country's debt through the collateral demand by banks in weaker ones, and that joint liability destroys this premium. Our paper has a different focus relative to these papers, such as the conditions of emergence of joint liability, PSI and contagion.

Bulow-Rogoff (1988) builds an infinite-horizon framework of a recurrent debt renegotiation among three players: the debtor country, creditor banks, and consumers in creditor countries, who benefit from the debtor country's exports and therefore are willing to contribute in order to avoid the debtor country's default and concomitant trade sanctions. The anticipation of future side-payments by consumers implies that bank lenders (the "market" in my model) are willing to lend more, which benefits the borrowing country. Bulow-Rogoff (1988)'s interesting analysis of repeated negotiation and private sector involvement (trade beneficiaries rather than investors in their model) does not address some of the main themes of this paper such as debt brakes, joint liability and contagion.

Finally, the paper offers some similarities with the literature on "cross-pledging": cross pledging of the revenues in several activities by a single agent (Diamond, 1984) and among agents (literature on group lending and microfinance<sup>11</sup>). It has been shown in the latter literature that group lending can increase entrepreneurs' access to capital either by mobilizing social capital or by inducing mutual monitoring. Relative to this literature, the paper adds bailouts (the group lending literature assumes that joint liability is the only vector of solidarity) and the requirement that the exercise of even contractual solidarity must respect the guarantor's willingness to pay constraint.

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<sup>10</sup>In the banking context, Rochet-Tirole (1996) derives optimal cross-exposures as the outcome of a trade-off between the incentive to monitor and the risk of contagion.

<sup>11</sup>See, e.g., Tirole (2006, section 4.6) for a review of that literature's main themes, as well as Tirole (2010) for a recent contribution to the economics of extended liability.

## 2 Collateral damage creates collateral

### 2.1 Baseline model

This section develops a model of joint lending by the private sector ( $M$ , the market) and the official sector ( $P$ , the principal) to a country ( $A$ , the agent). All parties are risk neutral. The official sector can be thought of as a deep pocket country or the international community. The private financial market is competitive. Both the agent and the principal honor their obligations if and only if they find it privately optimal to do so. In this “willingness-to-pay” model of sovereign borrowing, the agent’s incentive to repay is provided by the cost  $\Phi_A \geq 0$  that it incurs in case of default.

For the principal to be willing to rescue the agent, we assume that the principal incurs cost  $\phi_P$  whenever the agent defaults. Importantly, we distinguish between the “collateral damage cost” or “externality cost”  $\phi_P \geq 0$  incurred by the principal when  $A$  defaults and the larger cost  $\phi_P + \Phi_P > \phi_P$  born by the principal when it also defaults on its obligations.<sup>12</sup> Because the principal does not borrow in the basic model, it can only default by accepting to be jointly liable for the agent’s debt and by not honoring the associated commitment.

The default costs  $\Phi_A$  and  $\Phi_P$  admit a wide range of interpretations. Default costs may be associated with interruptions in the trade patterns, denial of trade credit, seizure of assets or other retaliatory moves, damages that default imposes on the domestic sector,<sup>13</sup> alliance shifts, FDI interruptions and so forth. As discussed in the introduction, the collateral damage cost  $\phi_P$  arises from the economic linkages studied in the contagion literature (reduced trade, impact of a foreign default on domestic banks, ...) and from political costs.

This modeling is a succinct way of capturing the idea that countries may want to prevent other countries’ default because of the concomitant collateral damage. We assume that the collateral damage is smaller than the country’s own damage:  $\Phi_A > \phi_P$ .

There are two periods,  $t = 1, 2$ . To simplify notation, there is no discounting between the two periods. The timing is described as in Figure 1.

*Date 1: borrowing.* At date 1, the agent borrows  $b = b_M + b_P$  from the market and the principal, respectively, and values this borrowing at  $Rb$ . The parameter  $R$  measures the intensity of the agent’s liquidity needs: current consumption needs or quality of his investment opportunities. The borrowing contract specifies debts  $d_M$  and  $d_P$  to be repaid

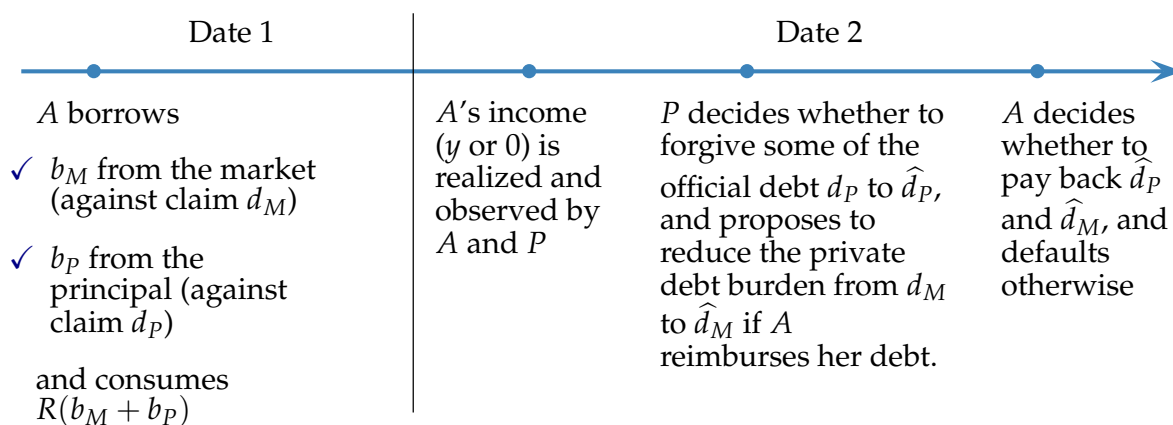
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<sup>12</sup>In this basic model,  $\Phi_A$  is the agent’s default cost when only the agent defaults. If the principal also defaulted (as will be the case in Section 3), the cost incurred by the agent might be higher; however multiple defaults do not occur in equilibrium in the basic model and so we do not introduce additional notation for the moment (the only result which might be affected by the existence of a  $\phi_A > 0$  is Proposition 2. But this proposition is then either unchanged or qualitatively the same).

Also, we assume that the agent bears no cost when he is bailed out. The agent could incur some (reputation) cost without this altering the basic insights.

<sup>13</sup>As in Gennaioli et al (2012) and Mengus (2012) for instance.





**Figure 1: Timing**

at date 2 to private investors and to the principal, respectively. The amount of borrowing is observable.  $Rb$  is to be interpreted as a private benefit for the agent; when  $R$  stands for the value of investment opportunities, one must be careful to distinguish for comparative statics purposes investments in non-tradables (which are indeed private benefits) and investments in tradables (that are likely to raise date-2 income available for repayment).

*Date 2: income realization.* At date 2, the agent receives a random income, equal to  $y$  with probability  $\alpha$  (good state of nature,  $G$ ) and  $0$  with probability  $1 - \alpha$  (bad state of nature,  $B$ ), where  $y > \Phi_A$ . Income “0” is to be interpreted more generally as some incompressible, minimum level of consumption below which the agent is not disposed to go. As is customary in willingness-to-pay<sup>14</sup> and insurance<sup>15</sup> models, the financial market does not observe the realization; equivalently, the market is uncertain as to whether  $A$  is willing to make sacrifices to reimburse the debt (i.e., as to the level of the incompressible level of consumption). The agent and the principal do observe the realization, although the assumption that the principal observes the realization only serves to simplify expressions: Appendix 5 shows that qualitatively similar insights hold when only the agent observes the income shock. We assume that the principal and the agent form a coalition at date 2 when deciding whether to reimburse the agent’s debt.<sup>16</sup>

<sup>14</sup>As well as in a number of standard corporate finance models (e.g., Bolton-Scharfstein 1990; Gale-Hellwig 1985; Townsend 1979).

<sup>15</sup>In the tradition of Holmström (1979). Were the state of nature verifiable, then contingent debt contracts could be written, that deliver a higher utility to the agent. The latter would then be tempted to renege in the *good* state of nature, as optimal insurance would call for debt forgiveness in the bad state and a high repayment in the good state. See Grossman-van Huyck (1988) for the view that if states of nature are verifiable, the sovereign’s ability to default partially or fully can, under some conditions, mimic an optimal state-contingent debt contract. Trebesch (2009) finds that domestic firms suffer more in their access to credit when the government has employed coercive actions instead of good faith debt renegotiations.

<sup>16</sup>This rules out Nash implementation schemes in this model, so the contract studied in Section 2.3 will indeed be optimal.

*Debt forgiveness and bailout.* Following  $A$ 's income realization, the principal can forgive some of its debt and bring it down to  $\hat{d}_P \leq d_P$ . Similarly, the principal can offer to bring conditional support  $d_M - \hat{d}_M$  provided that the agent reimburses the private investors.<sup>17</sup> The remaining private debt burden on  $A$  is then  $\hat{d}_M$  and the agent's effective debt burden is  $\hat{d} = \hat{d}_P + \hat{d}_M$ .

*Repayment decision.* Finally, the agent decides whether to repay its renegotiated liabilities  $\hat{d}_P$  and  $\hat{d}_M$  or to default.

*No-principal benchmark.*

Suppose that there is no principal. Equivalently, as will be shown later, the principal incurs no spillover cost ( $\phi_P = 0$ ). In this case, which is a special case of sections 2.2 and 2.3, the agent can borrow  $b_M = \alpha\Phi_A$  from the market, and reimburse  $d_M = \Phi_A$ , the highest credible reimbursement, in the good state, at the cost of default in the bad state. The agent then receives utility

$$\begin{aligned} U_A &= R(\alpha\Phi_A) + \alpha(y - \Phi_A) - (1 - \alpha)\Phi_A \\ &= (\alpha R - 1)\Phi_A + \alpha y. \end{aligned}$$

The agent alternatively can refrain from borrowing ( $b_M = 0$ ) and receive utility  $\alpha y$  (the linearity of the objective function implies that we can focus on these two alternatives). Thus the agent borrows from the market if and only if

$$\alpha R \geq 1.$$

*Debt forgiveness and bailouts.*

Let us return to the case of interest ( $\phi_P > 0$ ). In the last stage the agent reimburses its debts if and only if

$$\hat{d}_P + \hat{d}_M \leq \Phi_A,$$

and defaults otherwise.

In the state of nature in which the *agent receives no income*, the principal without loss of generality forgives official debt:  $\hat{d}_P = 0$ . The question is whether the principal is willing to foot the bill for the debt owed to the private sector. The principal bails out the agent if and only if

$$d_M \leq \phi_P$$

since the absence of agent income requires the principal to foot the bill for the entire private debt  $d_M$ . By contrast, when the private debt exceeds the collateral damage, the

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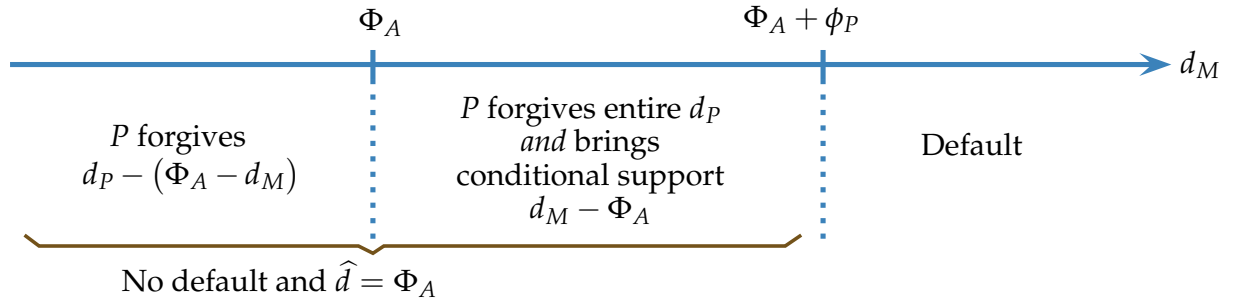
<sup>17</sup>Another form of conditionality (along the lines of the IMF programs) could be easily added to this framework: The principal could condition its support on the agent's undertaking some costly action that will generate some pledgeable income at some later date 3.

principal no longer stands by the agent.

In the state of nature in which the *agent receives income  $y$*  and  $d \leq \Phi_A$ , the principal knows that debt will be repaid and therefore does not intervene. Consider thus the case in which the agent defaults when the principal remains passive:  $d > \Phi_A$ .

- (a) If  $d_M \leq \Phi_A$ , then the principal just forgives  $d - \Phi_A$ . There is no default.
- (b) If  $\Phi_A < d_M \leq \Phi_A + \phi_P$ , the principal forgives the entire official debt and further offers conditional support  $d_M - \Phi_A$  so as to prevent default. The principal's cost is, as in case (a),  $d - \Phi_A$ .
- (c) Finally, if  $d_M > \Phi_A + \phi_P$ , then leaving aside debt forgiveness, the support,  $d_M - \Phi_A$ , needed to rescue the agent exceeds  $\phi_P$  and the principal prefers to incur the collateral damage from the agent's default. There is no bail out.

Figure 2 summarizes the outcome in the good state.



**Figure 2: Repayment and default behavior in the good state when  $d > \Phi_A$**

**Lemma 1 (repayment and default).**

- (i) In the bad state, the principal bails out the agent if  $d_M \leq \phi_P$  and lets the agent default otherwise.
- (ii) In the good state, the agent defaults if and only if the private debt exceeds  $\Phi_A + \phi_P$ . If  $d_M \leq \Phi_A + \phi_P$ , the principal prevents default by forgiving debt and possibly by bringing further support to repay private debt.

*Remark:* The extensive form depicted in Figure 1 creates a “soft budget constraint”, as the principal forgives its own claim and further bails out the agent if the latter’s market liability is not too large. An alternative extensive form would have the agent announce at date 2 its intended debt repayments, and then give the principal an opportunity to forgive its claim and reimburse the private investors’ shortfall, before default is pronounced. This would put the principal in an even weaker position. The qualitative results would be quite similar to those obtained below.

## 2.2 Borrowing from the private sector only (laissez faire)

Let us now investigate the agent's date-1 borrowing behavior, starting with the case in which  $A$  does not enter an agreement with  $P$  (and so  $b_P = d_P = 0$ ). Given the principal's behavior at date 2, the agent's optimal indebtedness is either  $d_M = \Phi_A + \phi_P$ , so as to benefit from the maximal bailout in the good state of nature, or  $d_M = \phi_P$ , so as to benefit from the maximal bailout in the bad state of nature. That is, the agent chooses between a risky, high-debt policy and a safe, low-debt one. The risky policy allows  $A$  to borrow  $b_M = \alpha(\Phi_A + \phi_P)$  but leads to a default in the bad state, while the safe policy raises  $b_M = \phi_P$  and generates no default. The agent's utility is

$$U_A^* = \max \left\{ R\alpha(\Phi_A + \phi_P) + \alpha(y - \Phi_A) - (1 - \alpha)\Phi_A; R\phi_P + \alpha(y - \phi_P) \right\} \quad (1)$$

We assume that

$$R \geq R_0 \equiv \min \left\{ \Phi_A / \alpha(\Phi_A + \phi_P), \alpha \right\},$$

so that the agent prefers borrowing to not borrowing, which yields utility  $\alpha y$ . Relative to this benchmark utility  $\alpha y$ , safe borrowing yields  $R\phi_P$  at date 1 and leads to an effective debt burden for the agent  $\phi_P$  with probability  $\alpha$ ; so the agent prefers safe borrowing to no borrowing if  $R \geq \alpha$ . Risky borrowing implies a sure cost  $\Phi_A$  (either in reimbursement or in default cost); but it allows immediate consumption  $R\alpha(\Phi_A + \phi_P)$ . Hence, if  $R\alpha(\Phi_A + \phi_P) \geq \Phi_A$ , the agent prefers risky borrowing to no borrowing. Note that  $R_0 < 1$ ; the agent may select a negative NPV borrowing strategy so as to benefit from the soft-budget constraint.

The agent chooses the risky policy if and only if

$$R[\alpha\Phi_A - (1 - \alpha)\phi_P] \geq \Phi_A - \alpha\phi_P. \quad (2)$$

In general, a higher nominal debt  $d_M$  may make it less credible that it will be reimbursed; and so an increase in liabilities may not bring in more money  $b_M$  for the agent. Indeed, the high-debt policy is also a high-borrowing one if and only if

$$\alpha\Phi_A > (1 - \alpha)\phi_P \quad (3)$$

If condition (3) is violated, then the safe policy both raises more income and generate less default cost, and so is always picked. If condition (3) is satisfied, then the high-debt policy is chosen whenever

$$R \geq \frac{\Phi_A - \alpha\phi_P}{\alpha\Phi_A - (1 - \alpha)\phi_P} \equiv R^* \quad (4)$$

We will use the convention that  $R^* = +\infty$  if (3) is violated. Note also that in the no-principal benchmark ( $\phi_P = 0$ ),  $R^*$  is indeed equal to  $1/\alpha$ . The principal's welfare is:

$$U_P^* = \begin{cases} -\phi_P & \text{if } R \geq R^* \\ -(1-\alpha)\phi_P & \text{if } R < R^* \end{cases} \quad (5)$$

The principal is always hurt in the bad state: either  $P$  shows solidarity with  $A$  and bails  $A$  out at level  $\phi_P$ , or  $P$  lets  $A$  default and then also incurs cost  $\phi_P$ . The principal is also hurt in the good state of nature under the risky, high-debt policy as  $\phi_P$  must be contributed by  $P$  to prevent default.

**Proposition 1 (laissez faire).** *Assume that  $R \geq R_0$ . When the agent borrows from the market and does not contract with the principal, the agent's optimal strategy is either a high-debt policy (borrowing  $\alpha(\Phi_A + \phi_P)$  and defaulting in the bad state) or a low-debt one (borrowing  $\phi_P$  and never defaulting, thanks to the principal's rescue in the bad state). The high-debt policy leads to a default in the bad state of nature and to a bailout in the good state of nature.*

- (i) *The agent picks the high-debt policy if  $R \geq R^*$ .*
- (ii) *The high-debt policy is more likely, the greater the probability of a good state.*

### 2.3 Optimal contract with the official sector

Suppose now that prior to borrowing at date 1 the agent makes a take-it-or-leave-it contractual offer to the principal. If the principal turns down the offer, the outcome is the unregulated one studied in the previous section; in particular, the principal's utility is given by (5). While the theory is easily generalized to more even distributions of bargaining power at date 1, giving no bargaining power to the principal is particularly interesting because it gives the best chance to joint-liability demands by the agent.

We adopt a mechanism design approach. The agent's contract offer to the principal specifies:

- ✓ A borrowing level  $b$  and its allocation between the market and the principal:  
 $b = b_M + b_P$ .
- ✓ A state-contingent reimbursement and its allocation. For  $\omega \in \{G, B\}$ , the agent *effectively* pays back  $d^\omega = d_M^\omega + d_P^\omega$ .<sup>18</sup>

Intuitively, the laissez-faire outcome can be improved upon regardless of the laissez-faire borrowing strategy. First, the risky borrowing strategy generates inefficient default and a collateral damage cost for the principal. The agent should arrange financing

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<sup>18</sup> $d_P^\omega$  can be negative (bailout). Note also that this notation refers to the *actual* repayments and does not imply that state-contingent debt can be issued.

so as to avoid default and in exchange receive a favorable treatment from the principal. Second, the safe borrowing strategy substantially constrains the agent's borrowing; the agent can credibly commit to reimburse  $\Phi_A$  in the good state, but, when the safe strategy is optimal, does not want to take on this commitment by fear of defaulting in the bad state. Transferring at least part of the liability from the market to the principal creates more flexibility in the level of repayment and allows the agent to borrow more without risking default. Proposition 2 establishes the validity of these intuitions.

**Proposition 2 (optimal contract).** *When the agent contracts with the principal at date 1 and  $R \geq 1$ ,*

(i) *an upper bound on the agent's utility is*

$$\widehat{U}_A = R(\alpha\Phi_A - U_P^*) + \alpha(y - \Phi_A);$$

(ii) *this upper bound is reached through the following mix of public and private financing:*

- ✓ *the agent borrows  $b_M = d_M^G = d_M^B = \phi_P$  from the market; the principal monitors this cap on market financing (debt brake) and spontaneously bails out the agent in the bad state of nature;*
- ✓ *the agent borrows  $b_P = \alpha\Phi_A - \phi_P - U_P^*$  from the principal, repays the principal  $d_P^G = \Phi_A - \phi_P$  in the good state of nature, and receives bailout money  $-d_P^B = \phi_P$  in the bad state of nature from the principal to repay its private creditors.*

*The agent never defaults.*

*Proof:*

(i) Consider the following program, consisting in maximizing the agent's default-free utility subject to incentive and participation constraints:

$$\max \left\{ U_A = Rb + \alpha(y - d^G) + (1 - \alpha)(-d^B) \right\}, \quad (\text{I})$$

where

$$b = b_M + b_P,$$

the participation constraints are satisfied:

$$\begin{aligned} -b_P + \alpha d_P^G + (1 - \alpha)d_P^B &\geq U_P^* \\ -b_M + \alpha d_M^G + (1 - \alpha)d_M^B &\geq 0, \end{aligned}$$

and the incentive constraints are satisfied:

$$\begin{aligned} d^G &\leq \Phi_A \\ d^B &\leq 0 \\ -d_P^\omega &\leq \phi_P + \Phi_P \quad \text{for } \omega \in \{G, B\}. \end{aligned}$$

The first two incentive constraints reflect the fact that the agent's repayment is constrained by its willingness to pay. The latter constraints represent the incentive constraints for the principal. The worst that possibly can happen to the principal if he refuses to pay anything is that both countries default, and so a lower bound on the principal's ex-post utility  $-(\phi_P + \Phi_P)$  (at this mechanism design stage, the method of implementation is irrelevant, but the reader may have in mind joint liability here). Adding up the participation constraints and replacing in  $U_A$  yields

$$U_A \leq R[\alpha d^G + (1 - \alpha)d^B - U_P^*] + \alpha(y - d^G) + (1 - \alpha)(-d^B).$$

The upper bound  $\hat{U}_A$  is reached when  $d^G$  and  $d^B$  take their highest values,  $\Phi_A$  and 0, respectively.

(ii) Computations are straightforward.

Two points are worth noticing, though. First, in the proposed implementation of the upper bound,  $d_P^G$  is positive. By contrast,  $-d_P^B = \phi_P$ . Partial or full joint-and-several liability (which would relax the potential commitment of the principal up to  $\Phi_P + \phi_P > \phi_P$ ) is not used and the principal spontaneously (that is, in the absence of contractual commitment) contributes  $\phi_P$  in the bad state of nature to rescue the agent.

Second, a control over private borrowing is in general required. Otherwise, the agent might well overborrow, preventing the optimum from being reached. To see this, suppose that the principal does not monitor that the agent borrows no more than the cap. Any increase in private debt leads to default in the bad state. So, conditional on increasing  $d_M$  beyond  $\phi_P$ , the agent might as well borrow as much as is consistent with the absence of default in the good state. So suppose that the agent issues an extra claim  $\Phi_A$  on the market, bringing total private debt to  $d_M = \Phi_A + \phi_P$ . In the good state, the principal forgives his own debt and further brings support  $\phi_P$  to enable the reimbursement of private debt. The agent thereby collects  $b_M = \alpha\Phi_A$  from private creditors since the extra borrowing generates default in the bad state. This strategy delivers utility

$$\begin{aligned} U_A &= R[2\alpha\Phi_A - \phi_P - U_P^*] + \alpha(y - \Phi_A) - (1 - \alpha)\Phi_A \\ &= \hat{U}_A + R[\alpha\Phi_A - \phi_P] - (1 - \alpha)\Phi_A. \end{aligned}$$

The term  $R[\alpha\Phi_A - \phi_P]$  represents the benefit from overborrowing in the market, while  $(1 - \alpha)\Phi_A$  corresponds to the expected cost of default associated with this strategy. And so, for  $\alpha$  large enough, the agent overborrows from the market.

Making official sector debt senior (Hellwig-Philippon 2011; Delpla-von Weizsacker 2010) is not sufficient to prevent overborrowing: In the absence of an explicit constraint, the agent may try to activate a bailout in the good state of nature.<sup>19</sup>

■

Finally, let us discuss the implementation of the optimal contract. First, because the principal's incentive constraints are not binding, the implementation of the optimal allocation developed in Proposition 2(ii) does not require an explicit joint-and-several liability. In the bad state of nature,  $P$  bails out  $A$ , but  $P$  need not be in default if he does not do so. Put differently,  $A$ 's unpaid debt does not necessarily become  $P$ 's debt. By contrast, we will see that joint-and-several liability can be strictly Pareto improving in the context of mutual insurance.

**Corollary 1 (no need for joint-and-several liability).** *While laissez-faire is dominated by a contractual relationship between the principal and the agent, the optimal contract can be implemented without the principal's being held legally liable for the agent's liabilities.*

Second, the implementation of the optimal allocation in Proposition 2(ii) involves mixed financing by the market and the official sector. Could the market be short-circuited and the entire loan be provided by the official sector (together with a debt ceiling) in an exclusive contract? The answer is often "no", even if we ignore the possibility that the "official sector" may have limited cash itself (this will indeed be the case in the next section, in which countries are both borrowers that co-insure each other). Consider the possibility of *borrowing exclusively from the official sector*. Under liability  $d_P = \Phi_A$ , the principal forgives the debt in the bad state. If, as we have assumed until now, the principal is committed to force default in the absence of repayment, the agent pays back  $\Phi_A$  in the good state, and so the optimum can be implemented through lending only by the official sector.

This conclusion, however, is not robust to our strong commitment assumption. To see this, note that the principal's policy of putting the agent in default when the agent does not repay is not time consistent, as the principal incurs cost  $\phi_P$  of doing so. Repayment is jeopardized by official sector's exclusivity in lending. *The threat of putting the country in default is less credible for the official sector* (which incurs cost  $\phi_P$  in case of default) than for the market (as a private investor bears no direct spillover externality from the country's default). This may jeopardize repayment, as shown by the

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<sup>19</sup>This argument is a variant of the classic dilution problem (e.g., Bizer-de Marzo, 1992; Segal 1999), but with a twist: Overborrowing is here motivated by the desire to trigger an uncontracted-for bailout.



following example. Suppose that when the agent refuses to pay back  $\hat{d}_p$ , the principal enforced default sanctions only with probability  $z < 1$  (instead of  $z = 1$  in the model). The mixed-financing implementation of Proposition 2 still operates as long as  $(1 - z)\Phi_A \leq \phi_P$ , that is as long as the attempt to pay only  $\phi_P$  to private creditors proves too costly to the agent. By contrast, with pure public financing, the agent never repays the principal as long as  $z < 1$ .

**Corollary 2 (pure official-sector financing).** *Pure official-sector financing cannot implement the optimum unless the principal's probability of enforcing default when its claim is not repaid is exactly 1.*

## 2.4 Discussion and some simple extensions

*Spreads.* Unsurprisingly a spread on the agent's sovereign debt appears in the absence of contractual agreement when the pressing liquidity needs (a high  $R$ ) induces the agent to opt for the risky strategy. Because in this model there is no shortage of stores of value, the agent's choice and/or the institutional arrangement has no impact on the principal's borrowing conditions: there is just no spread there. By contrast, if there were a shortage of safe financial instruments in the principal's economy, safe instruments' premium would increase due to a flight to quality, as in Bolton-Jeanne (2011).

*Eurobonds.* As mentioned in the introduction, a number of recent policy proposals by economists, think-tanks and politicians have proposed introducing limited solidarity through a two-tier borrowing structure: blue bonds, for which the Eurozone would be jointly liable, and red bonds, for which no such solidarity would operate.<sup>20</sup> Blue bond issues would be capped at a fraction of GDP (say 60 %). These proposals all insist on a number of features: budgetary supervision (a policy that in our model would be akin to controlling moral hazard on the choice of  $\alpha$ , as we discuss shortly), joint liability on the blue bonds, no bail-out clause on the red bonds, and seniority of blue bonds over red bonds.

While we noted that joint liability is not required in order to implement the optimal contract, we may wonder whether a Eurobond-style arrangement could not achieve the same outcome. A first observation is that the no bail-out clause for red bonds together with the absence of default in the optimal contract imply in our model that no red bonds should be issued. Thus it must be the case that the agent issues only blue bonds; so all issuing is in Eurobonds. Could Eurobonds achieve by themselves the op-

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<sup>20</sup>The particular terminology is due to Delpla-von Weizsacker (2010). See also the closely related Eurobill proposal of Hellwig-Philippon (2011).

timal outcome? Let  $d^B$  denote the amount of Eurobonds. Because they are guaranteed, the agent can borrow  $b = d^B$ . The following corollary is proved in Appendix 1:

**Corollary 3 (Eurobonds and more general joint-liability arrangements).**

(i) *The optimal contract (which from Corollary 1 can always be implemented without joint liability) is also implementable through a system of Eurobonds if and only if three conditions are all satisfied:  $R \geq R^*$ ,  $(1 - \alpha)\Phi_A \leq \phi_P$  and  $\alpha\Phi_A \leq \Phi_P$ .*

(ii) *Allowing a date-1 transfer from the agent to the principal, the optimum can be implemented through pure private-sector debt cum joint liability if and only if  $R \geq R^*$  and  $\max \{ \Phi_A, \alpha\Phi_A + \phi_P \} \leq \Phi_P + \phi_P$ .*

Intuitively, the principal is willing to accept Eurobonds only if it is in a very weak bargaining position. This is indeed the case if  $\phi_P$  is large and if  $R \geq R^*$ , so that the agent's threat of overleveraging is credible. It must also be the case that the principal's cost of default  $\Phi_P$  not be too small for the guarantee to be credible.

*Ex-post moral hazard.* The model is easily generalized to accommodate ex-post moral hazard. Suppose that after borrowing has taken place, the agent chooses the probability  $\alpha$  of a good state at cost  $g(\alpha)$  where  $g$  is increasing and convex. This moral hazard adds one constraint to Program (I): for instance, if default in the bad state is suboptimal (which no longer is a foregone conclusion):

$$g'(\alpha) = y - d^G + d^B.$$

The optimal allocation still involves no transfer in the bad state:  $d^B = 0$ , but the repayment constraint  $d^G \leq \Phi_A$  may no longer be binding so as to provide the agent with stronger incentives to avoid distress. Put differently, ex-post moral hazard unsurprisingly may well call for a tighter debt brake.

*Ex-ante moral hazard.* Suppose now that, "at date 0", i.e., before borrowing, the agent incurs effort cost  $\psi(e)$  in order to generate date-1 income  $e$  (to which borrowing will be added to yield date-1 consumption). Suppose further that the date-1 utility from consumption  $R(c_1)$  is concave in consumption  $c_1$  rather than linear. Straightforward computations yield the following condition for the risky borrowing strategy to be optimal under *laissez-faire*:

$$R(\alpha(\Phi_A + \phi_P) + e) - R(\phi_P + e) \geq \Phi_A - \alpha\phi_P. \quad (2')$$

An increase in  $e$  makes it *less* likely that the agent chooses the risky borrowing strategy.<sup>21</sup> Thus, the agent may want to choose a low  $e$  at date 0 in order to make it

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<sup>21</sup>Note that  $\alpha(\Phi_A + \phi_P) > \phi_P$  is a necessary condition for (2') to hold.

more credible that it will choose the risky borrowing strategy and (from Proposition 2) thereby extract better terms from the principal.

### 3 Contractual solidarity

#### 3.1 Modeling and laissez-faire

Consider now the two-country symmetric version of the model of Section 2. Both countries borrow at date 1. Country  $i$  values cash  $b_i$  available at date 1 at  $Rb_i$  where  $R > 1$ . At date 2, each country either has income  $y$  (is “intact” or “healthy”) or has no income (is “troubled”). The probability that  $k$  countries have income  $y$  is  $p_k$  (with  $\sum_{k=0}^2 p_k = 1$ ). By keeping these probabilities general, we allow arbitrary patterns of correlation between income shocks. Let  $\alpha \equiv p_2 + (p_1/2)$  denote the unconditional probability of being intact.

As earlier, we distinguish between a country’s own cost of defaulting,  $\Phi$ , and the (smaller) collateral damage this default imposes on the other country,  $\phi$  (where  $\phi < \Phi$ ).  $y$  is assumed large enough ( $y > \Phi + \phi$ ), so that a country’s willingness to pay rather than ability to pay is binding. Let

$$r \equiv \frac{\phi}{\Phi}.$$

The spillover-default cost ratio  $r$  reflects the scope for extortion under laissez-faire, and also measures the potential for increase in a country’s pledgeable income under ex-ante contracting.

Unlike in the asymmetric case studied in Section 2, in which the study of laissez-faire is crucial for the determination of the principal’s welfare in the status-quo of the negotiation and therefore for the features of the optimal contract, we do not here need to compute the (symmetric) laissez-faire outcome, as its exact value does not influence the optimal (symmetric) contract. For the sake of completeness, though, Appendix 5 computes the laissez-faire equilibrium.

**Proposition 3 (laissez-faire).** *The laissez-faire equilibrium is generically unique. Letting  $d_M^i$  denote country  $i$ ’s debt liability vis-à-vis the private capital market, it involves:*

- (i) no borrowing ( $d_M^i = 0$ ) if  $[R(1 - p_0) - \alpha]r \leq p_0$ ,  $R\alpha \leq 1$  and  $Rp_2r \leq 1 - Rp_2$ ;
- (ii) low debt ( $d_M^i = \phi$ ) if  $[R(1 - p_0) - \alpha]r \geq p_0$ ,  $(1 - p_0) - \alpha r \geq R[\alpha - (1 - p_0)r]$  and  $(Rp_1 - \alpha)r \geq (R - 1)p_2 - p_1$ ;
- (iii) medium debt ( $d_M^i = \Phi$ ) if  $R\alpha \geq 1$ ,  $(1 - p_0) - \alpha r \leq R[\alpha - (1 - p_0)r]$  and  $p_2(1 + r) \leq \alpha$ ;

(iv) *high debt* ( $d_M^i = \Phi + \phi$ ) if  $Rp_2r \geq 1 - Rp_2$ ,  $p_2(1+r) \geq \alpha$  and  $(Rp_1 - \alpha)r \leq (R-1)p_2 - p_1$ .

In the perfect correlation case ( $p_1 = 0$ , or equivalently  $\alpha = p_2$ ), the equilibrium is either no borrowing or high-debt, and the latter prevails iff

$$\alpha R(1+r) \geq 1,$$

that is more often (whenever  $r > 0$ ) than in the “no-principal benchmark” of Section 2.1 (in which borrowing occurred for  $\alpha R \geq 1$ ). And for  $r = 0$  (no spillovers), the equilibrium is either no borrowing ( $\alpha R \leq 1$ ) or medium borrowing ( $\alpha R \geq 1$ ).

### 3.2 Optimal contract

Let us now assume that the two countries agree on the levels of borrowing and on the extent of ex-post solidarity. Because of symmetry, joint liability no longer has re-distributive implications; but it creates a risk of domino effect and thereby increases default costs. Let us investigate the conditions under which joint liability emerges from an optimal pact. Let

$$\widehat{\Phi} \equiv \Phi + \phi$$

denote the total cost of a default. More generally, “hats” will stand for total costs. Let  $\widehat{\Phi}_0$  and  $\widehat{\Phi}_2$  denote the per-country total cost of a default (own default plus spillover) when  $k = 0$  and  $k = 2$ , respectively. For example,  $\widehat{\Phi}_k = 0$  if no country defaults and  $\widehat{\Phi}_k = \widehat{\Phi}$  if both countries default.

When  $k = 1$ , we will distinguish between the pain,  $\widehat{\Phi}_1^y$ , inflicted upon the country that has income  $y$ , and that,  $\widehat{\Phi}_1^0$ , inflicted upon the country with zero income. Let  $x_1 \in [0, 2]$  denote the expected number of defaults when  $k = 1$  (we do not restrict  $x_1$  to  $\{0, 1, 2\}$ , as a priori there could be stochastic defaults). Note that

$$\widehat{\Phi}_1^y + \widehat{\Phi}_1^0 = x_1 \widehat{\Phi}, \quad \text{and} \quad \left| \widehat{\Phi}_1^0 - \widehat{\Phi}_1^y \right| \leq \Phi - \phi. \quad (6)$$

Let  $\widehat{\Phi}_1 \equiv (\widehat{\Phi}_1^0 + \widehat{\Phi}_1^y)/2$  denote the per-country average pain when  $k = 1$ ; and let  $d_k$  denote the expected, per-country reimbursement to private creditors in state of nature  $k$ . Obviously,  $d_0 = 0$ .

We assume as earlier that the state of nature is known to sovereigns (although joint observability of a country’s income realization is not essential) but not observed by markets. The countries form a coalition in their report to the market. As often in mechanism design, the strategy for finding the optimal arrangement will consist in

considering a subconstrained program and checking that its solution can indeed be implemented.

Thus, consider the following program:

$$\max \left\{ R \left[ \sum_{k=0}^2 p_k d_k \right] - \sum_{k=0}^2 p_k (d_k + \widehat{\Phi}_k) \right\} \quad (\text{II})$$

s.t.

$$0 \leq \widehat{\Phi}_k \leq \widehat{\Phi} \quad \text{for all } k \quad (7)$$

$$d_2 + \widehat{\Phi}_2 \leq \widehat{\Phi}_0 \quad (8)$$

$$d_2 + \widehat{\Phi}_2 \leq d_1 + \widehat{\Phi}_1 \quad (9)$$

$$2d_1 + \widehat{\Phi}_1^y \leq \widehat{\Phi} \quad (10)$$

$$\text{either} \quad 2d_1 + \widehat{\Phi}_1^y + \widehat{\Phi}_1^0 \leq 2\widehat{\Phi}_0 \quad (11a)$$

$$\text{or} \quad 2d_1 + \widehat{\Phi}_1^y \leq \widehat{\Phi}_0 \quad (11b)$$

The objective function is the difference between a country's date-1 benefit derived from borrowing  $b = \sum_{k=0}^2 p_k d_k$ , and the date-2 cost, which includes monetary reimbursement and the pain associated with defaults and spillovers. Constraint (7) simply states that the pain inflicted upon a country cannot exceed  $\widehat{\Phi}$ . Constraints (8) and (9) are coalition incentive constraints when both countries have income. Constraint (8) prevents the countries from claiming they have no income (remember that by necessity  $d_0 = 0$ ). Constraint (9) similarly prevents the countries from letting one claim not to have income (and possibly compensating the other, who then has to foot the bill  $2d_1$ ). Constraint (10) says that when  $k = 1$ , the healthy country can always refuse to contribute. Its date-2 utility is then at least  $-\widehat{\Phi}$ , since  $\widehat{\Phi}$  is the worse pain that can be inflicted on the country, while compliance by definition means paying the entire debt  $2d_1$  and incurring default cost  $\widehat{\Phi}_1^y$ .

Constraint (11) can be understood as follows: condition (11a) is equivalent to the absence of a collusion gain for the two countries of declaring that both are distressed when only one actually is. If condition (11a) is violated, and so there is a surplus from colluding and misrepresenting the state of nature, then condition (11b) states that the healthy country must be compensated to participate in the misrepresentation, which is infeasible as the other country has no income.

The analysis of this program can be found in Appendix 5. We here content ourselves with an informal account. Let us assume that borrowing is desirable, which is actually the case if and only if  $R > (1 + p_0)/(1 - p_0) \geq 1$ . Because any borrowing

leads to some default (unless, perhaps,  $p_0 = 0$ ), the return on borrowing must strictly exceed 1 for borrowing to be worthwhile.

Second, assuming that borrowing is desirable, the program's linearity implies a number of simplifications. There is no default cost when both are intact:  $\widehat{\Phi}_2 = 0$ ; conversely, the default costs are maximal when both are distressed:  $\widehat{\Phi}_0 = \widehat{\Phi}$ . Furthermore, the analysis can focus on only two binding constraints:

$$d_2 \leq d_1 + \frac{x_1}{2} \widehat{\Phi} \quad (9')$$

and

$$2d_1 + \widehat{\Phi}_1^y \leq \widehat{\Phi} \quad (11b')$$

where, in the asymmetric state, the cost to the intact country when the other is distressed is minimal conditional on the number  $x_1$  of defaults:

$$\widehat{\Phi}_1^y = \begin{cases} x_1 \phi & \text{for } x_1 \leq 1 \\ \phi + (x_1 - 1) \Phi & \text{for } x_1 \geq 1. \end{cases}$$

Increasing  $x_1$  facilitates repayment when both countries are intact (condition (9')), but reduces the maximand and also makes it more difficult to obtain repayment in the asymmetric state (condition (11b')). At the optimum  $x_1$  is an integer:  $x_1^* \in \{0, 1, 2\}$ . Proposition 4 characterizes the three corresponding regions.

**Proposition 4 (contractual solidarity).** *Let  $r \equiv \frac{\phi}{\Phi}$  (spillover-default cost ratio) and  $\ell \equiv \frac{p_1}{p_2}$  (likelihood ratio). If  $R < (1 + p_0)/(1 - p_0)$ , then it is optimal for the countries not to borrow. If  $R > (1 + p_0)/(1 - p_0)$ , then countries borrow; there is no default when both are intact and full default when both are distressed. Furthermore:*

(1) *Solidarity region: If  $R(1 - \ell r) < 1 + \ell$ , then there is no equilibrium default when one country is distressed:  $x_1^* = 0$ . Each country takes on debt  $d_2 = (\Phi + \phi)/2$  and accepts to be jointly liable for the full amount of unpaid debt by the other country. Joint-and-several liability is required as  $d_2 > \phi$  and so an intact country would not spontaneously bail out a distressed one.*

(2) *PSI region: If  $R(1 - \ell r) > 1 + \ell > R(1 - \frac{\ell}{r})$ , then the distressed country, but not the intact one defaults:  $x_1^* = 1$ . Each country takes on debt  $d_2 = \Phi + (\phi/2)$ , but is not jointly liable for the other country's debt. There is private sector involvement, in the sense of a voluntary reduction in the debt owned to the intact country to  $\Phi < d_2$  when the distressed country defaults. This debt forgiveness is meant to prevent the intact country, which already incurs spillovers from the other country's default, from defaulting.*

(3) *Contagion region.* If  $R(1 - \frac{\ell}{r}) > 1 + \ell$ , then both countries default if at least one of them is in distress:  $x_1^* = 2$ . Both countries take a high level of debt  $d_2 = \hat{\Phi}$ ; so they default unless they are both intact.

Minor liquidity needs (low  $R$ ) and a high probability of a joint shock (high  $p_0$ ) both make borrowing suboptimal. As liquidity needs increase, say, borrowing combined with joint liability becomes desirable. Debt remains limited so as to make credible the assumption of a troubled country's debt by an intact one. As liquidity needs become more pressing, countries increase their indebtedness and abandon joint liability because solidarity is no longer credible; the troubled country defaults; perhaps surprisingly, the intact country is granted some debt forgiveness by the private sector: Because the troubled country defaults anyway, the intact country's incentives to repay must be re-established through debt reduction. Finally, for very large liquidity needs (or when shocks are highly correlated), countries choose a very risky strategy: they lever up a lot and default unless both are intact. Contagion occurs because a country's debt is at its highest possible level, namely the level at which an intact country is indifferent between reimbursing and entering a default that brings down both countries; if the other country is troubled and defaults anyway, then incentives for debt repayment by the intact country are reduced and that country prefers to default as well.

The ratio  $\ell$  can be interpreted as a measure of independence of country shocks,<sup>22</sup> while  $r$  measures the relative cost of spillovers of default to country cost of default. Figure 3 depicts the various regimes in the case  $\ell < r < 1/\ell$  (for which the four regions may exist for different values of  $R$ <sup>23</sup>). The following corollary provides the comparative statics with respects to  $\ell$  and  $r$ .

**Corollary 4 (comparative statics).**

- (i) *Solidarity in the form of joint liability is more likely to be optimal, the more independent the shocks (i.e., the larger  $\ell$  is).*
- (ii) *The solidarity and contagion regions expand when relative spillover costs ( $r$ ) increase.*

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<sup>22</sup>If  $\rho$  denotes the correlation and  $\alpha$  is the marginal probability of a good state for a country, then  $p_2 = \rho\alpha + (1 - \rho)\alpha^2$  and  $p_1 = (1 - \rho)2\alpha(1 - \alpha)$ . And so

$$\ell = \frac{2\alpha(1 - \alpha)}{\alpha^2 + \frac{\rho}{1 - \rho}\alpha}$$

Thus  $\ell$  decreases with  $\rho$ .

<sup>23</sup>Even so, some regimes may not exist. For example, if spillovers are small, then the solidarity region does not exist.

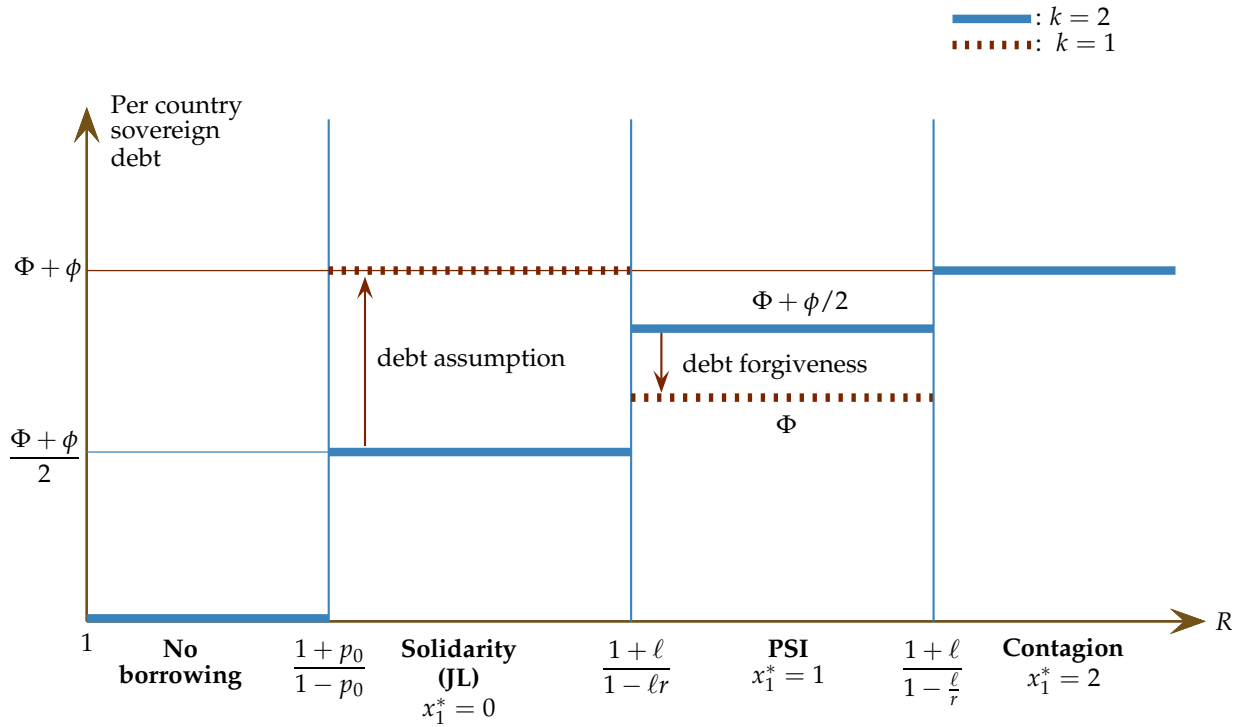


Figure 3: Optimal contracting behind the veil of ignorance

How does the correlation of the countries' shocks affect their welfare? When correlation increases, the solidarity region (if it exists) shrinks (and the contagion region becomes more likely). Furthermore, the countries' welfare within the solidarity region decreases; in a nutshell, there is less of a scope for mutual insurance. By contrast, correlation increases welfare within the PSI and contagion regions.

## 4 Endogenous spillovers

Spillovers such as trade and political disruptions can by and large be assumed exogenous. By contrast, banking exposures to a potentially distressed country are in part endogenous if only because banking supervision still operates at the domestic level.<sup>24</sup> This section extends the model of Sections 2 and 3, and asks whether countries would want to minimize spillovers as a first intuition would suggest. To study this question, we assume that country  $i$  chooses a degree of exposure  $z_i \in [0, 1]$  that determines the spillover from country  $j$ 's default:

$$\phi_i = \phi_0 + z_i(\phi - \phi_0),$$

<sup>24</sup>According to Basle rules. There are ongoing discussions as to the extent to which banking supervision will be centralized in Europe.



where  $\phi_0 < \phi$  stands for the non-controllable spillovers.<sup>25</sup>

### *Asymmetric insurance*

Consider first the model of Section 2. From Proposition 2, the outcome under contracting depends solely on the principal's utility in the absence of a contract ( $U_p^*$ ); this is due to the fact that the principal's incentive compatibility constraint is non-binding. In turn, the principal's utility under laissez-faire is given by (5), which implies that in each region ( $R \geq R^*(\phi_P)$ ) the principal is better off minimizing its exposure ( $z_P = 0$ ).

Furthermore  $\text{sign} \left( \frac{\partial R^*}{\partial \phi_P} \right) = \text{sign} (1 - \alpha - \alpha^2)$ . Thus, if  $1 < \alpha + \alpha^2$ ,  $z_P = 0$  is indeed the optimal policy, as minimizing the exposure both increases the principal's utility for a given borrowing strategy of the agent, but also makes the risky strategy less appealing to the agent.

By contrast, if  $1 > \alpha + \alpha^2$ , then the principal may not want to minimize exposure. It may be that being exposed to the agent ( $z_P > 0$ ) encourages the safe strategy, offsetting the direct cost for the principal of being exposed.<sup>26</sup>

### *Symmetric insurance.*

Let us now turn to the model of optimal contractual solidarity of Section 3. A change in spillovers moves the boundaries of the various regions as described in Corollary 4. More to the point, Appendix 2 studies the impact of an increase in  $\phi$  on the maximand.

In particular, in the solidarity region ( $x_1^* = 0$ ), increasing spillovers is costless as default does not occur in equilibrium; furthermore, by increasing collateral damages an increase in  $\phi$  increases borrowing capacity and allows both countries to borrow more. Similarly increasing spillovers is optimal in the contagion region. Each country commits to be dependent upon the other country's repaying its debt. Investing in the other country is akin to agree to an hostage and credibly commits the country to an extended solidarity. By contrast, the impact of an increase in  $\phi$  is ambiguous in the PSI region.

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<sup>25</sup>To illustrate this, suppose that the principal's banking sector invests  $b_{MP} (\leq b_M)$  in the agent's debt. Let  $d_{MP}$  denote the corresponding liability and  $\beta \geq 0$  the weight put by the principal on his banking sector. Then  $\phi_P = \phi_0 + \beta d_{MP}$ . A control by the principal of the volume of agent liabilities purchased by its banks amounts to choosing the spillover cost for the principal. One can for example suppose that the principal's banks have a slightly higher valuation for the agent's debt than other private-sector investors (for example, they share the same currency in the case of the Eurozone). They will then invest up to the cap set by the principal.

Another possible interpretation of the  $z_P = 0$  choice is that of an exit option.

<sup>26</sup>This happens for instance if  $R$  lies just above  $R^*(\phi_0)$ . By minimizing exposure, the principal then induces the risky borrowing strategy. By increasing  $\phi_P$  a bit above  $\phi_0$ , the principal induces the safe strategy and incurs expected loss slightly higher than  $(1 - \alpha)\phi_0$  instead of  $\phi_0$ .

**Proposition 5 (endogenous spillovers).** *Suppose that the collateral damages can be chosen in an interval  $[\phi_0, \phi]$ .*

(i) *One-way insurance: While the optimal contract can be implemented through maximal exposure ( $\phi$ ), in the absence of contract and conditional on the agent's choosing either the risky, high-debt policy or the safe, low-debt policy, the principal minimizes its exposure:  $\phi_P = \phi_0$ . Yet, if the probability of a bad state is high, the principal may in the absence of contract end up choosing  $\phi_P > \phi_0$  in order to incentivize the agent to choose the safe policy.*

(ii) *Two-way insurance (the mutual hostage insight): In the optimal contract, the countries always maximize their cross-exposure in the solidarity and contagion regions:  $\phi_i = \phi$ . By contrast, the impact on country welfare of an increase in spillovers in the PSI region is ambiguous.*

*Remark:* We have focused on the affected country's manipulation of the collateral damage. In practice, the *borrower* may also want to manipulate the collateral damage (for instance through a military program) by increasing it so as to benefit from the perception or the reality of implicit support.

## 5 Conclusion

Bailouts are driven by the fear that spillovers from the distressed country's default negatively affect the rescuer. This paper's first contribution was to provide formal content to the intuitive notion that collateral damages of a country's default are de facto collateral for the country.

The paper's second contribution was to unveil the conditions under which joint-and-several liability may emerge. Standard liquidity provision or risk sharing models presume that accord is reached behind the veil of ignorance. Once the veil of ignorance is lifted (as is currently the case in the Eurozone), healthy countries have no incentive to accept obligations beyond the implicit ones that arise from spillover externalities. Put differently, it is not in the self-interest of healthy countries to accept joint-and-several liability and assume the concomitant risk of a domino effect, even though they realize that they will be hurt by a default and thus will ex post show some solidarity in order to prevent spillovers; an ex-ante transfer from distressed countries to healthy ones to compensate them for, and make them accept the future liability is ruled out as it would just add to the distressed countries' indebtedness and thus the compensation would be in funny money.

Third, the paper showed that by contrast, in a more symmetrical, mutual-insurance context, contractual solidarity in the form of joint liability is optimal provided that

country shocks are sufficiently independent and spillovers costs sufficiently large relative to default costs. Joint liability increases both borrowing capability and the risk of contagion. Finally, the paper endogenized spillovers and provided conditions under which a country deliberately chooses to be exposed to another country's default.

On the theoretical front, the paper is only a first attempt at understanding the fundamentals of country solidarity, whether reluctantly provided or more pro-actively contracted for. There are many interesting alleys for future research in this area alone. For instance, one might extend the analysis of Section 3 to consider extended solidarity; first losses could be covered by an inner circle of countries within a solidarity area and macro shocks within this area might be partly insured by an outer solidarity area (rest of the world, IMF). Another fascinating topic for future analysis would result from asymmetries of information about collateral damages and the concomitant posturing behaviors in the international community.

On the policy front, one should investigate the likelihood of emergence of alternative solidarity zones. Recall for instance the puzzle stated in the introduction: both the bailout contributions and the policy debate about Eurobonds and the banking union mostly concern a very limited insurance pool, namely the Eurozone, while basic principles of insurance economics would call for a much broader solidarity area. Although the following suggestions are no substitute for a careful analysis, the model arguably sheds light on the puzzle. First, the monetary union has drastically increased the degree of financial integration among Eurozone countries.<sup>27</sup> Financial integration implies increased spillovers from default. And indeed, France and Germany have much larger exposure to, say, Italy than the UK, let alone the US and China.<sup>28</sup> Second, the establishment of the monetary union in large part was driven by a political project. In this sense, it reflects the presence of strong spillover effects; and abandoning the Euro, or letting some Eurozone countries default would have a substantial symbolic impact. These two factors are likely explanations for the otherwise peculiar risk-sharing arrangement.

Similarly, one may build on this paper to investigate the impact of fiscal unions. A fiscal union creates an automatic risk sharing mechanism and thus correlates income realizations; it further generates some extent of joint liability through the issuance of federal debt. And, as is well-known, the increase in correlation facilitates the conduct

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<sup>27</sup>Kalemli-Ozcan-et-al (2010) shows that this financial integration was driven more by the elimination of currency risk than by trade in goods.

<sup>28</sup>Interestingly, the ECB's three-year liquidity provision (LTRO) program has accelerated a trend toward "running for home": Spanish and Italian banks have increased their holdings of domestic government debt by 26 % and 31 %, respectively, in December 2011-January 2012. "More patriotic balance sheets" have several effects in my model: First the make the country less prone to default, as foreign debt decreases; second, the solidarity driver  $\phi_P$  decreases as well to the extent that domestic banks buy from Eurozone lenders (but not when they buy from lenders from outside the monetary zone). Note that if outside lenders do not change their government bonds holdings, then the outcome is just a swap and  $\Phi_A + \phi_P$  remains unchanged.

of monetary policy as well. Nonetheless, states still enjoy some degree of subsidiarity; the implications of fiscal federalism for solidarity are definitely worth investigating.

The paper has assumed that troubled countries can resort only to hard default to escape the burden of liabilities in adverse times. Either they are highly inflation averse or their commitment to a currency union precludes any debt monetization. In a companion paper (Tirole 2012), I take a first stab at deriving the implications of a “soft default” (devaluing the currency) and at studying the interaction between debt monetization and solidarity. First, I show that a currency union by precluding soft default generates more hard default, and that the choice between a currency union and an own currency exhibits a familiar trade-off between commitment and flexibility. A country’s having its own currency creates some automatic private sector involvement and makes other countries more willing to bail it out for a given shortfall between debt and willingness to pay; furthermore, rescuing countries optimally denominate their support in foreign rather than domestic currency.

Finally, the paper’s modeling and implications focused on its initial, international finance motivation. Its potential scope of applications however is broader. A corporation may guarantee a key supplier’s debts by integrating it as its division, or by keeping it independent and promising to cover its liabilities. Banks may enter various kinds of contractual agreements, including credit lines, which imply varying degrees of solidarity. Individuals choose between giving a helping hand to members of their family (children) or friends facing financial straits and more formally standing surety for them, thereby facilitating their access to credit or housing. Integrating the specificities of these other contexts would be of much interest.

I leave these and the many related topics on solidarity to future research.

## References

- Aguiar, Mark., and Gita Gopinath. 2006. "Defaultable Debt, Interest Rates and the Current Account." *Journal of International Economics*. 69: 64–83.
- Arteta, Carlos, and Galina Hale. 2008. "Sovereign Debt Crises and Credit to the Private Sector," *Journal of International Economics*. 74: 53–69.
- Bester, Helmut. 1987. "The Role of Collateral in Credit Markets with Imperfect Information." *European Economic Review*. 3: 887–899.
- Bizer, David, and Peter M. de Marzo. 1992. "Sequential Banking." *Journal of Political Economy*, 100: 41–61.
- Bolton, Patrick, and Olivier Jeanne. 2009. "Structuring and Restructuring Sovereign Debt: The Role of Seniority." *Review of Economic Studies*. 76(3): 879–902.
- Bolton, Patrick, and Olivier Jeanne. 2011. "Sovereign Default Risk and Bank Fragility in Financially Integrated Economies." *IMF Economic Review*. 59(2): 162–194.
- Bolton, Patrick, and Howard Rosenthal. 2001. "The Political Economy of Debt Moratoria, Bailouts and Bankruptcy." in *Defusing Default*, ed. Marco Pagano, 77–115. Washington DC: IDB Publication.
- Bolton, Patrick, and Howard Rosenthal. 2002. "Political Intervention in Debt Contracts." *Journal of Political Economy*; 110(5): 1103–34.
- Bolton, Patrick, and David Scharfstein 1990. "A Theory of Predation Based on Agency Problems in Financial Contracting." *American Economic Review*. 80: 93–106.
- Borensztein, Eduardo, and Ugo Panizza. 2009. "The Costs of Sovereign Default." *IMF Staff Papers*, 56: 683–741.
- Broner, Fernando, Alberto Martin, and Jaume Ventura. 2010. "Sovereign Risk and Secondary Markets." *American Economic Review*, 100: 1523–55.
- Bulow, Jeremy, and Kenneth Rogoff. 1988. "Multilateral Negotiations for Rescheduling Developing Country Debt: A Bargaining Theoretic Framework." *IMF Staff Papers*, 35. Reprinted in *Analytical Issues in Debt*, eds J. Frenkel, M. Dooley, and P. Wickham, (IMF, Washington D.C. 1989).
- Bulow, Jeremy, and Kenneth Rogoff. 1989a. "A Constant Recontracting Model of Sovereign Debt." *Journal of Political Economy*, 97(1): 155–178.

- Bulow, Jeremy, and Kenneth Rogoff. 1989b. "Sovereign Debt: Is to Forgive to Forget?," *American Economic Review*, 79(1): 43–50.
- Calvo, Guillermo. 1988. "Servicing the Public Debt: The Role of Expectations." *American Economic Review*, 78(4): 647–661.
- Claessens, Stijn, Ashoka Mody, and Shahin Vallee. 2012. "Paths to Eurobonds," IMF Working Paper 12/172. <http://www.bruegel.org/publications/publication-detail/publication/733-paths-to-eurobonds>
- Cole, Harold L., and Patrick J. Kehoe. 1995. "The Role of Institutions in Reputation Models of Sovereign Debt." *Journal of Monetary Economics*, 35(1): 45–64.
- Cole, Harold L., and Patrick J. Kehoe. 1998. "Models of Sovereign Debt: Partial vs. General Reputations." *International Economic Review*, 39(1): 55–70.
- Cole, Harold L., and Timothy J. Kehoe. 2000. "Self-Fulfilling Debt Crises." *Review of Economic Studies*, 67(1): 91–116.
- Corsetti, Giancarlo., Bernardo Guimaraes, and Nouriel Roubini. 2006. "International Lending of Last Resort and Moral Hazard: A Model of IMF's Catalytic Finance." *Journal of Monetary Economics*, 53(3): 441-471.
- Cruces, Juan J., and Christoph Trebesch. 2011. "Sovereign Defaults: The Price of Haircuts." CESifo Working Paper Series 3604, CESifo Group Munich.
- Dellas, Harris. , and Dirk F. Niepelt. 2012. "Credibility For Sale." Mimeo, University of Bern and Gerzensee. <http://www.eief.it/files/2012/10/dirk-niepelt.pdf>
- Delpla, Jacques, and Jakob von Weizsacker. 2010. "The Blue Bond Proposal." Bruegel Policy Brief 420. <http://www.bruegel.org/publications/publication-detail/publication/509-eurobonds-the-blue-bond-concept-and-its-implications>
- Diamond, Douglas W. 1984. "Financial Intermediation and Delegated Monitoring." *Review of Economic Studies*, 51(3): 393–414.
- Eaton, Jonathan. 1996. "Sovereign Debt, Reputation, and Credit Terms." *International Journal of Finance and Economics*, 1: 25–36.
- Eaton, Jonathan J., and Mark Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies* 48(2): 289–309.
- Eaton, Jonathan J., Mark Gersovitz, and Joseph E. Stiglitz. 1986. "The Pure Theory of Country Risk." *European Economic Review*, 30: 481–513.

- Euro-nomics Group (Brunnermeier, Markus, Luis Garicano, Philip R. Lane, Marco Pagano, Ricardo Reis, Tano Santos, Stijn Van Nieuwerburgh, and Dimitri Vayanos). 2011. "ESBies: A Realistic Reform of Europe's Financial Architecture." VoxEU.org, 25 October. <http://www.voxeu.org/article/esbies-realistic-reform-europes-financial-architecture>
- Farhi, Emmanuel, and Jean Tirole. 2012. "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts." *American Economic Review*, 102(1): 60–93.
- Fernandez, Raquel, and Robert W Rosenthal. 1990. "Strategic Models of Sovereign-Debt Renegotiations." *Review of Economic Studies*, 57(3): 331–349.
- Gale, Douglas., and Martin F. Hellwig. 1985. "Incentive Compatible Debt Contracts: The One Period Problem." *Review of Economic Studies*, 52: 647–663.
- Gennaioli, Nicola., Alberto Martin, and Stefano Rossi. 2011. "Sovereign Default, Domestic Banks and Financial Institutions." mimeo, CREI, Barcelona.
- Grossman, Herschel, and John Van Huyck. 1988. "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation." *American Economic Review*, 78(5): 1088–1097.
- Hellwig, Christian, and Guido Lorenzoni. 2009. "Bubbles and Self-Enforcing Debt." *Econometrica*, 77: 1137–1164.
- Hellwig, Christian, and Thomas Philippon. 2011. "Eurobills, not Eurobonds." VoxEU.org, December 2, 2011. <http://www.voxeu.org/article/eurobills-not-eurobonds>
- Holmström, Bengt. 1979. "Moral Hazard and Observability." *Bell Journal of Economics*, 10(1): 74–91.
- Jeske, Karsten. 2006. "Private International Debt with Risk of Repudiation." *Journal of Political Economy*, 114 (3): 576–93.
- Kalemli-Ozcan, Sebnem., Elias Papaionnou, and Jose-Luis Peydro. 2010. "What Lies Beneath the Euro's Effect on Financial Integration? Currency Risk, Legal Harmonization, or trade?" *Journal of International Economics*, 81: 75–88.
- Kletzer, Kenneth, and Brian Wright. 2000. "Sovereign Debt as Intertemporal Barter." *American Economic Review*, 90: 621–639.

- Krugman, Paul. 1985. "International Debt Strategies in an Uncertain World." in *International Debt and the Developing Countries*, eds M. A. Smith, and A. Cuddington, 79–100. Washington DC: The World Bank.
- Mengus, Eric. 2012. "Foreign Borrowing, Portfolio Allocation and Bailouts." Mimeo, TSE. <http://www.crei.cat/conferences/ief-workshop/Mengus.pdf>
- Mitchener, Kris J., and Marc Weidenmier. 2010. "Supersanctions and Sovereign Debt Repayment." *Journal of International Money and Finance*, 29: 19–36.
- Obstfeld, Maurice, and Kenneth Rogoff. 1996. *Foundations of International Macroeconomics*. Cambridge, MA: MIT press.
- Persson, Torsten, and Guido Tabellini. 1996. "Federal Fiscal Constitutions: Risk Sharing and Moral Hazard." *Econometrica*, 64(3): 623–646.
- Philippon, Thomas. 2012. "Yes, Europe Can Become a Stable Currency Area." Mimeo, NYU.
- Rochet, Jean-Charles, and Jean Tirole. 1996. "Interbank Lending and Systemic Risk." *Journal of Money, Credit and Banking*, 28: 733–762.
- Rose, Andrew. 2005. "One Reason Countries Pay their Debts: Renegotiation and International Trade." *Journal of Development Economics*, 77(1): 189–206.
- Roubini, Nouriel. 2004. "Private Sector Involvement in Crisis Resolution and Mechanisms for Dealing with Sovereign Debt Problems." in *Fixing Financial Crises in the Twenty-First Century*, ed. Andrew Haldane. 101–42. London: Routledge.
- Sachs, Jeffrey. 1983. "Theoretical Issues in International Borrowing." NBER Working Paper No. 1189.
- Segal, Ilya R. 1999. "Contracting with Externalities." *Quarterly Journal of Economics* 114(2): 337–388
- Sturzenegger, Federico, and Jeromin Zettelmeyer. 2007. *Debt Defaults and Lessons from a Decade of Crises*. Cambridge: MIT Press.
- Tirole, Jean. 2003. "Inefficient Foreign Borrowing: A Dual-and Common-Agency Perspective." *American Economic Review*, 93(5): 1678–1702.
- Tirole, Jean. 2006. *The Theory of Corporate Finance*, Princeton: Princeton University Press.
- Tirole, Jean. 2010. "From Pigou to Extended Liability: On the Optimal Taxation of Externalities Under Imperfect Capital Markets," *Review of Economic Studies*, 77: 697–729.



- Tirole, Jean. 2012. "Hard vs. Soft Default: Debt Monetization and International Bailouts." Mimeo, TSE.
- Townsend, Robert M. 1979. "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, 21: 417–425.
- Trebesch, Christoph. 2009. "The Cost of Aggressive Sovereign Debt Policies: How Much is the Private Sector Affected?" IMF Working Paper 09/29.
- Weinschelbaum, Federico, and José Wynne . 2005. "Renegotiation, Collective Action Clauses and Sovereign Debt Markets." *Journal of International Economics*, 67: 47–72.
- Wright, Mark L.J. 2002. "Reputations and Sovereign Debt." Stanford University manuscript. <http://sciie.ucsc.edu/conference/2002/wrightpaper.pdf>
- Wright, Mark L.J. 2006. "Private Capital Flows, Capital Controls and Default Risk." *Journal of International Economics*, 69(1): 120–49.

## Appendix 1: Proof of Corollary 3

Suppose that the agent contracts liabilities *solely with the market* ( $d_P = 0$ ). From Proposition 2, the agent must borrow  $b = \alpha\Phi_A - U_P^*$  and reimburse  $d^G = \Phi_A$  and  $d^B = 0$ . Unlike in Section 2.2, we allow the principal to contract with the agent on other aspects such as the debt level or some date-1 lump-sum payment. If the country's liability  $d_M$  is strictly smaller than  $\Phi_A$ , then  $d^G < \Phi_A$  and so the scheme does not mobilize enough country collateral to implement the optimum. By contrast, if  $d_M \geq \Phi_A$ , then in the absence of joint liability, the principal does not rescue the agent in the bad state and so the latter defaults. So pure market debt cannot implement the optimum in the absence of joint liability.

(i) Suppose first that  $R < R^*$ . The principal's participation constraint requires that  $-(1 - \alpha)d^B \geq U_P^* = -(1 - \alpha)\phi_P$  and so  $d^B \leq \phi_P < b^* = \alpha\Phi_A + (1 - \alpha)\phi_P$ , where  $b^*$  is the optimal borrowing level as given by Proposition 2: The agent underborrows relative to the optimum.

Suppose next that  $R \geq R^*$ . The principal's participation constraint is looser:  $d^B \leq \phi_P / (1 - \alpha)$ , while optimal borrowing is now  $b^* = \alpha\Phi_A + \phi_P$ . Requiring  $d^B = b^*$  yields a necessary condition for Eurobonds to implement the optimum:  $(1 - \alpha)\Phi_A \leq \phi_P$ , which is equivalent to  $d^B = b^* \geq \Phi_A$ . So let us assume that  $(1 - \alpha)\Phi_A \leq \phi_P$ . The principal's utility is

$$-\alpha(b^* - \Phi_A) - (1 - \alpha)b^* = -\phi_P = U_P^*.$$

Finally, the principal must be willing to rescue the agent in the bad state of nature:

$$b^* \leq \Phi_P + \phi_P \iff \alpha\Phi_A \leq \Phi_P.$$

(ii) Suppose that the agent borrows  $b$ , transfers  $t = b - b^* \geq 0$  to the principal at date 1, and consumes  $b^*$ . Credible solidarity in the bad state (which guarantees the absence of default) requires that

$$b \leq \Phi_P + \phi_P.$$

Suppose first that  $b \leq \Phi_A$ . Then the principal's participation constraint writes

$$[b - b^*] - (1 - \alpha)b = U_P^*.$$

When substituting for this expression of  $b$ , the condition  $b \leq \Phi_A$  is infeasible. So assume that  $b \geq \Phi_A$ . Then

$$[b - b^*] - \alpha(b - \Phi_A) - (1 - \alpha)b = U_P^*$$

or

$$b^* = \alpha\Phi_A - U_p^*,$$

which is feasible only if  $U_p^* = -\phi_P$  or  $R \geq R^*$ . So, assuming  $R \geq R^*$ , one must find  $b \geq b^*$  such that  $b \geq \Phi_A$ ,  $b \leq \Phi_P + \phi_P$  and  $b \geq b^*$ . These conditions boil down to

$$\max \left\{ \Phi_A, \alpha\Phi_A + \phi_P \right\} \leq \Phi_P + \phi_P. \quad \blacksquare$$

## Appendix 2: Proofs of Propositions 4 and 5

To solve Program (II), let us first note that at its optimum there is no point punishing countries that announce truthfully that they both are healthy:

$$\widehat{\Phi}_2 = 0.$$

Another preliminary result that comes from inspecting this program is that for a given  $x_1$ , i.e., for a given total punishment  $\widehat{\Phi}_1^y + \widehat{\Phi}_1^0 = x_1 \widehat{\Phi} = 2\widehat{\Phi}_1$  when only one of the countries is healthy, it is optimal to minimize  $\widehat{\Phi}_1^y$  as this relaxes constraints (10) and (11b) without altering the rest. So we can without loss of generality assume that

$$\widehat{\Phi}_1^y \equiv \Phi_1^y(x_1) \equiv \begin{cases} x_1\phi & \text{for } x_1 \leq 1 \\ \phi + (x_1 - 1)\Phi & \text{for } x_1 \geq 1. \end{cases}$$

(a) Let us first assume that (11b) is binding, and so ignore (11a). We further ignore condition (8) and verify ex post that it is indeed verified. From (7), we can also ignore (10), which is implied by (11b). It is then clear that (9) and (11b) must be binding, otherwise  $d_1$  or  $d_2$  or both would tend to infinity, which would violate some of the constraints, e.g., (8). And so

$$d_1 = \frac{\widehat{\Phi}_0 - \Phi_1^y(x_1)}{2}$$

and

$$d_2 = \frac{\widehat{\Phi}_0 - \Phi_1^y(x_1)}{2} + \frac{x_1\widehat{\Phi}}{2}$$

The maximand can then be written as:

$$U^* = \max_{\{x_1, \widehat{\Phi}_0\}} \left\{ (R-1) \left[ (p_1 + p_2) \frac{\widehat{\Phi}_0 - \Phi_1^y(x_1)}{2} + p_2 \frac{x_1\widehat{\Phi}}{2} \right] - p_1 \frac{x_1\widehat{\Phi}}{2} - p_0\widehat{\Phi}_0 \right\} \quad (11)$$

Either the optimal punishment when both are distressed is equal to 0 and then the maximand is also equal to 0. Or

$$(R - 1) \left( \frac{p_1 + p_2}{2} \right) > p_0 \quad (12)$$

and then the optimal punishment when both are distressed is maximal:

$$\hat{\Phi}_0 = \hat{\Phi}.$$

Maximizing with respect to  $x_1$  yields:

$$\begin{aligned} x_1 = 0 & \quad \text{if} \quad (R - 1) \left[ \Phi - \frac{p_1}{p_2} \phi \right] < \frac{p_1}{p_2} \hat{\Phi} \\ x_1 = 1 & \quad \text{if} \quad (R - 1) \left[ \Phi - \frac{p_1}{p_2} \phi \right] > \frac{p_1}{p_2} \hat{\Phi} > (R - 1) \left[ \phi - \frac{p_1}{p_2} \Phi \right] \\ x_1 = 2 & \quad \text{if} \quad (R - 1) \left[ \phi - \frac{p_1}{p_2} \Phi \right] > \frac{p_1}{p_2} \hat{\Phi} \end{aligned} \quad (13)$$

Finally, we verify that constraints (8) and (11b) are satisfied. They can be rewritten as:

$$\frac{\hat{\Phi} - \Phi_1^y(x_1)}{2} + \frac{x_1 \hat{\Phi}}{2} \leq \hat{\Phi} \quad \text{for} \quad (8)$$

and

$$\hat{\Phi} - \Phi_1^y(x_1) + x_1 \hat{\Phi} \leq 2\hat{\Phi} \quad \text{for} \quad (11b)$$

So we need only to check that

$$x_1 \hat{\Phi} - \Phi_1^y(x_1) \leq \hat{\Phi} \quad \text{for all} \quad x_1,$$

which is straightforward (the left-hand side is an increasing function of  $x_1$  and takes value  $\hat{\Phi}$  for  $x_1 = 2$ ).

*Implementation*

✓ When  $x_1^* = 0$ , then

$$d_1 = d_2 = \frac{\hat{\Phi}}{2}.$$

The intact country takes on the entire debt of the distressed country (reimburses  $2d_1 = 2d_2$ ).

Joint-and-several liability is required as  $d_1 = \frac{\Phi + \phi}{2} > \phi$ , and so there is no spontaneous bailout in state 1. Finally, note that, again in state of nature 1, the intact country does not want to default and receive  $-\hat{\Phi} = -2d_1$ .

✓ When  $x_1^* = 1$ , then

$$d_1 = \frac{\Phi}{2} \quad \text{and} \quad d_2 = \Phi + \frac{\phi}{2}.$$

In state of nature 1, the intact country owes  $\Phi$  and suffers spillover  $\phi$ . It therefore cannot improve its welfare by defaulting itself (it would then have utility  $-(\Phi + \phi)$ ). Would the intact country want to rescue the distressed country by paying  $2d_2$ ? Its utility would then be  $-2d_2 = -2\Phi - \phi < -\Phi - \phi$ , and so there is indeed no bailout ( $x_1 = 1$ ).

✓ When  $x_1^* = 2$ , then

$$d_1 = 0 \quad \text{and} \quad d_2 = \hat{\Phi} = \Phi + \phi.$$

Again, in state 1, the intact country does not want to bail out the distressed country as

$$-2d_2 = -2\hat{\Phi} < -\hat{\Phi}.$$

(b) Second, let us investigate the possibility that (11a) is binding. We can then ignore (8), which is implied by (9) and (11a). Condition (9) must be binding, otherwise  $d_2$ , and thereby the maximand, could be increased:

$$d_2 = d_1 + \hat{\Phi}_1.$$

Similarly (11a) must be satisfied with equality. Otherwise  $\hat{\Phi}_0$  could be reduced (if  $\hat{\Phi}_0 = 0$ , the maximand is equal to 0 anyway):

$$d_1 + \hat{\Phi}_1 = \hat{\Phi}_0.$$

Substituting  $d_1$  and  $d_2$ , we can rewrite the program as

$$\max \left\{ \left[ (R-1)(p_2 + p_1) - p_0 \right] \hat{\Phi}_0 - p_1 R \hat{\Phi}_1 \right\}$$

subject to constraint (10), rewritten as:

$$2\hat{\Phi}_0 \leq \hat{\Phi} + \hat{\Phi}_1^0. \quad (10')$$

Assume  $(R-1)(p_2 + p_1) - p_0 > 0$  (otherwise the value of the program is 0). Then (10') must be binding, and so the maximand becomes, up to a constant,

$$\max_{\{x_1\}} \left\{ \left[ (R-1)(p_2 + p_1) - p_0 \right] \frac{\Phi_1^0(x_1)}{2} - p_1 R x_1 \widehat{\Phi} \right\}.$$

Again the optimal  $x_1$  is either 1 or 2, if it is not equal to 0. For  $x_1 = 1$ , then  $d_1 = \Phi/2$  and  $d_2 = \Phi + (\phi/2)$ , which is the same solution as in case (a).

For  $x_1 = 2$ , then  $d_1 = 0$  and  $d_2 = \widehat{\Phi}$ , again as in case (a). So the solutions considered in case (a) cover the entire range of solutions. ■

### *Endogenous spillovers*

From (11), and assuming that (12) (which is independent of  $\phi$ ) is satisfied, the change in the maximand  $U^*$  is given by:

$$\frac{\partial U^*}{\partial \phi} = \begin{cases} (R-1) \left( \frac{p_1 + p_2}{2} \right) - p_0 & \text{in the solidarity region} \\ (R-1) \left( \frac{p_2}{2} \right) - \frac{p_1}{2} - p_0 & \text{in the PSI region} \\ (R-1) (p_2) - p_1 - p_0 & \text{in the contagion region} \end{cases}$$

From (12),  $\frac{\partial U^*}{\partial \phi} > 0$  in the *solidarity region*. In that region,  $x_1^* = 0$ , and so an increase in  $\phi$  has no impact on the two constraints (9') and (11b'). Hence increasing  $\phi$  is optimal whenever borrowing is.

The impact of an increase in  $\phi$  is ambiguous in the *PSI and contagion regions*. Because  $x_1^* > 0$  in these regions, increasing  $\phi$  is costly when  $k = 1$  or  $2$ , while it has no impact on constraint (11b') and relaxes constraint (9').

For example, in the *PSI region*,

$$\frac{\partial U^*}{\partial \phi} \begin{cases} > 0 & \text{for } p_0 \text{ small} \\ < 0 & \text{for } p_0 \text{ large (close to } \frac{R-1}{R+1} \text{), } r \text{ small and } R \text{ close to } 1 + \ell. \end{cases}$$

In the *contagion region*,  $\partial U^* / \partial \phi < 0$  requires that  $p_2 R < 1$ . Furthermore, to be in the contagion region, it must be the case that investment be desirable:

$$R \geq \frac{1 + p_0}{1 - p_0}$$

and that

$$R \left( 1 - \frac{\ell}{r} \right) \geq 1 + \ell.$$

Because  $r \leq 1$ , the latter condition requires that

$$R(2p_2 + p_0 - 1) \geq 1 - p_0.$$

Putting the three conditions together,  $\partial U^* / \partial \phi < 0$  requires that

$$1 < \max \left\{ \frac{1 + p_0}{1 - p_0}, \frac{1 - p_0}{2p_2 + p_0 - 1} \right\} < \frac{1}{p_2},$$

which can be checked is impossible. ■

### Appendix 3: Asymmetric information within the official sector

We have assumed homogeneous information between the official sector and debtor countries. More generally the official sector could receive a noisy signal regarding the debtor country's willingness to pay. This Appendix checks the robustness of our results in the polar case of complete absence of a signal: The principal of Section 2 has no better information than the market about the debtor country's willingness to pay. The timing is the same as in Figure 1, except that only  $A$  observes the realized income.

*Laissez-faire.*

Suppose first that the agent borrows from the market without contracting with the principal. Suppose that the agent has borrowed  $d_M \leq \Phi_A$  in the international financial market. The principal is willing to cover this debt as long as

$$d_M \leq (1 - \alpha)\phi_P$$

because failure to guarantee the country's liabilities results in a default with probability  $1 - \alpha$ . Similarly, suppose that  $d_M \in (\Phi_A, \Phi_A + \phi_P]$ . Then the principal is willing to cover  $d_M - \Phi_A$  (conditionally on the agent's repaying its private liabilities) in order to bring probability of default from 1 to  $(1 - \alpha)$ : the principal obtains  $-\phi_P$  by not intervening and  $-\alpha(d_M - \Phi_A) - (1 - \alpha)\phi_P$  by offering a conditional guarantee.

As in section 2.2, the agent optimally chooses between:

- ✓ a safe debt policy at level  $d_M = (1 - \alpha)\phi_P$ , which never leads to default, and
- ✓ a risky debt policy at level  $\Phi_A + \phi_P$ , which results in default with probability  $1 - \alpha$ .

The low-debt policy is less attractive than under symmetric information between  $P$  and

$A$ , while the attractiveness of the high-debt policy is unchanged. The agent's utility is

$$U_A^{**} = \max \left\{ R(1 - \alpha)\phi_P + \alpha y; \right. \\ \left. R\alpha \left( \Phi_A + \phi_P \right) + \alpha(y - \Phi_A) - (1 - \alpha)\Phi_A \right\}$$

and so the agent chooses the risky policy if and only if

$$R \geq R^{**} \equiv \frac{\Phi_A}{\alpha\Phi_A - (1 - 2\alpha)\phi_P}$$

(with the convention that  $R^{**} = +\infty$  if  $\alpha\Phi_A \leq (1 - 2\alpha)\phi_P$ ). Note that  $R^* \geq R^{**}$  if and only if  $(1 - \alpha)\Phi_A - (2\alpha - 1)\phi_P \geq 0$ . The principal's utility is

$$U_P^{**} = \begin{cases} -\phi_P & \text{if } R \geq R^{**} \\ -(1 - \alpha)\phi_P & \text{if } R < R^{**} \end{cases}$$

*Optimal contract.*

Like in the case of *laissez faire*, the analysis is remarkably similar to that of symmetric information between the principal and the agent. We obtain an upper bound on the agent's expected utility and show that this upper bound can indeed be implemented.

The twist with respect to the analysis of Section 2.3 is that some default must be induced in the bad state in order to induce repayment in the good state (Gale-Hellwig 1985; Townsend 1979). Let  $x$  denote the probability of default in the bad state. Then incentive compatibility requires that

$$y - d^G \geq y - x\Phi_A \quad \text{or} \\ d^G \leq x\Phi_A.$$

The modified program is then (with the notation of Section 2.3)

$$U_A = Rb + \alpha(y - d^G) + (1 - \alpha)(-d^B - x\Phi_A)$$

where

$$b = b_M + b_P,$$

the participation constraints are satisfied:

$$-b_P + \alpha d_P^G + (1 - \alpha)(d_P^B - x\phi_P) \geq U_P^{**} \\ -b_M + \alpha d_M^G + (1 - \alpha)d_M^B \geq 0$$



and the incentive constraints are satisfied:

$$d^G \leq x\Phi_A$$

$$d^B \leq 0.$$

We ignore the principal's incentive constraints (they will be satisfied in the implementation). Substituting, one obtains:

$$U_A \leq \max_{\{d^G \leq x\Phi_A, d^B \leq 0\}} \left\{ R \left[ \alpha d^G + (1 - \alpha)(d^B - x\phi_P) - U_P^{**} \right] + \alpha(y - d^G) + (1 - \alpha)(-d^B - x\Phi_A) \right\}.$$

So, provided that  $R \geq 1$ ,  $d^B = 0$  and  $d^G = x\Phi_A$ . Furthermore, letting

$$R^{***} \equiv \frac{\Phi_A}{\alpha\Phi_A - (1 - \alpha)\phi_P} > R^{**},$$

$$x^* = \begin{cases} 0 & \text{if } R < R^{***} \\ 1 & \text{if } R \geq R^{***}. \end{cases}$$

The implementation of the optimal allocation is described in the following proposition.

**Proposition 6 (asymmetric information).** *When the principal does not observe the agent's income realization and  $R > 1$ ,*

(i) *Laissez faire. The agent picks a high-debt policy ( $d_M = \Phi_A + \phi_P$ ) when  $R \geq R^{**}$  and a low-debt one ( $d_M = (1 - \alpha)\phi_P$ ) if  $R < R^{**}$ . Default occurs in the bad state under the high-debt policy. The principal's expected utility is  $U_P^{**} = -\phi_P$  if  $R \geq R^{**}$  and  $-(1 - \alpha)\phi_P$  if  $R < R^{**}$ .*

(ii) *Optimal contracting. The optimal contract between the principal and the agent can be implemented as follows:*

- ✓ *If  $R < R^{***}$ , the principal pays the agent  $b_P = -U_P^{**}$  for not taking on any liability.*
- ✓ *If  $R \geq R^{***}$ , the agent takes on liability  $d_M = \Phi_A$  and defaults in the bad state. The principal enforces the debt ceiling  $d_M$ , transfers to the agent  $b_P = \alpha\phi_P$  at date 1, and takes no debt claim in exchange.*

## ONLINE APPENDIX

### Appendix 4: Proof of Proposition 3 (laissez-faire under the veil of ignorance)

This appendix solves for the laissez-faire equilibrium when countries are ex-ante symmetrical. Like in the asymmetric case we assume that each country ex post offers some conditional support toward the repayment of the other country's sovereign debt after the state of nature is realized (the outcome is qualitatively the same if the two countries negotiate instead). At date 2, country  $i$  owes  $d_M^i$  to the private international capital market.

#### *Ex-post behavior*

(i) If  $k = 0$  (both are in distress), no support can be brought and so country  $i$  default unless  $d_M^i = 0$ .

(ii) If  $k = 1$  (say, country  $i$  is intact and country  $j$  is in distress), then country  $i$  takes two independent decisions:

- ✓ repay its own debt, which it does iff  $d_M^i \leq \Phi$ ;
- ✓ rescue country  $j$ , which it does iff  $d_M^i \leq \phi$ .

(iii) If  $k = 2$  (both countries are intact), debt repayment and bailout decisions are again separable. Country  $i$  rescues country  $j$  iff  $\Phi \leq d_M^i \leq \Phi + \phi$ , by contributing  $d_M^i - \Phi$ . Outside this range, country  $i$  does not intervene and so country  $j$  defaults iff  $d_M^j > \Phi + \phi$ . Symmetrically, country  $i$  defaults iff  $d_M^i > \Phi + \phi$ . Interestingly, both countries may bring each other support so as to prevent negative externalities.

To sum up, the ex-post behavior is similar to that in the asymmetric case. Country  $i$ 's optimal debt level can only take one of four values:  $d_M^i \in \{0, \phi, \Phi, \Phi + \phi\}$ .

#### *Ex-ante borrowing equilibrium*

Let  $U(d_M^i, d_M^j)$  denote country  $i$ 's total utility when it chooses liability  $d_M^i$  and the other country chooses liability  $d_M^j$ .

#### (i) *No borrowing equilibrium*

In the no-borrowing equilibrium, each country obtains

$$U(0, 0) = \alpha y.$$

For this to be an equilibrium, and from our earlier remark on optimal strategies, it must be the case that

$$U(0,0) \geq \max \{U(\phi,0), U(\Phi,0), U(\Phi + \phi,0)\}$$

where

$$\begin{aligned} U(\phi,0) &= R[(1 - p_0)\phi] + (p_2 + \frac{p_1}{2})(y - \phi) - p_0\Phi \\ &= \alpha y + R(1 - p_0)\phi - \alpha\phi - p_0\Phi, \end{aligned}$$

$$U(\Phi,0) = R[\alpha\Phi] + \alpha(y - \Phi) - (1 - \alpha)\Phi,$$

and

$$\begin{aligned} U(\Phi + \phi,0) &= R[p_2(\Phi + \phi)] + p_2(y - \Phi) + \frac{p_1}{2}(y - \Phi) + \frac{p_1}{2}(-\Phi) + p_0(-\Phi). \\ &= \alpha y + Rp_2(\Phi + \phi) - \Phi \end{aligned}$$

And, so the no-borrowing equilibrium exists iff

$$[R(1 - p_0) - \alpha]r \leq p_0 \quad (14)$$

$$R\alpha \leq 1 \quad (15)$$

and

$$Rp_2r \leq 1 - Rp_2. \quad (16)$$

(ii) *Low-debt equilibrium*

In a low-debt equilibrium, each country has utility:

$$\begin{aligned} U(\phi,\phi) &= R[(1 - p_0)\phi] + p_2(y - \phi) + \frac{p_1}{2}(y - 2\phi) + \frac{p_1}{2}(0) - p_0(\Phi + \phi) \\ &= \alpha y + R(1 - p_0)\phi - \phi - p_0\Phi. \end{aligned}$$

The equilibrium condition is

$$U(\phi,\phi) \geq \max \{U(0,\phi), U(\Phi,\phi), U(\Phi + \phi,\phi)\},$$

where

$$\begin{aligned} U(0,\phi) &= p_2y + \frac{p_1}{2}(y - \phi) + \frac{p_1}{2}(0) - p_0\phi \\ &= \alpha y - (1 - \alpha)\phi, \end{aligned}$$

$$U(\Phi, \phi) = R[\alpha\Phi] + p_2(y - \Phi) + \frac{p_1}{2}(y - \Phi - \phi) + \frac{p_1}{2}(-\Phi) + p_0(-\Phi - \phi),$$

and

$$\begin{aligned} U(\Phi + \phi, \phi) &= R[p_2(\Phi + \phi)] + p_2(y - \Phi) + \frac{p_1}{2}(y - \Phi - \phi) + \frac{p_1}{2}(-\Phi) + p_0(-\Phi - \phi) \\ &= \alpha y + Rp_2(\Phi + \phi) - \Phi - (1 - \alpha)\phi. \end{aligned}$$

The low-debt equilibrium exists iff

$$[R(1 - p_0) - \alpha]r \geq p_0, \quad (17)$$

$$(1 - p_0) - \alpha r \geq R[\alpha - (1 - p_0)r] \quad (18)$$

and

$$(Rp_1 - \alpha)r \geq (R - 1)p_2 - p_1 \quad (19)$$

(iii) *Medium-debt equilibrium*

In a medium-debt equilibrium

$$U(\Phi, \Phi) = R[\alpha\Phi] + p_2(y - \Phi) + \frac{p_1}{2}(y - \Phi - \phi) + \frac{p_1}{2}(-\Phi) + p_0(-\Phi - \phi),$$

The equilibrium condition is

$$U(\Phi, \Phi) \geq \max \{U(0, \Phi), U(\phi, \Phi), U(\Phi + \phi, \Phi)\},$$

which translates into:

$$R\alpha \geq 1 \quad (20)$$

$$(1 - p_0) - \alpha r \leq R[\alpha - (1 - p_0)r] \quad (21)$$

and

$$p_2(1 + r) \leq \alpha. \quad (22)$$

(iv) *High-debt equilibrium*

In a high-debt equilibrium,

$$\begin{aligned} U(\Phi + \phi, \Phi + \phi) &= R[p_2(\Phi + \phi)] + \alpha(y - \Phi - \phi) + (1 - \alpha)(-\Phi - \phi) \\ &= \alpha y + (Rp_2 - 1)(\Phi + \phi). \end{aligned}$$

Furthermore

$$U(0, \Phi + \phi) = \alpha y - \phi,$$

$$U(\Phi, \Phi + \phi) = R(\alpha\Phi) + \alpha y - [\Phi + \phi],$$

and

$$U(\phi, \Phi + \phi) = R[(p_2 + p_1)\phi] + p_2(y - 2\phi) + \frac{p_1}{2}(y - 2\phi) + \frac{p_1}{2}(-\phi) + p_0(-\Phi - \phi).$$

And so the high-debt equilibrium exists iff:

$$Rp_2r \geq 1 - Rp_2, \quad (23)$$

$$p_2(1 + r) \geq \alpha, \quad (24)$$

and

$$(Rp_1 - \alpha)r \leq (R - 1)p_2 - p_1. \quad (25)$$

The equilibrium exists and is (generically) unique.