DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

Disaster Risk and Preference Shifts in a New Keynesian Model

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Working Paper 614

December 2016

ABSTRACT

In RBC models, "disaster risk shocks" reproduce countercyclical risk premia but generate an increase in consumption along the recession and asset price fall, through their effects on agents' preferences (Gourio, 2012). This paper offers a solution to this puzzle by developing a New Keynesian model with such a small but time-varying probability of "disaster". We show that price stickiness, combined with an elasticity of intertemporal substitution smaller than unity, restores procyclical consumption and wages, while preserving countercyclical risk premia, in response to disaster risk shocks. The mechanism then provides a rationale for discount factor first- and second-moment ("uncertainty") shocks.

Keywords: disaster risk, rare events, uncertainty, Epstein-Zin-Weil preferences, asset pricing, DSGE models, New Keynesian models, business cycles, risk premium.
JEL classification: D81, D90, E20, E31, E32, E44, G12, Q54.

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NON-TECHNICAL SUMMARY[3]

During the 2007-2009 financial crisis, risk premia increased significantly in advanced economies and central banks implemented various unconventional monetary policies in order to reduce them. However, generating realistic risk premia along with expected macroeconomic variations in response to shocks is particularly challenging in standard economic models. We show that incorporating a time-varying "disaster risk" in a New Keynesian model allows to reproduce countercyclical risk premia together with the expected co-movement of macroeconomic variables, in particular consumption, investment, and wages, along the recession. In that respect, we improve some of the macroeconomic predictions from real business cycle (RBC) analysis of disaster risk and generalize the mechanism into a policy-friendly framework.

A central feature of our model relies on the likelihood that rare disasters hit the economy. Rare disasters are large adverse events associated with a low probability, such as great recessions, wars, terrorist attacks or natural catastrophes. They can lead to important declines in production, consumption, capital, or productivity. Rietz (1988) and Barro (2006) showed that accounting for these events can explain the high level of risk premia observed in the data, incompatible with previous standard asset pricing models (equity premium puzzle). Furthermore, Gabaix (2011), Gourio (2012), and Wachter (2013) developed dynamic models which allow the probability of disasters to be time-varying. In particular, Gourio (2012) introduced a small time-varying probability of disaster, defined as an event that destroys a large share of the existing capital stock and productivity, into RBC model. An interesting implication is that an increase in the probability of disaster, without occurrence of the disaster itself, suffices to trigger a recession and replicate key asset pricing regularities.

However, this literature faces two limitations. First, an increase in disaster risk generates a recession and a drop in stock prices, but it also increases consumption which seems counterfactual. Second, the model generates output and investment drops only under the assumption that the agents have a very large adaptability to substitute consumption across periods in response to shocks. In other terms, the elasticity of intertemporal substitution (EIS) parameter needs to be set to a value strictly greater than unity. Should the EIS be lower than unity, the results are completely reversed. In particular, a rise in the probability of disaster would then generate a boom in output and investment. Empirical evidence on the EIS is mixed, yet values below unity are realistic and conventionally adopted in macroeconomic calibrations. Therefore, such a contrasting response of output from changes in disaster risk at the unity threshold seems particularly puzzling.

In this paper, we introduce a small time-varying probability of disaster à la Gourio (2012) into an otherwise standard New Keynesian model. To the best of our knowledge, we are the first to do so.

[3] We thank George-Marios Angeletos, Guido Ascari, Pierpaolo Benigno, Nick Bloom, Ambrogio Cesa-Bianchi, Fabrice Collard, Luca Dedola, Martin Ellison, Francesco Furlanetto, Xavier Gabaix, François Gourio, Oren Levintal, Vivien Lewis, Julien Matheron, Antonio Mele, Afrasiab Mirza, Stefan Niemann, Salvatore Nisticò, Juan Carlos Parra-Alvarez, Julien Penasse, Johannes Pfeifer, Antti Ripatti, Kjetil Storesletten, Fabien Tripier, Philippe Weil, Raf Wouters, and Francesco Zanetti, as well as participants at numerous seminars and conferences, for fruitful discussions. M. Isoré is grateful to the Yrjö Jahnsson Foundation for financial support on this project.

The main result of our paper consists in demonstrating that price stickiness, combined with an EIS below unity, is able to restore procyclicality of the main macroeconomic quantities. In particular, we show that agents become more "patient" in response to disaster risk shocks. Their propensity to save increases while their consumption declines, such that deflation follows. Yet, higher savings do not immediately translate into higher investment, such that output also drops. As a result, an increase in disaster risk leads to simultaneous falls in investment, consumption, prices, and output as capital becomes riskier. As for asset prices, we observe a 'flight-to-quality' effect that is visible through the drop in the risk-free rate when the disaster risk shock hits, as well as the increase in the risk premium. Hence, while improving Gourio (2012)'s predictions for the macroeconomic variables, we preserve the strength of his mechanism for accounting for the countercyclicality of the risk premia.

The intuition works as follows. Let us consider for instance the case of an EIS below unity. An increase in disaster risk decreases agents' propensity to consume such that savings go up. Because of price stickiness, firms cannot deflate their good prices as much as they would like to face reduced consumption and thus demand fewer factors of production, capital and labor. Therefore, despite precautionary motives, all macroeconomic quantities go down with output. Since the return on capital is riskier after the increased probability of disaster, the risk premium is countercyclical. Overall, we thus show that introducing a time-varying disaster risk à la Gourio (2012) into a full-fleshed New Keynesian model is critical, not just to enrich the macroeconomic setting and spectrum of potential policy analysis, but because it literally conditions most of the qualitative effects associated with a change in disaster risk, for a given value of the EIS.

Following Gourio (2012)'s solution method, the effects of disaster risk on preferences are embedded into agents' discount factor. However, this discount factor goes up in our model, as agents' patience increases when the EIS is below unity, whereas it was going down in Gourio (2012)'s. Combined with price stickiness, the responses of aggregate macroeconomic and financial variables to a disaster risk shock in turn resemble the responses to a preference shock (Christiano et al. (2011)) and a second-moment "uncertainty" shock (Basu and Bundick (2015)). In that sense, we show how Gourio (2012)'s mechanism of disaster risk can be conciliated with these other shocks from the New Keynesian literature, recently found to drive the economy into the zero lower bound and secular stagnation. We thus provide a milestone model which could be used for further investigation of monetary policy responses to changes in the probability of disaster events.

RÉSUMÉ : Risque de désastre et changement de préférences dans le modèle néo-keynésien

Dans les modèles de cycles réels, un choc de « risque de désastre » permet bien de reproduire la contra-cyclicité des primes de risque mais génère une hausse de la consommation, simultanément à une récession et une chute de prix des actifs, via ses effets sur les préférences des agents (Gourio, 2012). Cet article répond à ce problème théorique en développant un modèle néo-keynésien comprenant une telle probabilité de « désastre » faible mais variable dans le temps. Nous montrons que la rigidité des prix, associée à une élasticité de substitution inter-temporelle inférieure à un, rétablit la pro-cyclicité de la consommation et des salaires tout en préservant la contra-cyclicité des primes de risque en réponse aux chocs de risque de désastre. Le mécanisme fournit alors une source possible de chocs sur les premier et second moments du facteur d'escompte, respectivement dits chocs de préférences et d'« incertitude ».

Mots-clés : risque de désastre, événements rares, incertitude, préférences Epstein-Zin-Weil, évaluation des actifs financiers, modèles DSGE, modèles néo-keynésiens, cycles économiques, prime de risque.

1 Introduction

Recent years have seen renewed interest in the economic impact of 'rare events'. In particular, Gabaix (2011, 2012) and Gourio (2012) have introduced a small but time-varying probability of 'disaster', defined as an event that destroys a large share of the existing capital stock and productivity, into real business cycle (RBC) models. The main result is that an increase in the probability of disaster, without occurrence of the disaster itself, suffices to trigger a recession and replicate key asset pricing regularities.

However, this literature faces two limitations. First, an increase in disaster risk generates a recession and a drop in stock prices, but it also increases consumption. Yet, recent estimations from option price tails document that disaster risk tends to increase in periods of financial distress and recessions (Siriwardane (2015)), while such episodes are themselves correlated with contemporaneous declines in consumption (see Albuquerque et al. (2015) for e.g). Second, the model predictions for output and asset pricing rely on the elasticity of intertemporal substitution (EIS) being set to a value strictly greater than unity, but are completely reversed otherwise.[1] In particular, a rise in the probability of disaster generates a boom in output should the EIS be lower than unity, everthing else equal. Empirical evidence on the EIS is mixed, yet values below unity are realistic and conventionally adopted in macroeconomic calibrations, whether the models feature Epstein-Zin-Weil preferences or not.[2] Therefore, such a contrasting response of output from changes in disaster risk at the unity threshold seems particularly puzzling.

In order to address these caveats, we introduce a small time-varying probability of disaster à la Gourio (2012) into an otherwise standard New Keynesian model. To the best of our knowledge, we are the first to do so.[3] The contribution is threefold. First, we nest Gourio (2012)'s mechanism of

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[1]Barro (2009) shows that the aggregate stock market declines with the probability of disaster only when the EIS is greater than one with Epstein-Zin-Weil preferences. Gourio (2012)'s recessionary effects of the disaster risk rely on the same condition.

[2]See Section 4 for further related discussion.

[3]Two previous attempts of disaster risk into a New Keynesian model include Isoré and Szczerbowicz (2013), considering the capital depreciation effect of disaster risk only, and Brede (2013) where the 'disaster state' is permanent and deterministic, i.e the economy entering a disaster state stays there forever. In contrast, we keep the essence of Gourio (2012) in considering disaster risk as a time-varying source of uncertainty here. Finally, despite a title close to ours, Andreasen (2012) studies skewed shock distributions in a DSGE model, which quite differ from the formalization of disaster risk we adopt here.

disaster risk into a New Keynesian setup and shed light on the critical role of the EIS in driving the sign of macroeconomic responses to a disaster risk shock. In particular, when prices are flexible, we obtain an output boom with an EIS below unity, and a recession together with an increase in consumption, wages, and prices with an EIS above unity. Second, we show that price stickiness provides a simple solution to this puzzle. Indeed, it allows to conciliate the recessionary effects of disaster risk with an EIS below unity, as well as to restore the expected comovements of other macroeconomic and asset pricing variables. In that respect, we improve some of macroeconomic predictions from Gourio (2012)'s mechanism of disaster risk and generalize the analysis to a policy-friendly framework. Third, we conciliate this mechanism with the New Keynesian literature on discount factor shocks. Changes in disaster risk produce a mix of first- and second-moment effects on agents' (endogenized) discount factor. Yet, in Gourio (2012)'s RBC model, they both contribute to agents' impatience, such that lower savings then cause the recession. In contrast, increased patience drives the recession via lower consumption levels, in our setup. This result is very much in line with the New Keynesian literatures on (*i*) exogenous 'preference shocks' defined as shocks to the level of the discount factor (Smets and Wouters, 2003, Christiano et al., 2011, for e.g), and (*ii*) 'uncertainty shocks' defined as shocks to the volatility of the discount factor (Basu and Bundick, 2015). Therefore, it supports the interpretation of disaster risk as a potential source of uncertainty in future economic conditions.

The intuition works as follows. The occurrence of a disaster would destroy parts of the capital stock and productivity.[4] The probability of such an event going up suffices to create a mix of depreciation and uncertainty effects on the future return on capital. Agents' response depends on the relative valuation of substitution and income effects, which is determined by the value of the EIS. Indeed, as known since Leland (1968) and Sandmo (1970), an increase in interest rate risk increases (decreases) agents' propensity to consume (save), and thus reduces savings (consumption), if and only if the EIS is larger than 1.[5] In a RBC setup, the response of savings drives investment, and therefore

[4]Disasters can be alternatively modeled as large declines in consumption (Barro (2006), Wachter (2013)). Yet, in models including a production sector, they are most often represented by a drop in capital and/or productivity (Barro (2009), Gabaix (2011), Gourio (2012)). We follow Gourio (2012) so as to nest his results as a particular case of our model.

[5]Weil (1990) shows that a large EIS implies that the elasticity of savings to a 'certainty-

output, despite an opposite response in consumption. Hence, with an EIS above unity, agents invest less when disaster risk goes up, and a recession ensues. Under some assumptions, Gourio (2012) captures this mechanism by endogenizing the discount factor as a negative function of disaster risk when the EIS is above unity. Should the EIS be below unity, the exact opposite holds, i.e the economy is booming as disaster risk goes up.

The additional presence of sticky prices does not alter the effect of the EIS on agents' willingness to consume/save. However, the aggregate dynamics become more impacted by the demand side (consumption) than the supply side (savings) of the economy. Take for instance the case of an EIS below unity such that consumption goes down and savings go up in response to an increase in disaster risk. Because of price stickiness, firms cannot deflate as much as they would like to face reduced consumption and thus demand less factors of production, capital and labor. Therefore, despite precautionary motives, all quantities co-move and output is driven down. Since the return on capital is riskier, the risk premium remains countercyclical.[6] Overall, we thus show that introducing a time-varying disaster risk à la Gourio (2012) into a full-fleshed New Keynesian model is critical, not just to enrich the macroeconomic setting and spectrum of potential policy analysis, but because it literally conditions most of the qualitative effects associated with a change in disaster risk, for a given value of the EIS.

The remainder is as follows. The rest of the introduction briefly reviews the literature and our related contribution. Section (2) presents the model. Section (3) describes the solution method and emphasizes the analytical role of the EIS in driving the qualitative results from disaster risk shocks. Section (4) discusses the calibration and steady-state values. In particular, Tallarini (2000)'s "observational equivalence" holds in the presence of disaster risk when the EIS is equal to unity, but not for any other value above or below that threshold. Section (5) simulates a positive shock on the probability of disaster. We find that the combination of sticky prices and an EIS below unity is able to generate a recession, jointly with a drop in consumption,

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equivalent' interest rate is positive, i.e savings decrease in the aggregate interest rate risk. On the contrary, a small EIS implies that savings go up with interest rate risk.

[6]Basu and Bundick (2015) show that "uncertainty" shocks (as exogenous second-moment discount factor shocks) also generate very different qualitative economic responses depending on price stickiness/flexibility. In particular, they argue that only stickiness generates a positive co-movement of consumption and investment, as observed in the data.

investment, hours and wages, a deflation, and an increase in the risk premium. We also show that all other combinations of EIS below/above unity and price flexibility/stickiness generate either a boom or a negative macroeconomic comovements along the recession. Section (6) concludes.

Literature review.

Our paper is related to two main strands of literature. First, the literature on 'rare events', sometimes called economic 'disasters', which emerged as a response to the equity premium puzzle (Mehra and Prescott, 1985). Indeed, Rietz (1988), Barro (2006), and Barro and Ursúa (2008) introduced a small constant probability of an economic disaster into endowment economies and showed it able to replicate the size of the risk premium.[7] Yet, the volatility and countercyclicality of the risk premium, as well as the stock price-dividend ratio, return predictability and the bond risk premia, were still not accounted for.[8] Therefore, Gabaix (2008) first considered the probability of disaster to be *time-varying* into an endowment economy, before Gabaix (2011, 2012) and Gourio (2012, 2013) further introduced it into RBC models.[9]

Although successful in reproducing the size, volatility, and countercyclicality of risk premia, these RBC models still leave room for improvement in the responses of the macroeconomic variables to disaster risk shocks. Indeed, macroeconomic quantities are unresponsive to changes in disaster risk in Gabaix (2012), in line with Tallarini (2000)'s "observational equivalence" where only asset prices are affected by aggregate risks. Gourio (2012) shows that this result holds only for an EIS strictly equal to unity, but not for any other value as disaster risk shocks then impact agents' preferences and subsequently macroeconomic variables. However, some responses seem counterfactual, such as consumption in particular. Empirical analyses on the effects of disaster risk variations are quite new but tend to suggest that consumption decreases and becomes more volatile with disaster risk (Marfè and Penasse, 2016, for e.g).[10] More evidence tend to associate estimated time series of dis-

[7]Numerous alternative solutions have been provided to this puzzle but we choose not to enumerate them here for concision as this is not the main focus of our paper.

[8]In Barro and Ursúa (2008), the price-dividend ratio and the risk-free rate are constant.

[9]An Epstein-Zin-Weil (EZW) utility function is a necessary ingredient, that we also adopt here, yet not sufficient to address the equity premium puzzle alone. Especially, Weil (1989) must set the risk aversion coefficient to 45 and the EIS to 0.1 in order to obtain a reasonable match with the data. Yet, it matters for dissociating the degree of risk aversion from the value of the EIS. Here as in Weil (1990), risk aversion will impact the size of the macroeconomic responses to aggregate risks while the EIS will determine their sign.

[10]We are not aware of any opposite evidence as of to date.

aster risk to periods of recessions and stock price falls (Siriwardane, 2015), while such episodes are commonly associated with lower consumption.[11] We thus build on Gourio (2012)'s approach here but try to restore the expected sign of some quantities, consumption, wages, and output prices, in particular.

Second, our paper is related to literatures on preference and uncertainty shocks. Indeed, following Gourio (2012)'s approach, disaster risk creates a mix of first- and second-moment effects on agents' discount factor in the model. Preference shocks have also been praised as an early solution to capture asset pricing phenomena, such as the equity premium, the bond term premium, and the weak correlation between stock returns and fundamentals, that supply-driven shocks alone could not (Campbell and Ammer (1993), Cochrane (2011)). The intuition is that they affect agents' demand for riskfree assets, and thus generate a good fit for riskfree rate variations, independently of cash flows (Campbell (1986), Schorfheide et al. (2014)).

However, these models, as well as Gourio (2012), find that a *negative* preference shock, i.e a decrease in agents' discount factor, is associated with recessions and stock price falls. This sharply contrasts with the New Keynesian literature where similar effects stem from a *positive* preference shock. Indeed, a higher degree of patience drive agents' demand for risky assets down and favor 'fly-to-quality' effects.[12] Thereby, positive preference shocks are very successful in driving the economy to the zero lower bound (ZLB) on the nominal policy rate in New Keynesian setups (Eggertsson and Woodford (2003), Christiano et al. (2011), Erceg and Linde (2012), Eggertsson et al. (2014)).[13] Empirical variance decompositions support positive preference shocks as a major determinant of the nominal interest rate (Smets and Wouters (2003) and Ireland (2004)). Our model suggests that Gourio (2012)'s mechanism of disaster risk, which endogenizes preference shifts, can be seen as a rationale for *positive* – rather than *negative* – preference shocks when the combination of an EIS below unity and sticky prices is at play. We

[11] Siriwardane (2015) estimates time series of a"risk-neutral" probability of disaster from information contained in the cross-section of option and equity returns. He finds it strongly (negatively) correlated to periods of recessions and lower stock prices .

[12] Smets and Wouters (2003)'s DSGE model of the euro area include such a preference shock for instance. Smets and Wouters (2007), estimated on US data, consider a slightly different shock which directly affects agents' preference for riskfree assets (Fisher, 2015).

[13] As the discount factor increases, agents' propensity to consume decreases, putting downward pressure on real factor prices, real marginal cost, and inflation. In turn, the interest rate diminishes to reduce deflationary pressures.

thus restore its compatibility with that branch of the literature.

Finally, second-moment discount factor shocks, sometimes referred to as 'uncertainty' shocks, matter. Bloom (2009) first stressed the sharp recessionary effects of uncertainty shocks, formalized as productivity volatility shocks. More recently, Basu and Bundick (2015) found a simultaneous decline in output, consumption, investment, and hours worked, in response to second-moment discount factor shocks when price stickiness is at play into a New Keynesian setup.[14] [15] Our responses resemble theirs but stem from an endogenous mechanism of preference shifts induced by disaster risk. In that sense, changes in disaster risk can be seen as a rationale for uncertainty shocks. This idea is supported empirically by Baker and Bloom (2013), for instance, who use rare events, such as natural disasters, terrorist attacks, political coups d'état and revolutions to instrument for changes in the level and volatility of stock-market returns. They argue that some shocks, like natural disasters, lead primarily to a change in stock-market levels (first-moment shocks), while other shocks like coups d'état lead mainly to changes in stock-market volatility (second-moment shocks).

2 Model

2.1 Households

Households derive utility from consumption and leisure, accumulate capital and bonds, own monopolistic competition firms to which they rent capital and labor force, earn profits, and pay lump-sum taxes.

Households' Epstein-Zin-Weil preferences are given by

$$\tilde{V}_t = \left[\left[C_t \left(1 - L_t\right)^{\varpi} \right]^{1-\psi} + \beta_0 \left(E_t \tilde{V}_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \right]^{\frac{1}{1-\psi}} \tag{1}$$

[14] In a similar spirit, Leduc and Liu (2016) find that nominal rigidities amplify the effect of uncertainty shocks on the unemployment rate through declines in aggregate demand.

[15] Bloom (2009) and Basu and Bundick (2015) consider the VIX and alternative measures of uncertainty to support their predictions with a VAR analysis. We are unfortunately not able to reproduce it with estimated measures of disaster risk such as Marfè and Penasse (2015) or Siriwardane (2015) since those series are very new and not available yet. However, in a recent discussion of our paper (Banque de France, October 2016), Julien Penasse showed that consumption indeed declines in response to one standard deviation disaster risk shock using their estimated series of disaster risk (Marfè and Penasse, 2016). This discussion is available on request.

where C is consumption, L labor supply, β_0 the discount factor, γ the coefficient of risk aversion, and $1/\tilde{\psi}$ the elasticity of intertemporal substitution (EIS) with $\tilde{\psi} = 1 - (1 + \varpi)(1 - \psi)$. Capital accumulates as

$$K_{t+1} = \left[(1 - \delta_t) K_t + S\left(\frac{I_t}{K_t}\right) K_t \right] e^{x_{t+1} \ln(1-\Delta)} \tag{2}$$

where, within the brackets, K stands for capital, I for investment, $\delta_t = \delta_0 u_t^\eta$ for the depreciation rate of capital as a function of the utilization rate u of capital, and where $S\left(\frac{I_t}{K_t}\right) = \frac{I_t}{K_t} - \frac{\tau}{2}\left(\frac{I_t}{K_t} - \frac{\bar{I}}{\bar{K}}\right)^2$ are convex capital adjustment costs. This bracketed part is standard in the New Keynesian literature. In addition, x is an indicator variable capturing the occurrence of a "disaster". Specifically, $x_{t+1} = 1$ with probability θ_t, in which case a large share Δ of the existing capital stock is destroyed, otherwise $x_{t+1} = 0$.[16]

The probability θ_t of disaster is itself small (0.009 in steady state) but time-varying, following a first-order autoregressive process as

$$\log \theta_t = (1 - \rho_\theta) \log \bar{\theta} + \rho_\theta \log \theta_{t-1} + \sigma_\theta \varepsilon_{\theta_t} \tag{3}$$

where $\bar{\theta}$ is the mean, ρ_θ the persistence, and ε_θ i.i.d innovations.[17]

Finally, households' budget constraint reads as

$$C_t + I_t + T_t + \frac{B_{t+1}}{p_t} \leq \frac{W_t}{p_t} L_t + \frac{P_t^k}{p_t} u_t K_t + D_t + \frac{B_t(1 + r_{t-1})}{p_t} e^{x_t \ln(1-\Delta)} \tag{4}$$

where W denotes the (nominal) wage rate, p the good price, B one-period bonds issued by the public authority, r the corresponding interest rate, P_t^k is the (nominal) rental rate of capital, D the (real) dividends from monopolistic firms' (real) profits, and T lump-sum taxes to the public authority.

As Gabaix (2012) and Gourio (2012), we assume that bonds are also subject to the disaster risk. A general justification is that sovereign debt

[16] Gourio (2012) also considers the size Δ of disasters as a random variable, but we abstract from this feature as it is unessential to our results. Also note that this specification of capital accumulation is reminiscent of Gertler and Kiyotaki (2010)' 'capital quality shock', yet the latter does not affect producivity growth, unlike the disaster risk shock.

[17] These parameters are calibrated according to empirical estimations of disaster risk (Section 4). Our qualitative results are yet essentially insensitive to these values, including when persistence in disaster risk is nil (Section 5). Also, although nothing technically prevents the probability of disaster to exceed unity with perturbation methods (featuring normally distributed innovations), the calibrated mean and variance are so low that this is extremely unlikely. In our impulse analysis (Section 5), the bound is clearly not met.

can indeed be risky during tail events in the sense that it becomes subject to partial default, as we have observed for Greece in the last financial crisis, Argentina in the early 2000s, and UK in the Great Depression, as for a few examples. Conditional on no disaster, bonds are however riskfree, unlike capital. A more specific reason for the presence of risky bonds is a technical trick making the problem particularly easy to solve. Indeed, when the event destruction size Δ is the same for both assets and productivity, then the *detrended* system will not be directly impacted by the *large* disaster event (x) itself but only by the *small* probability of disaster (θ), which is our variable of interest, as explained more extensively in Section 4.[18]

Overall, households maximize their utility (1) subject to their capital accumulation (2) and budget (4) constraints, given the time-varying disaster risk (3). Optimality conditions to this problem, given in Appendix, still contain the presence of the disaster event (x) at this stage. However, only the disaster probability (θ) will remain in the detrended equilibrium system, following Gourio (2012)'s solution method described further in Section 4.

2.2 Asset pricing

From the Epstein-Zin-Weil preferences defined above (1), the (real) stochastic discount factor can be calculated as

$$Q_{t,t+1} \equiv \frac{\partial \tilde{V}_t / \partial C_{t+1}}{\partial \tilde{V}_t / \partial C_t} = \beta_0 \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varpi(1-\psi)} \frac{V_{t+1}^{-\chi}}{\left(E_t V_{t+1}^{1-\chi} \right)^{\frac{-\chi}{1-\chi}}} \tag{5}$$

Then, asset pricing orthogonality conditions give the *riskfree rate*, R^f, as $E_t\left(Q_{t,t+1}\right) = 1/R^f_{t+1}$ known at time t, and the (real) *rate of return on capital*, $R^{k,real}_{t+1}$, such that $E_t\left(Q_{t,t+1} R^{k,real}_{t+1}\right)$ where the time-t expectation operator accounts, together with other possible shocks, for a disaster realization in $t+1$. Note that the riskfree rate defined here is not the yield on the one-period bonds which are again risky, but rather a "natural" (gross) interest

[18] This feature is not essential to our results but makes the solution tractable. One can relax it and use projection methods to solve for the model instead, but with qualitatively similar results as Gourio (2012) shows. Alternatively, one could as well take a stand on whether the economy is currently in a disaster ($x = 1$) regime or not ($x = 0$) and generate (regime-contingent) impulse response functions to changes in disaster risk.

rate. Thus, the *risk premium* on capital holdings can now be defined as

$$E_t(Risk\ premium_{t+1}) \equiv E_t\left(\frac{R^{k,real}_{t+1}}{R^f_{t+1}}\right)$$

which will be nil with a first-order approximation, constant with second order, and time-varying with third and higher orders.

2.3 Firms

The structure of production considered here is quite standard as for a New Keynesian model. However, it plays crucial role for our results: unlike Gourio (2012)'s centralized economy flexible-price model, the decentralized economy featuring monopolistic competition and sticky prices allows to obtain recessionary effects from a disaster risk shock when the EIS is smaller than unity. The nominal price friction makes the reponse of output affected mostly by the demand side (consumption) rather than the supply side (savings) of the economy. Thus, the drop in consumption associated with a rise in disaster risk when the EIS is below unity will generate here recession and deflation.

Firms are operating in two sectors, final good production and intermediate good production. The former market is competitive, while the latter is monopolistic. They are briefly described below, see Appendix for details.

2.3.1 Final good production

The final good is an aggregate of intermediate goods j as given by

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}}\, dj\right)^{\frac{\nu}{\nu-1}}$$

where ν is the elasticity of substitution among intermediate goods. Profit maximization gives a demand curve which is decreasing in the price of intermediate good j relative to the aggregate price index $(p_{j,t}/p_t)$ as

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

2.3.2 Intermediate sector

Intermediate sector firms use households' capital and labor to produce goods j, according to a Cobb-Douglas function with labor-augmenting productivity.

In each period, they optimize the quantities of factors they want to use, taking their prices as given, subject to the production function and the aggregate demand function at a given output price. Thet also set their price optimally at frequency determined by a constant Calvo probability.[19]

The intra-temporal problem (cost minimization problem) is thus

$$\min_{L_{j,t},\tilde{K}_{j,t}} \; W_t L_{j,t} + P_t^k \tilde{K}_{j,t}$$

$$\text{s.t.} \quad \tilde{K}_{j,t}^{\alpha}(z_t L_{j,t})^{1-\alpha} \geq \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

where W stands for the (non-detrended) nominal wage rate, P^k the capital rental rate, and $\tilde{K} \equiv uK$ the effective capital (with u the utilization rate of capital). The first-order conditions, expressed in detrended terms, are

$$(L_{j,t}:) \quad w_t = mc_{j,t}^{nom}(1-\alpha)\left(\frac{\tilde{k}_{j,t}}{L_{j,t}}\right)^{\alpha}$$

$$(\tilde{K}_{j,t}:) \quad P_t^k = mc_{j,t}^{nom}\alpha\left(\frac{\tilde{k}_{j,t}}{L_{j,t}}\right)^{\alpha-1}$$

in which the Lagrange multiplier denoted $mc_{j,t}^{nom}$ can be interpreted as the (nominal) marginal cost associated with an additional unit of capital or labor. Rearranging further gives an optimal capital to labor ratio which is the same for all intermediate firms in equilibrium.

Let's now consider the inter-temporal problem of a firm that gets to update its price in period t and wants to maximize the present-discounted value of future profits. Given the (real) profit flows that read as $\frac{p_{j,t}}{p_t}Y_{j,t} - mc_t^* Y_{j,t}$ and the demand function $Y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$, the maximization problem is

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s}\left(\left(\frac{p_{j,t}}{p_{t+s}}\right)^{1-\nu} Y_{t+s} - mc_{t+s}^*\left(\frac{p_{j,t}}{p_{t+s}}\right)^{-\nu} Y_{t+s}\right)$$

[19] It can be that firms adjust their price more frequently when disaster risk goes up. In Isoré and Szczerbowicz (2013), we considered both time- and state-dependent price-adjustment settings. In the latter case, the probability of not adjusting one firm's price can be thought of as a decreasing function of disaster risk. Yet, the results are not significantly different, such that we chose not to include this additional channel of disaster risk here.

where discounting includes both households' stochastic discount factor, $Q_{t,t+s}$, and the probability ζ^s that a price chosen at time t is still in effect at time s. After some simplification, the first-order condition is

$$p_{j,t}^* = \frac{\nu}{\nu - 1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu} Y_{t+s} mc_{t+s}^*}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s}}$$

which depends on aggregate variables only, so that $p_t^* = p_{j,t}^*$. Increases in this optimal price from one period to another will give us the reset inflation rate while increases in the current price level p_t defines the current inflation rate. (see Appendix for more details.)

2.4 Public authority

Bonds clears with public debt issued by a public authority which raises taxes from the households. The public authority also sets up the nominal interest rate on bonds following a Taylor type rule as

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) \left[\Phi_\pi (\pi_t - \bar{\pi}) + \Phi_Y (y_t - y^*) + r^* \right] \tag{6}$$

2.5 Equilibrium

The equilibrium consists of the optimality conditions and constraints of households, firms, and the public authority. They are reported in Section (A.1) of the Appendix.

3 Solution method

We here follow Gourio (2012) in detrending the equilibrium system by the productivity level z_t so as to eliminate the disaster event x_{t+1} itself, while keeping the disaster risk θ_t, in that system. This will not only bring some insightful analytical results on the impact of the EIS, but it will also allow us to use standard perturbation methods on the detrended system in order to simulate the impact of a (small) shock to the (small) probability of disaster θ_t. In the absence of any large event x itself, perturbation methods become accurate to account for the response of macroeconomic quantities to a disaster risk shock already from a first order approximation, and for the responses of the time-varying risk premia from the third order.[20]

This detrending method requires one additional assumption which is to assume that productivity is affected by disaster events, if any, in the same proportion as the assets (physical capital and bonds).[21] Specifically,

$$\frac{z_{t+1}}{z_t} = e^{\mu+\varepsilon_{z,t+1}+x_{t+1}\ln(1-\Delta)} \tag{7}$$

where μ is a technological trend and $\varepsilon_{z,t+1}$ are i.i.d normally distributed innovations with zero mean.

The rest of this Section describes the detrending of some key equations and stresses the intuition. The detrending of the full equilibrium system is relegated to the Appendix.

As emphasized in Gourio (2012), the law of motion for capital is straightforward. Indeed, using (7), (2) can be rewritten as

$$k_{t+1} = \frac{(1-\delta_t)k_t + S\left(\frac{i_t}{k_t}\right)k_t}{e^{\mu+\varepsilon_{z,t+1}}} \tag{8}$$

with $\delta_t = \delta_0 u_t^{\eta}$ and with $S\left(\frac{i_t}{k_t}\right) = \frac{i_t}{k_t} - \frac{\tau}{2}\left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}}\right)^2$, where a lower case letter denote a productivity-detrended variable (for e.g, $k_t = K_t/z_t$). Note that indeed, the disaster indicator x_{t+1} has now vanished from (8) as opposed to its non-detrended counterpart (2).

Another important effect of that detrending can be seen from the Epstein-Zin-Weil utility function. Indeed, denoting $v_t \equiv \left(\tilde{V}_t/z_t\right)^{1-\psi}$, (1) becomes

$$v_t = \left[c_t(1-L_t)^{\varpi}\right]^{1-\psi} + \beta(\theta)e^{(1-\psi)\mu}\left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{\frac{1-\gamma}{1-\psi}}\right]^{\frac{1-\psi}{1-\gamma}} \tag{9}$$

where

$$\beta(\theta) = \beta_0\left[1-\theta_t+\theta_t e^{(1-\gamma)\ln(1-\Delta)}\right]^{\frac{1-\psi}{1-\gamma}} \tag{10}$$

[20] Fernández-Villaverde and Levintal (2016) analyze the accuracy of perturbation and projections methods in models with disasters. However, the shock they consider is a disaster *size* shock, for which they find that perturbation methods are not accurate below the fifth order, which is not surprising as the size is potentially large. We are not subject to this criticism as long as we consider small deviations in a (small) disaster *probability*.

[21] Labor productivity may indeed decrease during financial crises (e.g Hughes and Saleheen, 2012), as well as during wars or natural disasters as people may find themselves not necessarily matched with jobs requiring their specific skills. Total factor productivity may also decrease as firms facing severe financing constraints may reduce their R&D expenditures (Millard and Nicolae, 2014). Gabaix (2012) and Gourio (2012)'s RBC models with disaster risk both assume that the destruction share is the same than for the assets.

is a time-varying pseudo-'discount factor' as a function of the time-varying disaster risk. The appealing feature of this method is that the whole dynamic effect of potential disasters is now captured by the presence of this time-varying probability in agents' discount factor. As discussed in Gourio (2012), a change in θ_t creates a mix of first- and second-moment effects on $\beta(\theta)$ which reinforce one another in affecting macroeconomic quantities and asset prices.[22] In that respect, this change in θ_t is expected to resemble both a 'preference shock' (in level) à la Smets and Wouters (2003) or Christiano et al. (2011), for examples, and a second-moment preference ('uncertainty') shock à la Basu and Bundick (2015). Before turning to the general equilibrium effect of disaster risk shocks, let us have a further look at the crucial role of the EIS on the sign of $\beta(\theta)$. Indeed, recall that we denoted the EIS as $1/\tilde{\psi}$ with $\tilde{\psi} = 1 - (1+\varpi)(1-\psi)$, such that (10) can be rewritten as

$$\beta(\theta) = \beta_0 \left[1 - \theta_t \left(1 - e^{(1-\gamma)\ln(1-\Delta)}\right)\right]^{\frac{1-1/EIS}{(1-\gamma)(1+\varpi)}} \tag{11}$$

This expression makes it clear that the unity threshold of the EIS determines the sign of the effect of the probability of disaster (θ) on the discount factor, i.e on agents' propensity to save or consume. Indeed, when $EIS < 1$, $\beta(\theta)$ increases in θ, such that the households become more patient and save more with disaster risk. On the contrary, when $EIS > 1$, $\beta(\theta)$ decreases in θ and impatient agents tend to consume more in response to changes in disaster risk. Note that this role of the EIS in determining the sign of $\beta(\theta)$ holds regardless of the degree of risk aversion of the agents, including risk neutrality, i.e for all $\gamma \geq 0$, and regardless of price flexibility or stickiness. Finally, for $EIS = 1$, $\beta(\theta) = \beta_0$ such that disaster risk will not impact macroeconomic quantities at all. This case is the so-called 'Tallarini (2000)'s equivalence', stating that macroeconomic quantities are determined irrespectively of the level of aggregate risk or risk aversion whenever $EIS = 1$.

In the general equilibrium, agents' choice for saving or consuming in response to disaster risk shocks of course affects output. But the sign of this output response differs whether prices are flexible or sticky. Indeed, when prices are flexible, a higher propensity to save translates in higher investment and thus higher output. Hence, the flexible-price case either predicts that disaster risk decreases output (while increasing consumption) with $EIS > 1$

[22] See the last paragraph on page 13 of Gourio (2012) for more about this.

(Gourio, 2012) or increases output with $EIS < 1$. On the contrary, a sticky environment makes firms unable to adjust their price to their optimal level immediately, affecting their demand of factors of production (capital and labor) so as to maximize their profits (or minimize their loss). Specifically, when disaster risk increases agents' propensity to save (when $EIS < 1$), consumption drops but firms cannot deflate prices as much as they would like. As a result, they demand less capital such that investment also falls despite higher savings. The model is then able to predict the contemporaneous drop in all key macroeconomic variables in response to a disaster risk shock.

4 Calibration and steady-state analysis

4.1 Effect of the EIS on the balanced growth path

Using chain rules, the stochastic discount factor (20) can be rewritten as[23]

$$Q_{t,t+1} = \frac{z_t}{z_{t+1}} \left(\frac{c_{t+1}}{c_t}\right)^{-\psi} \left(\frac{1-L_{t+1}}{1-L_t}\right)^{\varpi(1-\psi)} \beta(\theta_t) e^{(1-\psi)\mu} \frac{e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{-\chi}}{\left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi}\right]^{\frac{-\chi}{1-\chi}}}$$

which, along the balanced growth path, becomes

$$Q(x') = \beta_0 \frac{e^{-\psi\mu - \gamma x' \ln(1-\Delta)}}{\left(E e^{(1-\gamma)x' \ln(1-\Delta)}\right)^{\frac{\psi-\gamma}{1-\gamma}}}$$

where $v'(k')$ cancel out from the previous expression as they are independent from x'.[24] Therefore, the orthogonality condition $E(Q(x')R^k(x')) = 1$ gives the return on capital (along the balanced growth path) as

$$E(R^k(x')) = \frac{E(e^{x' \ln(1-\Delta)})}{\beta_0 e^{-\psi\mu} \left(E e^{(1-\gamma)x' \ln(1-\Delta)}\right)^{\frac{1-\psi}{1-\gamma}}}$$

and the constant riskfree rate from $R^f = 1/E(Q(x'))$ as

[23] Note that this expression is still a function of the future disaster state, x_{t+1}, as part of the productivity growth term (z_t/z_{t+1}). We momentaneously use it to derive the asset pricing equations, but later combine it with the Euler equation on bonds such that only the probability θ of disaster remains in our final equilibrium set (calculations in Appendix).

[24] Indeed, capital accumulation for the next period, k', is independent from the occurrence x' of a disaster in its detrended version (8). Thus, so is $v'(k')$, as in Gourio (2012). Moreover, the arrival of x' only depends on the current probability θ.

$$R^f = \frac{\left(Ee^{(1-\gamma)x' \ln(1-\Delta)}\right)^{\frac{\psi-\gamma}{1-\gamma}}}{\beta_0 e^{-\psi\mu} E\left(e^{-\gamma x' \ln(1-\Delta)}\right)}$$

Note that the riskfree rate decreases in the disaster risk (along the balanced growth path), and the smaller the EIS, the larger the drop.[25] This results is well known in the literature and often justifies the need for a use of an EIS larger than unity in order to limit the fall in the riskfree rate (Tsai and Watcher (2015).[26] However, in our general equilibrium setup, price stickiness makes the drop in the riskfree rate relatively lower when the EIS is below unity rather than above.

Finally, the balanced growth path risk premium is given by

$$\frac{E(R^k(x'))}{R^f} = \frac{E(e^{x' \ln(1-\Delta)})E(e^{-\gamma x' \ln(1-\Delta)})}{E\left(e^{(1-\gamma)x' \ln(1-\Delta)}\right)}$$

which, as expected, depends positively on the disaster risk, and the larger the risk aversion, the larger the effect. If agents were risk neutral, i.e when $\gamma = 0$, the risk premium is unaffected by changes in the probability of disaster. Note that the EIS does not directly impact the value of the risk premium along the balanced growth path, in line with Gourio (2012), although its dynamic response to the disaster risk shock will eventually differ under alternative values of the EIS as a general equilibrium effect (See Section 5).[27]

4.2 Calibration

Table 1 summarizes our baseline calibration values. We also proceed to various sensitivity analyses presented in Section 5.3.

The first part of Table 1 is related to the calibration of disaster risk. We follow empirical estimates of the literature on historical data. Evidence is

[25]See the Appendix for calculation details.

[26]An increase in disaster risk directly reduces the price of equities as it lowers expected cash flows. But meanwhile, it causes an increase in precautionary savings which diminishes the risk-free rate, and in turn tends to increase the price of equities by increasing demand relatively to the supply of equities. Whether this latter effect offsets the former depends on the value of EIS. Indeed, the smaller the EIS, the larger the precautionary savings and the drop in the risk free rate. For example, Berkman et al. (2011) show that the probability of disasters, defined as political crises, is negatively correlated with stock prices. Evidence of this kind encouraged the asset pricing literature to adopt an EIS larger than one.

[27]Gourio (2012) finds the same expression for the risk premium along the balanced growth path (Proposition 5). However, general equilibrium effects will differ here.

Table 1: **Baseline calibration values (quarterly)**

Disaster risk		
$\bar{\theta}$	mean probability of disaster	0.009
Δ	size of disaster	0.22
ρ_θ	persistence of (log) disaster risk	0.9
σ_θ	std. dev. of innovations to (log) disaster risk	0.436
Utility function		
β_0	discount factor	0.99
$1/\tilde{\psi}$	elasticity of intertemporal subtitution	0.5
γ	risk aversion coefficient	3.8
ϖ	leisure preference	2.33
Investment		
δ_0	capital depreciation rate	0.02
τ	capital adjustment cost	0.3
$\bar{u}$	utilization rate of capital	1
Production		
α	capital share of production	0.33
ζ_0	Calvo probability	0.8
ν	elasticity of substitution among goods	6
μ	trend growth of productivity	0.005
Public authority		
ψ_π	Taylor rule inflation weight	1.5
ψ_Y	Taylor rule output weight	0.5
$\bar{\pi}$	target inflation rate	0.005
ρ_r	interest rate smoothing parameter	0.85

quite mixed as for the size and frequency of disasters, and highly depends on the definitions and methods used for the estimations. Barro (2006) found a quarterly probability of disasters of 0.72%, with an average disaster size of 43%, over the twentieth century. Further extending Barro (2006)'s model with data on consumption, particularly relevant for asset pricing models, Barro and Ursúa (2008) found a quarterly probability of disasters of 0.9% with a mean disaster size of 22%, on international time series since 1870.[28] We adopt the latter estimates for our baseline values, although our qualitative results are robust to alternative values for these parameters (See Section 5). Persistence in the log disaster risk, ρ_θ, is set to 0.9 following Gourio (2012), which implies, after normalizing the standard deviation of log disaster risk to 1, a standard deviation of innovations of $(1-\rho_\theta^2)^{1/2} \approx 0.436$.

[28] On US data, Gourio (2013) found smaller disasters (15%) with a probability of 0.5%.

The second part of Table 1 refers to the EZW utility function. The instantantaneous utility specification is borrowed from Gourio (2012) and we thus adopt the same valuation of leisure ($\varpi = 2.33$). We also keep his degree of risk aversion coefficient ($\gamma = 3.8$), as it seems aligned with recent estimates in presence of disaster risk. Indeed, Barro and Jin (2011) found a mean close to 3, with a 95% confidence interval for values from 2 to 4. Yet, we show in the sensitivity analysis, that the degree of risk aversion affects only the size but not the sign of the responses, confirming early theoretical results from Leland (1968), Sandmo (1970), and Weil (1990), for instance.

The only parameter in the second block of Table 1 which is crucial for our results and departs from Gourio (2012) is the value of the EIS. The empirical evidence is quite mixed. Hall (1988)'s seminar paper found this parameter to be close to zero, and a subsequent literature has provided further support for values smaller than one (see e.g Campbell and Mankiw (1989), Ludvigson (1999), Yogo (2004)).[29] Yet, at least two types of concerns about these estimates have been raised. First, agents' heterogeneity matters: the EIS tends to be larger for richer households (Blundell et al. (1994), Attanasio and Browning (1995)) and stockholders (Mankiw and Zeldes (1991), Vissing-Jorgensen (2002)), but much smaller for liquidity-constrained households (Bayoumi (1993)). Second, the presence of uncertainty is an important concern for the estimates, although this is still under debate (Bansal and Yaron (2004) Bansal et al. (2012), Beeler and Campbell (2012)).[30] Turning to the theory, most of the macroeconomic RBC and DSGE models take a value smaller than one. In particular, this holds whether preferences are of Greenwood-Hercowitz-Huffman (GHH) type, potentially augmented with external habits (for e.g Smets and Wouters (2003, 2007)), or Epstein-Zin-Weil where the EIS parameter does not need to be the inverse of the risk aversion coefficient (for e.g Rudebusch and Swanson (2012), Caldara et al. (2012)). Yet, a notable exception are macro-finance models aiming at matching asset

[29] Havránek et al. (2015) have collected 2,735 estimates of EIS reported in 169 studies to explore estimation differences across countries and methodologies from a meta analysis. They find a mean estimate around 0.5, and typically lying between 0 and 1. They suggest that the type of utility function does not affect much the results.

[30] Bansal and Yaron (2004) argue that ignoring time-varying consumption volatility leads to a downward bias in the macro estimates. Using an instrumental variable approach, Beeler and Campbell (2012) recognize the existence of the downward bias but do not find it large enough to question the low estimated values. Bansal et al. (2012) further criticize the high sensitivity of their results to the samples, assets, and instruments they used.

pricing moments, where the EIS value is generally set above unity in order to replicate the level, volatility, and cyclicality of financial returns and the equity premium. In that respect, models of disaster risk (Barro and Ursúa (2008), Gourio (2012), Nakamura et al. (2013)) generally set the EIS above unity (generally, at 2).[31] Our approach here is not to impose any particular value of the EIS ex ante, but rather to select the one that gives the plausible responses to a disaster risk shock. In particular, we keep 0.5 as it is the inverse from Gourio (2012)'s value of 2, and indeed find exactly opposed responses of the macroeconomic variables. Yet, any value of the EIS up to unity would generate results qualitatively similar to our baseline responses, while any value above unity would reproduce Gourio (2012)'s. In that sense, we are able to make Gourio (2012)'s mechanism of disaster risk compatible with more standard macroeconomic calibrations.

The rest of Table 1 mostly follows the New Keynesian literature. Two values are still especially worth commenting here. First, we set the quarterly Calvo probability of firms not changing their price the common value of $\zeta = 0.8$. This may seem strong in the presence of aggregate risk, yet our qualitative results are unchanged under a large span of lower degrees of price stickiness (0.3-0.8), even though their size of course is. With low values of stickiness ($<$0.3), the responses become qualitatively similar to the pure flexible-price case ($\zeta = 0$). Here as for the EIS, we do not need to take a stand on the degree of price stickiness/flexibility in the presence of disaster risk ex ante, but rather aim at experimenting several combination of EIS and price stickiness values before selecting the most plausible one. Finally, the capital adjustment cost parameter is calculated so as to match the elasticity of the ratio I/K with respect to the Tobin's q, in a standard way. This implies a low value of $\tau = 0.3$ here, consistent with the literature on uncertainty where nonconvex adjustment cost functions tend to become more relevant, as shown by Bloom (2009) in particular.

[31] In their review of disaster risk models, Tsai and Wachter (2015) explain that a high EIS indeed prevents the risk-free rate from declining too much, and thus asset prices to rise, by reducing the precautionary saving effect in response to a disaster risk shock. However, estimating Wachter (2013)'s model with disaster risk, Irarrazabal and Parra-Alvarez (2015) find that appropriate levels of equity premium and volatility of government bonds can be compatible with an EIS below unity, while price-dividend volatility still requires a higher EIS. Without disaster risk but also trying to disentangle the role of the EIS in replicating asset pricing moments, Yang (2016) compares a model with habit in consumption with a model of long-run risk. He finds that the former requires an EIS equal to unity whereas the latter requires an EIS strictly above unity in order to match the equity premium.

4.3 Steady-state values in the presence of disaster risk

Table 2 shows the steady-state values obtained under our calibration for some selected variables. In particular, we compare the economy without disaster, i.e having either a probability of disaster ($\bar{\theta}$) or a size of disaster (Δ) equal to zero, to the economy with disaster (for two example sizes, $\Delta = 0.22$ and $\Delta = 0.40$). This is reported here for three different cases: flexible prices ($\zeta = 0$) and EIS $= 2$ (economy à la Gourio), flexible prices and EIS $= 0.5$, sticky prices ($\zeta = 0.8$) and EIS $= 0.5$ (baseline scenario).

The role of the EIS is particularly worth discussing here. In the economy with an EIS below 1, agents have a high propensity to consume the certainty-equivalent income (see Weil (1990)). Thus, steady-state consumption has to be lower in the economy with disaster risk than the economy without. Intuitively, one can think that agents make precautionary savings if they expect a potential disaster to arrive. The same reasoning holds for providing more labor and capital initially in an economy that will be potentially affected by a disaster. Thus current output is higher. One can also see this higher 'degree of patience' in the (time-varying) discount factor and the stochastic discount factor. This holds whether prices are flexible or sticky.

On the contrary, with an EIS larger than 1, agents do not make so much precautionary savings and precautionary labor supply. As a result, investment and output are lower, and by wealth effect so is consumption, when disaster risk is present in the economy versus not. Note that in both cases, the return on capital is of course decreasing in disaster risk. As one can also expect, the risk premium is nil in all cases as the agents make financial arbitrage with perfect foresight at the steady-state.

Finally, as the EIS tends to unity, steady-state values in economies with and without disaster risk tend to be equal to one another. Indeed, when the EIS is equal to 1, the time-varying discount factor ($\beta(\theta)$) boils down to the usual discount factor β_0 (in equation (6)), and the disaster risk does not affect the economic outcomes anymore. This results is referred to as Tallarini (2000)'s "observational equivalence", stating that the macroeconomic quantities are unaffected by the amount of risk in the economy. Again, here as in Gourio (2012), this holds if and only if the EIS is equal to one. In all other cases, quantities differ from the economy without disaster risk.

Table 2: Steady-state values for selected variables.

	no disaster risk $\Delta = 0.22$ and $\bar{\theta} \to 0$ or $\Delta = 0$ and $\bar{\theta} = 0.9\%$	**baseline disaster risk** $\Delta = 0.22$ $\bar{\theta} = 0.9\%$	**larger disaster risk** $\Delta = 0.4$ $\bar{\theta} = 0.9\%$
EIS = 0.5, sticky prices ($\zeta = 0.8$)			
output (detrended)	0.614	0.625	0.651
consumption (detrended)	0.499	0.505	0.518
investment (detrended)	0.115	0.121	0.133
labor	0.228	0.229	0.232
capital (detrended)	4.608	4.820	5.332
$\beta(\bar{\theta})$	0.990	0.991	0.993
Tobin's q	1	1	1
wage	1.505	1.525	1.570
capital rental rate	0.037	0.036	0.034
stochastic discount factor	0.984	0.986	0.990
return on capital	1.017	1.014	1.010
(gross) risk premium	1	1	1
EIS = 0.5, flexible prices ($\zeta = 0$)			
output (detrended)	0.614	0.626	0.652
consumption (detrended)	0.499	0.505	0.518
investment (detrended)	0.115	0.120	0.133
labor	0.228	0.229	0.232
capital (detrended)	4.604	4.818	5.333
$\beta(\bar{\theta})$	0.990	0.991	0.993
Tobin's q	1	1	1
wage	1.506	1.526	1.572
capital rental rate	0.037	0.036	0.034
stochastic discount factor	0.984	0.986	0.990
return on capital	1.017	1.014	1.010
(gross) risk premium	1	1	1
EIS = 2, flexible prices ($\zeta = 0$)			
output (detrended)	0.642	0.635	0.623
consumption (detrended)	0.513	0.510	0.504
investment (detrended)	0.128	0.125	0.119
labor	0.230	0.230	0.229
capital (detrended)	5.129	5.008	4.766
$\beta(\bar{\theta})$	0.990	0.990	0.989
Tobin's q	1	1	1
wage	1.554	1.543	1.521
capital rental rate	0.034	0.035	0.036
stochastic discount factor	0.986	0.987	0.988
return on capital	1.014	1.013	1.012
(gross) risk premium	1	1	1

5 Impulse Response Functions

In this Section, we simulate the effects of a positive shock to the probability of disaster (θ) from its steady-state value. As described earlier, this shock is small and the large disaster event (x) is absent from the detrended model, such that we can use perturbation methods. We use a third-order approximation, where the risk premium starts interacting with the macroeconomic variables, unless otherwise stated. Our goal is mainly qualitative, in analyzing the responses of macroeconomic and asset pricing variables to this disaster risk shock. As we consider a small departure from the small probability of disaster risk, specifically a change from $\bar{\theta} = 0.009$ to 0.01, the size of our responses will be naturally small as well. A larger shock would give a feel for the absolute magnitude of the effects but with potentially large errors since the detrended model remains nonlinear. Thus, we restrict our analysis to the sign of the effects, and their relative size under different assumptions, while the reader can refer to Gourio (2012) for magnitude purposes.[32]

This Section continues as follows. First, we present the effects of a disaster risk shock under the baseline scenario, combining an EIS below unity and price stickiness. Then, we show the results under the alternative combinations of EIS and price stickiness or flexibility, including Gourio (2012)'s case where prices are purely flexible and the EIS equal to 2. Finally, we test our baseline combination under alternative calibration values. We also simulate the responses of the model to a standard monetary policy shock, in order to make sure that our model is able to replicate well-known perturbation responses in spite of the presence of disaster risk.

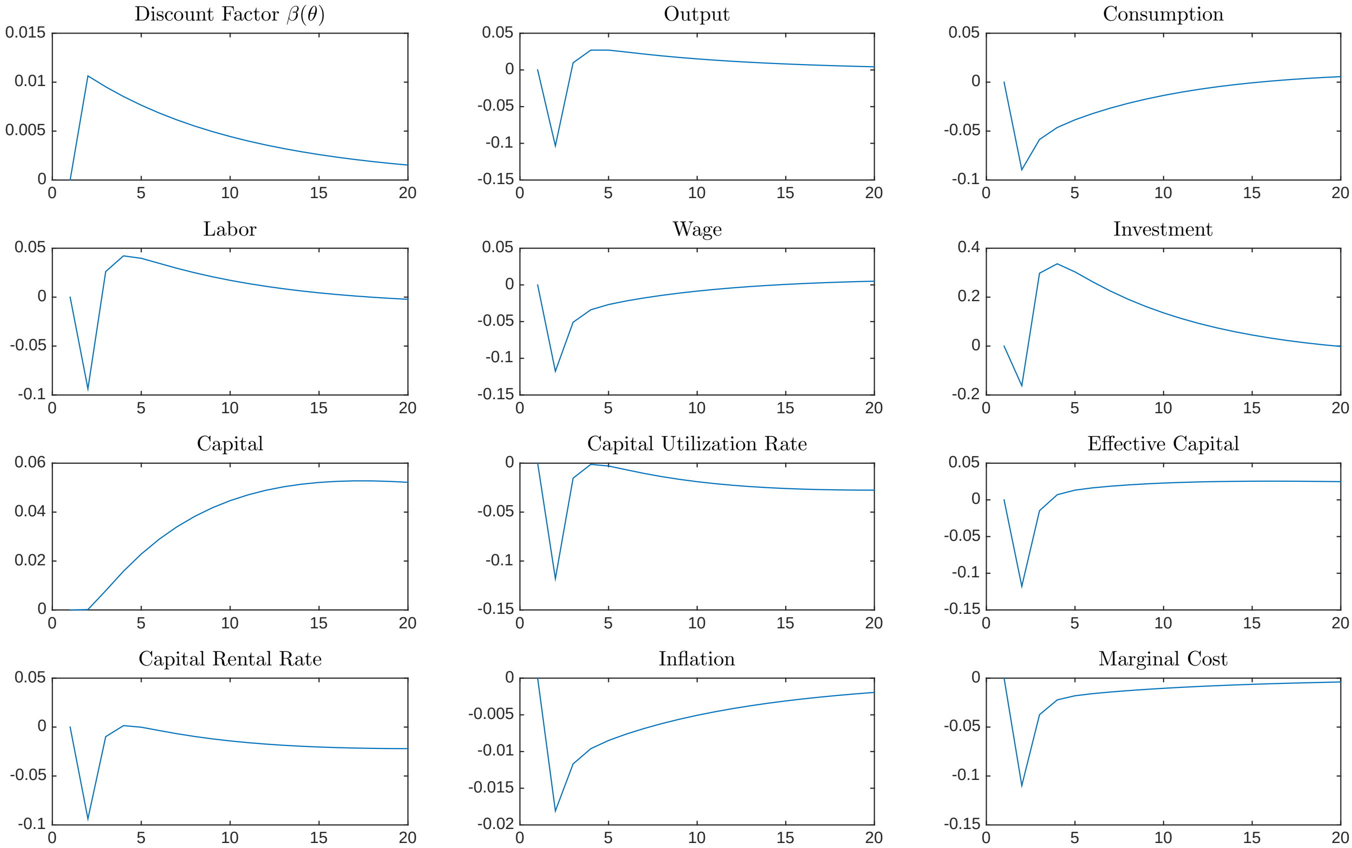
5.1 Effect of a disaster risk shock: the baseline scenario

Figures 1 and 2 depict the responses to a rise in θ from 0.9% to 1% in the baseline scenario (EIS = 0.5 and $\zeta = 0.8$).

With an EIS below unity, the endogenous discount factor $\beta(\theta)$ increases in response to higher disaster risk. As agents become more "patient", their propensity to save increases such that their consumption drops, naturally associated with deflation. Yet, the higher savings do not immediately translate into higher investment, such that output also drops. This staggered effect

[32]Indeed, also solving his model with the use of projection methods, Gourio (2012) considers a deviation of the disaster risk from 0.72% to 4%.

Figure 1: Responses of main macroeconomic variables to a positive disaster risk shock; baseline scenario (EIS $= 0.5$ and $\zeta = 0.8$)



Effect of a change in θ from 0.9% to 1%. Percentage change from the steady-state (vertical axis). Third-order approximation.

on investment comes from price stickiness: firms cannot deflate their prices as much as they would like in the face of depressed consumption, and thus demand less factors of production. Indeed, as one can see on Figure 1, the utilization rate of capital, and thus the effective capital, as well as the rental rate of capital, decline. This makes the households less willing to invest. For the same reason, labor also drops, as well as wages. This overall generates positive co-movements between the main macroeconomic aggregates, i.e consumption, investment, output, labor, wages, and inflation.

A noteworthy phenomenon is the overshooting in labor, investment, and to a lower extent in output, in the subsequent periods. Both labor and investment responses are affected by two opposite channels. One is the lower demand for factors from firms discussed above. The other is a precautionary motive from the households when the disaster risk goes up. Indeed, the agents want to limit the decrease in their consumption by acquiring more capital and increasing their labor supply when a disaster becomes more likely (under the assumption of an EIS below one). These upward pressures tend to dominate the recessionary effects in the subsequent periods as more and more firms eventually turn to change their prices.[33] This overshooting result is reminiscent of the uncertainty literature. Indeed, Basu and Bundick (2015) obtain a similar effect from a second-moment discount factor shock in the presence of sticky prices for instance. This same effect holds in Bloom (2009) although it results from the nonconvex adjustment cost function rather than price stickiness in his model. The intuition is similar though in the sense that firms enter a zone of "inactivity" as uncertainty shocks hit. Bloom (2009) argues that first-moment shocks generate long-lasting recessions while second-moment shocks indeed typically create sharp but short-lived recessions.

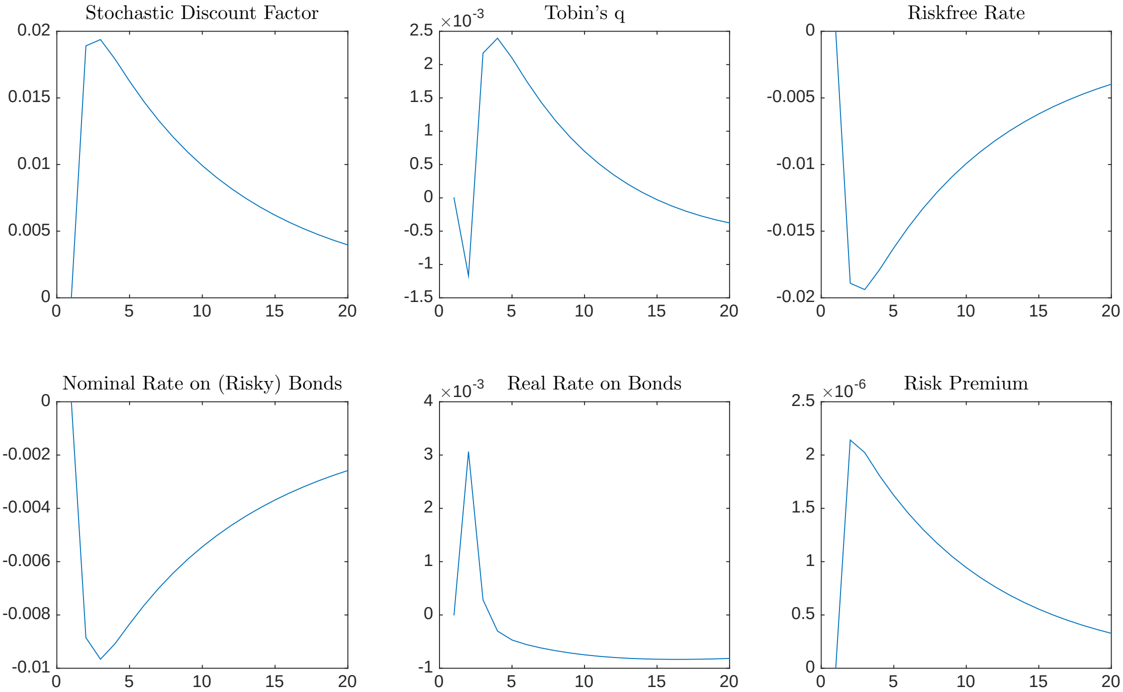
As for asset prices in Figure 2, we observe a 'flight-to-quality' effect that is visible through the drop in the riskfree rate when the disaster risk shock hits,[34] as well as an increase in the risk premium. Hence, while improving Gourio (2012)'s predictions for the macroeconomic variables, we preserve the strength of his mecanism for accounting for the countercyclicality of the risk premia. We further comment on the relative size of the financial variable responses with Gourio (2012)'s in the next subsection.

Overall, our results are vey much in line with the preference shocks literatures, whether as first- (Smets and Wouters, 2003, for instance) or second-moment 'uncertainty' (Basu and Bundick, 2015) effects. This is not surprising as $\beta(\theta)$ indeed features a mix of first- and second-moment effects, as

[33]Another reason amplifying the size of this overshooting is that agents cannot buy any riskfree asset here, such that risky bonds or risky capital are the only savings vehicles available. That participates in investment bouncing back quite rapidly and strongly after the shock. We could relax this assumption and make the bonds riskfree. However, Gourio (2012)'s detrending method could then not be perfectly applied and we would have to condition our responses on the current state of the economy, within a 'disaster' regime or not, when generating the impule response functions. This would be straightforward to apply but we chose to present the unconditional impulse response functions, closer to Gourio (2012)'s spirit.

[34]Note again that the riskfree rate is not the interest rate on bonds which are risky in this model, but merely the inverse of agents' stochastic discount factor.

Figure 2: Responses of main financial variables to a positive disaster risk shock; baseline scenario (EIS = 0.5 and $\zeta = 0.8$)



Effect of a change in θ from 0.9% to 1%. Percentage change from the steady-state (vertical axis). Third-order approximation.

explained in Gourio (2012). Yet, as long as the EIS was above unity, both effects played downward: agents become more impatient as the disaster risk shock hits (see next subsection). On the contrary, having an EIS below unity makes the first- and second-moment effects on $\beta(\theta)$ upward. Combined with price stickiness, this leads to a recession. In that respect, we reinforce the interpretation of Gourio (2012)'s mechanism of disaster risk as a potential source of preference shifts and uncertainty shocks.

5.2 Effect of a disaster risk shock: other scenarii

In Figures 3 and 4, we consider the same shock under alternative scenarii as follows:

(i) Flexible prices and EIS = 2, a decentralized version of Gourio (2012)'s economy, in order to nest his results as a particular case;

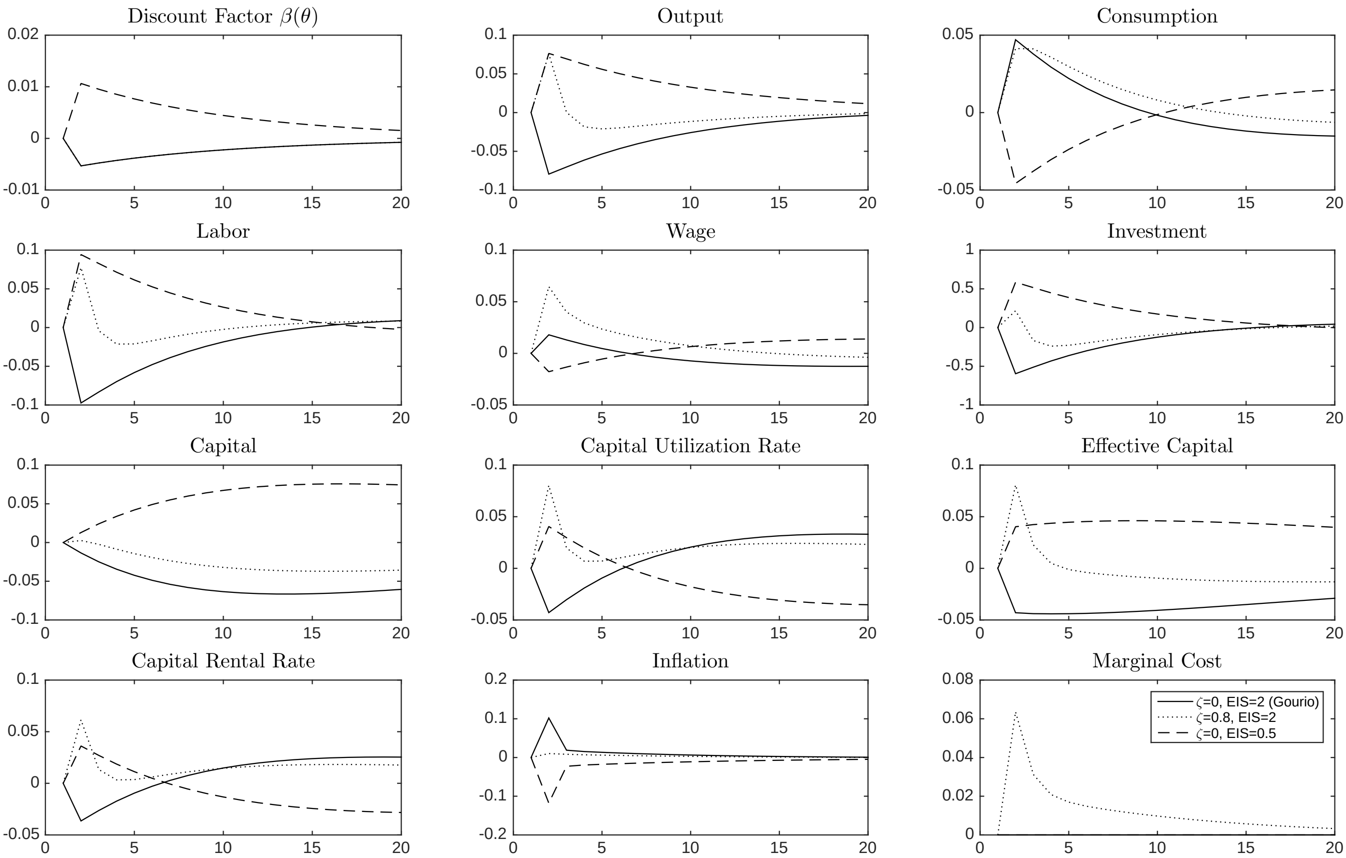
(ii) Flexible prices and EIS = 0.5 as a value more compatible with the standard RBC and New Keynesian literatures;

(iii) Sticky prices and EIS = 2, as a mirroring experiment.

Scenario (i) is depicted by the solid line in Figures 3 and 4. This reproduces the effects of a disaster risk shock in an economy à la Gourio (2012), with flexible prices ($\zeta = 0$) and an EIS of 2, yet here in a decentralized economy version. In contrast with the baseline scenario, an EIS above unity here makes the agents more impatient ($\beta(\theta)$ decreases) when θ increases. Hence, they save and invest less, driving the economy into a recession, despite an increase in consumption. Since prices are fully flexible, there is no staggered effect on firms' side, unlike in the previous case. Agents choose to work less, despite precautionary motives at play, since the higher return on today's capital already sustains their higher consumption level, such that the equilibrium wage increases. This also contrasts with the sticky case where the wage goes down. These results are all identical to Gourio (2012), although without explicit wage and capital rental rate as for any centralized economy. Among the financial variables in Figure 4, first note that the risk premium goes up. The order of magnitude is higher here than in our scenario, but recall that this holds for an EIS above unity. Moving from an EIS larger than one to an EIS smaller than one always increases the response of the risk premium for a given degree of price stickiness/flexibility (comparing scenarii (i) with (ii) on the one hand, and (iii) with baseline, on the other hand). Finally, the riskfree rate goes down here, as in our scenario. However, note that the drop is more pronounced here, which is a common puzzling feature of finance asset pricing models (Tsai and Wachter, 2015). This suggests that sticky prices may provide a solution to avoid an excessive fall in the riskfree rate while allowing the EIS to be smaller than one.

Scenario (ii) is depicted by the dashed line in the same figures. Here, we consider that prices are still flexible, but the EIS is below unity. It turns out that all macroeconomic responses become the exact opposite of the Gourio (2012)-like solid line. Indeed, the economy enters a boom, driven by the fact that a low EIS increases agents' propensity to save and thus to invest. Only the stochastic discount factor, the riskfree rate, and the risk premium responses, remain qualitatively unchanged in all cases.

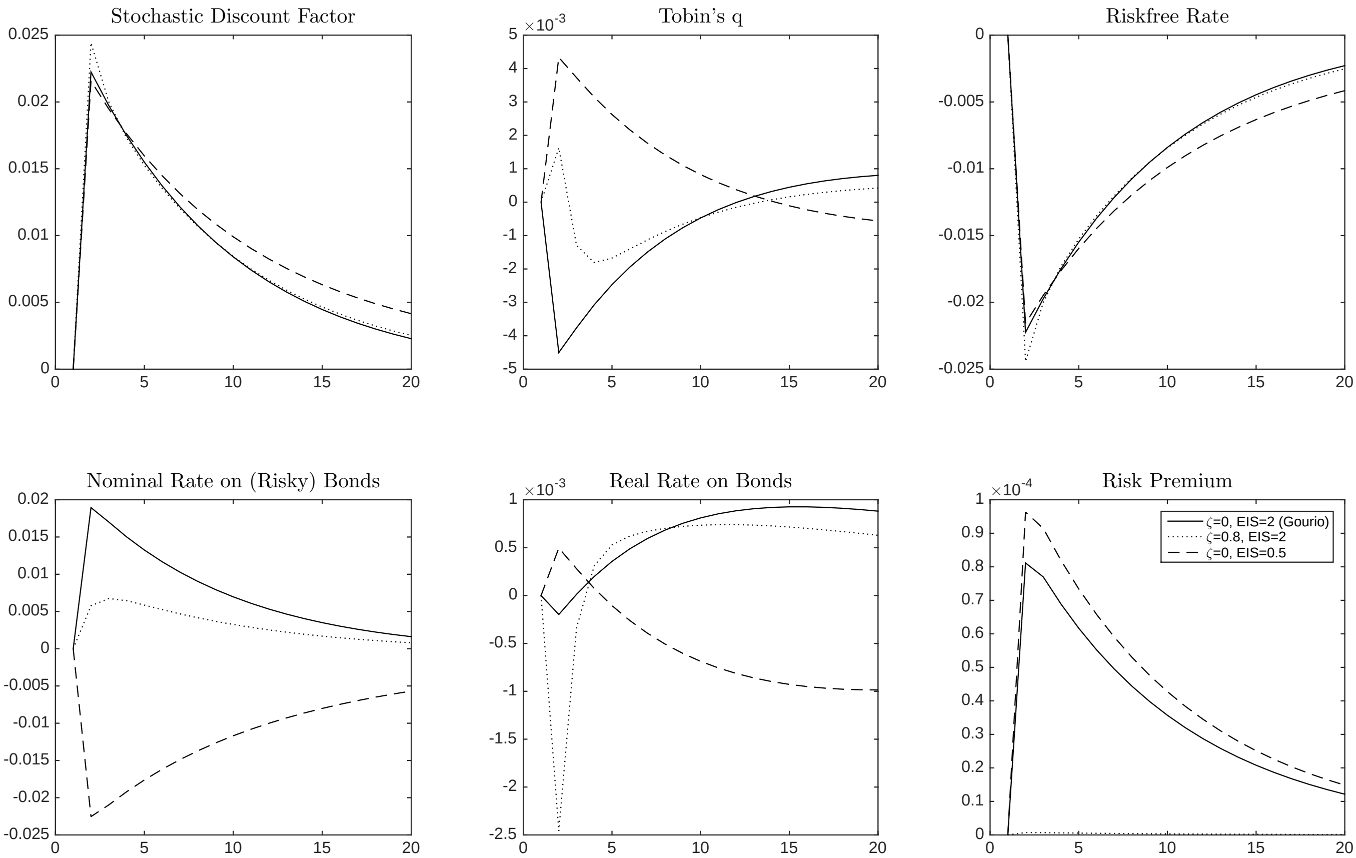
Figure 3: Responses of main macroeconomic variables to a disaster risk shock; other scenarii (EIS $= 2$, $\zeta = 0$; EIS $= 2$, $\zeta = 0.8$, EIS $= 0.5$, $\zeta = 0$)



Effect of a change in θ from 0.9% to 1%. Percentage change from the steady-state (vertical axis). Third-order approximation.

Scenario (*iii*) is finally represented with the dotted line. Here, we experiment the mirroring combination of sticky prices (as in our baseline) and EIS above unity (as in Gourio (2012)). In that case again, the economy enters a boom, driven by agents' impatience. As in our baseline scenario, price stickiness makes all macroeconomic variables co-move, while financial variables preserve their sign. This case provides an interesting counterfactual exercise to confirm, on the one hand, the effect of the EIS on the discount factor and the propensity to consume/save, and on the other hand, the fact that aggregate dynamics are more impacted by the demand (supply) side of the

Figure 4: Responses of main financial variables to a disaster risk shock; other scenarii (EIS $=2$, $\zeta=0$; EIS $=2$, $\zeta=0.8$, EIS $=0.5$, $\zeta=0$)



Effect of a change in θ from 0.9% to 1%. Percentage change from the steady-state (vertical axis). Third-order approximation.

economy when price stickiness (flexibility) is at play, for a given value of the EIS. We can also observe that the riskfree rate happens to decrease more than in the baseline scenario where the EIS was low. This suggests that the asset pricing literature argument of a EIS larger than unity being necessary to limit the fall of the riskfree rate may not hold under sticky prices.

5.3 Sensitivity analysis

Figure 5 replicates the baseline scenario under alternative values for some other parameters. The qualitative results remain essentially unaffected.

The first subfigure presents alternative values for the risk aversion coefficient. Our baseline value is standard under Esptein-Zin specification, i.e lying typically between 3 and 4. Barro and Jin (2011)'s estimates in the presence of disaster risk are particularly informative. Yet, we try alternative values of γ within the range allowed by the model, i.e compatible with an (endogenous) discount factor $\beta(\theta)$ below unity. When the agents are closer to risk neutrality ($\gamma = 1.5$), the upward precautionary labor supply effect causing the overshooting in labor, investment, and output, turns to be more moderate. On the contrary, high risk aversion (here, $\gamma = 6$) systematically amplifies the responses, yet without questioning the qualitative effects.

Next subfigures are related to the fixed part of the discount factor β_0 (which would be the standard discount factor in the absence of disaster risk), the mean disaster size Δ, and the mean probability of disaster $\bar{\theta}$. All three have size but not sign effects.

Only the persistence in disaster risk, ρ, in the last subfigure, seems to have a qualitative impact on investment. As discussed previously, investment is driven by two opposite effects and it happens that the upward pressure (the precautionary motive) tends to outweight the downward pressure (firms' lower demand for factors of production) when the persistence is lower. Once again, should the households be allowed to buy the riskfree asset in our model setup, this effect may vanish as precautionary savings would find another vehicle than investment. The size of the subsequent overshooting is however negatively correlated with the persistence in disaster risk. Further, this different response of investment is itself not sufficiently strong to modify the impact response of output to the disaster risk shock.

Finally, Figure 6 displays the responses of selected variables to a monetary policy shock under the baseline scenario. These responses are quite standard and inform us about the validity of the model to well-known shocks.

6 Conclusion

This paper has developed a New Keynesian model featuring a small but time-varying probability of rare events à la Gourio (2012). In line with the RBC literature, we find that an increase in disaster risk, without occurrence of the disaster itself, is able to generate a recession and a countercyclical risk premium. However, the New Keynesian setup improves the RBC responses along two dimensions. First, price stickiness makes the recessionary effects of disaster risk compatible with a standard value of the elasticity of intertemporal of substitution below unity, and then also allows consumption, wage, and output prices to co-move as expected. Second, changes in disaster risk then resemble an *increase* in agents' discount factor, instead of a *decrease* as in Gourio (2012). Indeed, with an EIS below unity, agents (endogenously) become more patient in response to a disaster risk shock. Combined with price stickiness, the responses of aggregate macroeconomic and financial variables in turn resemble preference shocks (Smets and Wouters (2003), Christiano et al. (2011)) and second-moment "uncertainty" discount factor shock (Basu and Bundick (2015)). In that sense, we show how to conciliate Gourio (2012)'s mechanism of disaster risk with the New Keynesian literatures on preference and uncertainty shocks.

Preference and uncertainty shocks have lately been praised for pushing the economy into zero lower bound and secular stagnation situations. To the extent that disaster risk shocks can provide a potential rationale for these shocks, such an extension could be of interest for further research. Moreover, it could be informative to study variations in the term premium due to the disaster risk, and interactions between the short-term risk premium with the long-term end of the yield curve. Finally, it would be worth investigating the effectiveness and desirability of policies, such as unconventional monetary policies, to be implemented in response to variations in disaster risk.

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7 Appendices

A Households' problem with disaster risk

A.1 Utility maximization and first-order conditions

Rewriting the Epstein-Zin preferences

$$\tilde{V}_t = \left[\left[C_t \left(1 - L_t\right)^{\varpi}\right]^{1-\psi} + \beta_0 \left(E_t \tilde{V}_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \right]^{\frac{1}{1-\psi}}$$

with the notations $\tilde{V}_t = V_t^{\frac{1}{1-\psi}}$ and $\chi = 1 - \frac{1-\gamma}{1-\psi}$ as

$$V_t = \left[C_t \left(1 - L_t\right)^{\varpi}\right]^{1-\psi} + \beta_0 \left(E_t V_{t+1}^{1-\chi} \right)^{\frac{1}{1-\chi}} \tag{12}$$

the Lagrangian for households' problem reads as

$$\begin{aligned} \mathcal{L} &= \left[C_t \left(1 - L_t\right)^{\varpi}\right]^{1-\psi} + \beta_0 \left(E_t V_{t+1}^{1-\chi} \right)^{\frac{1}{1-\chi}} \\ &+ E_t \left\{ \Lambda_t^B \left[\frac{W_t}{p_t} L_t + \frac{B_t(1+r_{t-1})}{p_t} e^{x_t \ln(1-\Delta)} + \frac{P_t^k}{p_t} u_t K_t + D_t - C_t - I_t \right.\right. \\ &- \left.\left. \frac{B_{t+1}}{p_t} - T_t \right] + \Lambda_t^C \left[(1 - \delta_0 u_t^{\eta}) K_t + S\left(\frac{I_t}{K_t}\right) K_t - \frac{K_{t+1}}{e^{x_{t+1}\ln(1-\Delta)}} \right] \right\} \end{aligned}$$

with $S\left(\frac{I_t}{K_t}\right) = \frac{I_t}{K_t} - \frac{\tau}{2}\left(\frac{I_t}{K_t} - \frac{\bar{I}}{\bar{K}}\right)^2$ the capital adjustment cost function, and Λ_t^B and Λ_t^C the Lagrangian multipliers associated with the budget constraint and capital accumulation constraint, respectively.

The first-order conditions are thus

$$(C_t :) \quad E_t\left(\frac{\Lambda_{t+1}^B}{\Lambda_t^B}\right) = \left(\frac{C_{t+1}}{C_t}\right)^{-\psi} \left(\frac{1-L_{t+1}}{1-L_t}\right)^{\varpi(1-\psi)} \beta_0 \frac{V_{t+1}^{-\chi}}{\left(E_t V_{t+1}^{1-\chi}\right)^{\frac{-\chi}{1-\chi}}} \tag{13}$$

$$(L_t :) \quad \frac{1-L_t}{C_t} = \frac{\varpi}{W_t^{real}} \tag{14}$$

$$(B_{t+1} :) \quad E_t\left(\frac{\Lambda_{t+1}^B}{\Lambda_t^B} \frac{1+r_t}{1+\pi_{t+1}} e^{x_{t+1}\ln(1-\Delta)}\right) = 1 \tag{15}$$

$$(u_t :) \quad \frac{\Lambda_t^C}{\Lambda_t^B} = \frac{P_t^k/p_t}{\delta_0 \eta u_t^{\eta-1}} \tag{16}$$

$$(I_t:) \quad \frac{\Lambda_t^C}{\Lambda_t^B} = \frac{1}{1-\tau\left(\frac{I_t}{K_t}-\frac{\bar{I}}{\bar{K}}\right)} \tag{17}$$

$$(K_{t+1}:) \quad E_t\left\{\Lambda_{t+1}^B \frac{P_{t+1}^k}{p_{t+1}}u_{t+1} + \Lambda_{t+1}^C\left[1-\delta_0 u_{t+1}^\eta + \tau\frac{I_{t+1}}{K_{t+1}}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)\right.\right.$$
$$\left.\left. -\frac{\tau}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)^2\right]\right\} = \frac{\Lambda_t^C}{E_t(e^{x_{t+1}\ln(1-\Delta)})}$$

where Λ_t^C can be substituted out using (16), such that it is rewritten as

$$E_t\left\{\frac{\Lambda_{t+1}^B}{\Lambda_t^B}P_{t+1}^{k,real}\left\{u_{t+1}+\frac{1}{\delta_0\eta u_{t+1}^{\eta-1}}\left[1-\delta_0 u_{t+1}^\eta+\tau\frac{I_{t+1}}{K_{t+1}}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)\right.\right.\right.$$
$$\left.\left.\left. -\frac{\tau}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)^2\right]\right\}\right\} = \frac{P_t^{k,real}}{\delta_0\eta u_t^{\eta-1}}E_t\left(\frac{1}{e^{x_{t+1}\ln(1-\Delta)}}\right) \tag{18}$$

or equivalently, using the expression for the Tobin's q as

$$E_t\left\{\frac{\Lambda_{t+1}^B}{\Lambda_t^B}\left[\frac{P_{t+1}^{k,real}u_{t+1}}{q_t}+\frac{q_{t+1}}{q_t}\left[1-\delta_0 u_{t+1}^\eta+\tau\frac{I_{t+1}}{K_{t+1}}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)\right.\right.\right.$$
$$\left.\left.\left. -\frac{\tau}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)^2\right]\right]\right\} = \frac{1}{E_t(e^{x_{t+1}\ln(1-\Delta)})} \tag{19}$$

A.2 The stochastic discount factor and asset pricing

The stochastic discount factor, $Q_{t,t+1} \equiv \frac{\partial\tilde{V}_t/\partial C_{t+1}}{\partial\tilde{V}_t/\partial C_t}$, is here given by

$$Q_{t,t+1} = \beta_0\left(\frac{C_{t+1}}{C_t}\right)^{-\psi}\left(\frac{1-L_{t+1}}{1-L_t}\right)^{\varpi(1-\psi)}\frac{V_{t+1}^{-\chi}}{(E_tV_{t+1}^{1-\chi})^{\frac{-\chi}{1-\chi}}} \tag{20}$$

where $\tilde{V}_t = V_t^{\frac{1}{1-\psi}}$ and $\chi = 1-\frac{1-\gamma}{1-\psi}$ for notational ease, and further from (13),

$$Q_{t,t+1} = E_t\left(\frac{\Lambda_{t+1}^B}{\Lambda_t^B}\right) \tag{21}$$

The standard asset pricing orthogonality condition that determines the real return on asset i as $E_t\left[Q_{t,t+1}R^i_{t+1}\right] = 1$ here gives

$$E_t\left[Q_{t,t+1}R^f_{t+1}\right] = 1 \tag{22}$$

for the *riskfree rate*, R^f, and

$$E_t(Q_{t,t+1}R^{k,real}_{t+1}) = E_t\left(\frac{\Lambda^B_{t+1}}{\Lambda^B_t}R^{k,real}_{t+1}\right) = 1 \tag{23}$$

for the (real) *rate of return on capital*, $R^{k,real}_{t+1}$. Using the first-order condition on capital (18), we can further write

$$\begin{aligned} R^{k,real}_{t+1} &= e^{x_{t+1}\ln(1-\Delta)}\frac{P^{k,real}_{t+1}}{P^{k,real}_t}\delta_0\eta u_t^{\eta-1}\left\{u_{t+1} + \frac{1}{\delta_0\eta u_{t+1}^{\eta-1}}\left[1-\delta_0 u^\eta_{t+1}\right.\right. \\ &\left.\left. +\tau\frac{I_{t+1}}{K_{t+1}}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right) - \frac{\tau}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)^2\right]\right\} \end{aligned} \tag{24}$$

or equivalently, using the Tobin's q notation, as

$$\begin{aligned} R^{k,real}_{t+1} &= e^{x_{t+1}\ln(1-\Delta)}\left\{\frac{P^{k,real}_{t+1}u_{t+1}}{q_t} + \frac{q_{t+1}}{q_t}\left[1-\delta_0 u^\eta_{t+1}\right.\right. \\ &\left.\left. +\tau\frac{I_{t+1}}{K_{t+1}}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right) - \frac{\tau}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)^2\right]\right\} \end{aligned}$$

which stresses out the impact of the disaster event on the return on capital. Finally, the *risk premium* is defined, in gross terms, as the ratio of the real return on capital to the riskfree rate, i.e

$$E_t(Premium_{t+1}) \equiv E_t\left(\frac{R^{k,real}_{t+1}}{R^f_{t+1}}\right) \tag{25}$$

A.3 Detrending the households' optimality conditions

In this subsection, we detail our solution method. Following Gourio (2012), it consists in detrending the set of equilibrium equations so as to get rid of the disaster event variable x_{t+1} assuming that the productivity growth is subject to disasters in the same proportion as the assets, i.e.

$$\frac{z_{t+1}}{z_t} = e^{x_{t+1}\ln(1-\Delta)+\mu+\varepsilon_{z,t+1}} \tag{26}$$

where $x_{t+1} = 1$ with probability θ_t, and $x_{t+1} = 0$ otherwise. This leaves us with a system where no large shock such as a disaster realization can occur, and thus where perturbations can be used as a solution method to simulate innovations on the (small) probability of disaster alone.

A.3.1 The utility function

Because of the Epstein-Zin-Weil recursive formulation, the first-order condition on consumption (13) contains the utility function term v, which requires including the utility function as part of the equilibrium set. Following Gourio (2012), we here below reexpress the utility function in detrended terms so as to obtain a time-varying discount factor $\beta(\theta)$. First, rewrite (12) as

$$v_t \equiv \frac{V_t}{z_t^{1-\psi}} = [c_t(1-L_t)^{\varpi}]^{1-\psi} + \frac{\beta_0}{z_t^{1-\psi}}\left(E_t\left(z_{t+1}^{1-\psi}v_{t+1}\right)^{1-\chi}\right)^{\frac{1}{1-\chi}}$$

$$= [c_t(1-L_t)^{\varpi}]^{1-\psi} + \frac{\beta_0}{z_t^{1-\psi}}\left(E_t\left(z_t^{(1-\psi)}e^{[\mu+\varepsilon_{z,t+1}+x_{t+1}\ln(1-\Delta)](1-\psi)}v_{t+1}\right)^{1-\chi}\right)^{\frac{1}{1-\chi}}$$

$$= [c_t(1-L_t)^{\varpi}]^{1-\psi} + \beta_0\left(E_t e^{[\mu+\varepsilon_{z,t+1}+x_{t+1}\ln(1-\Delta)](1-\psi)(1-\chi)}v_{t+1}^{1-\chi}\right)^{\frac{1}{1-\chi}}$$

By independence of the disaster event, the expectation is decomposed as

$$v_t = [c_t(1-L_t)^{\varpi}]^{1-\psi} + \beta_0 E_t\left[e^{(1-\gamma)x_{t+1}\ln(1-\Delta)}\right]^{\frac{1}{1-\chi}} e^{(1-\psi)\mu} E_t\left[e^{(1-\gamma)\varepsilon_{z,t+1}}v_{t+1}^{1-\chi}\right]^{\frac{1}{1-\chi}}$$

with $(1-\gamma) = (1-\psi)(1-\chi)$ from earlier definition, where the first expectation operator is conditional on the disaster risk and information at time t whereas the second expectation operator is only conditional on information at time t. Moreover, since a disaster ($x = 1$) is realized with probability θ and there is no disaster ($x = 0$) otherwise $(1-\theta)$, we can further establish that

$$\beta_0 E_t\left[e^{(1-\gamma)x_{t+1}\ln(1-\Delta)}\right]^{\frac{1}{1-\chi}} = \beta_0\left[(1-\theta_t) + \theta_t e^{(1-\gamma)\ln(1-\Delta)}\right]^{\frac{1}{1-\chi}} \equiv \beta(\theta) \tag{27}$$

which defines a time-varying discount factor $\beta(\theta)$ as a function of the (time-varying) disaster risk, and finally allows to rewrite (12) as

$$v_t = [c_t(1-L_t)^{\varpi}]^{1-\psi} + \beta(\theta_t)e^{(1-\psi)\mu}\left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{28}$$

A.3.2 The household's constraints

First, as in Gourio (2012), the law of motion of capital can simply be rewritten in detrended terms as

$$k_{t+1} = \frac{(1-\delta_t)k_t + S\left(\frac{i_t}{k_t}\right)k_t}{e^{\mu+\varepsilon_{z,t+1}}} \tag{29}$$

where x is not present anymore. On the contrary, for the budget constraint we obtain

$$c_t + i_t + \frac{b_{t+1}(z_{t+1}/z_t)}{p_t} + \frac{T_t}{z_t} \leq \frac{w_t}{p_t}L_t + \frac{b_t(1+r_{t-1})e^{x_t \ln(1-\Delta)}}{p_t} + \frac{P_t^k}{p_t}u_t k_t + \frac{D_t}{z_t}$$

where the term z_{t+1}/z_t, and thus the present of disaster is still present, but can be replaced by the aggregate resource constraint ($y_t = c_t + i_t$) in the equilibrium system.

A.3.3 The stochastic discount factor and FOC on consumption

Applying derivation chain rules, the stochastic discount factor (20) can be reexpressed as

$$Q_{t,t+1} = \frac{z_t}{z_{t+1}}\left(\frac{c_{t+1}}{c_t}\right)^{-\psi}\left(\frac{1-L_{t+1}}{1-L_t}\right)^{\varpi(1-\psi)}\beta(\theta_t)e^{(1-\psi)\mu}\frac{e^{(1-\gamma)\varepsilon_{z,t+1}}v_{t+1}^{-\chi}}{\left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}}v_{t+1}^{1-\chi}\right]^{\frac{-\chi}{1-\chi}}}$$

Therefore, by (21), the first-order on consumption becomes

$$E_t\left(\frac{\Lambda_{t+1}^B}{\Lambda_t^B}\right) = \frac{z_t}{z_{t+1}}\left(\frac{c_{t+1}}{c_t}\right)^{-\psi}\left(\frac{1-L_{t+1}}{1-L_t}\right)^{\varpi(1-\psi)}\beta(\theta_t)e^{(1-\psi)\mu}\frac{e^{(1-\gamma)\varepsilon_{z,t+1}}v_{t+1}^{-\chi}}{\left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}}v_{t+1}^{1-\chi}\right]^{\frac{-\chi}{1-\chi}}}$$

However, since the term z_t/z_{t+1} is not expected but realized here, we need to further define

$$\tilde{Q}_{t,t+1} \equiv Q_{t,t+1}\frac{z_{t+1}}{z_t} \tag{30}$$

so as to rewrite it as

$$E_t\left(\frac{\Lambda^B_{t+1}}{\Lambda^B_t}\right)=\frac{Q_{t,t+1}}{\tilde{Q}_{t,t+1}}\left(\frac{c_{t+1}}{c_t}\right)^{-\psi}\left(\frac{1-L_{t+1}}{1-L_t}\right)^{\varpi(1-\psi)}\beta(\theta_t)e^{(1-\psi)\mu}\frac{e^{(1-\gamma)\varepsilon_{z,t+1}}v_{t+1}^{-\chi}}{\left[E_te^{(1-\gamma)\varepsilon_{z,t+1}}v_{t+1}^{1-\chi}\right]^{\frac{-\chi}{1-\chi}}}$$

$$\Leftrightarrow\quad \tilde{Q}_{t,t+1}=\left(\frac{c_{t+1}}{c_t}\right)^{-\psi}\left(\frac{1-L_{t+1}}{1-L_t}\right)^{\varpi(1-\psi)}\beta(\theta_t)e^{(1-\psi)\mu}\frac{e^{(1-\gamma)\varepsilon_{z,t+1}}v_{t+1}^{-\chi}}{\left[E_te^{(1-\gamma)\varepsilon_{z,t+1}}v_{t+1}^{1-\chi}\right]^{\frac{-\chi}{1-\chi}}} \tag{31}$$

A.3.4 Other first-order conditions

The definition of a 'detrended' stochastic discount factor (30) further helps to rewrite the optimality conditions on capital and bonds. Multiplying both sides of the first-order condition on capital (18) by $E_t\left(e^{x_{t+1}\ln(1-\Delta)}\right)$ and including it into the same expectation operator by the independence of x, we get

$$E_t\left\{\frac{\Lambda^B_{t+1}}{\Lambda^B_t}e^{x_{t+1}\ln(1-\Delta)}P^{k,real}_{t+1}\left\{u_{t+1}+\frac{1}{\delta_0\eta u_{t+1}^{\eta-1}}\left[1-\delta_0u^\eta_{t+1}+\tau\frac{I_{t+1}}{K_{t+1}}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)\right.\right.\right.$$
$$\left.\left.\left.-\frac{\tau}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)^2\right]\right\}\right\}=\frac{P^{k,real}_t}{\delta_0\eta u_t^{\eta-1}}$$

$$\Leftrightarrow\quad E_t\left\{\frac{\Lambda^B_{t+1}}{\Lambda^B_t}\frac{z_{t+1}}{z_t}\frac{1}{e^{\mu+\varepsilon_{z,t+1}}}P^{k,real}_{t+1}\left\{u_{t+1}+\frac{1}{\delta_0\eta u_{t+1}^{\eta-1}}\left[1-\delta_0u^\eta_{t+1}+\tau\frac{I_{t+1}}{K_{t+1}}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)\right.\right.\right.$$
$$\left.\left.\left.-\frac{\tau}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)^2\right]\right\}\right\}=\frac{P^{k,real}_t}{\delta_0\eta u_t^{\eta-1}}$$

$$\Leftrightarrow\quad E_t\left\{\tilde{Q}_{t,t+1}P^{k,real}_{t+1}\left\{u_{t+1}+\frac{1}{\delta_0\eta u_{t+1}^{\eta-1}}\left[1-\delta_0u^\eta_{t+1}+\tau\frac{I_{t+1}}{K_{t+1}}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)\right.\right.\right.$$
$$\left.\left.\left.-\frac{\tau}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\frac{\bar{I}}{\bar{K}}\right)^2\right]\right\}\right\}=\frac{P^{k,real}_t}{\delta_0\eta u_t^{\eta-1}}e^{\mu+\varepsilon_{z,t+1}}$$

$$\Leftrightarrow \quad E_t \left\{ \tilde{Q}_{t,t+1} P_{t+1}^{k,real} \left\{ u_{t+1} + \frac{1}{\delta_0 \eta u_{t+1}^{\eta-1}} \left[1 - \delta_0 u_{t+1}^{\eta} + \tau \frac{i_{t+1}}{k_{t+1}} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right) \right.\right.\right.$$
$$\left.\left.\left. - \frac{\tau}{2} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right)^2 \right] \right\} \right\} = \frac{P_t^{k,real}}{\delta_0 \eta u_t^{\eta-1}} e^{\mu + \varepsilon_{z,t+1}} \tag{32}$$

Given (21), the first-order condition on bonds (15) reads as

$$E_t \left(\frac{\Lambda_{t+1}^B}{\Lambda_t^B} \frac{1 + r_t}{1 + \pi_{t+1}} \frac{z_{t+1}}{z_t} \right) = E_t \left(\tilde{Q}_{t,t+1} \frac{1 + r_t}{1 + \pi_{t+1}} \right) = e^{\mu + \varepsilon_{z,t+1}} \tag{33}$$

Finally, it is straightforward to rewrite the first-order conditions on labor (14), capital utilization rate (16), and investment (17), respectively as

$$\frac{1 - L_t}{c_t} = \frac{\varpi}{w_t^{real}} \tag{34}$$

$$\frac{P_t^{k,real}}{\delta_0 \eta u_t^{\eta-1}} = \frac{1}{1 - \tau \left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}} \right)} \tag{35}$$

$$\frac{1}{1 - \tau \left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}} \right)} = q_t \tag{36}$$

A.3.5 Asset Pricing

From (30) together with (33), the stochastic discount factor is

$$E_t \left(\tilde{Q}_{t,t+1} \right) = E_t \left(Q_{t,t+1} \frac{z_{t+1}}{z_t} \right) = E_t \left(Q_{t,t+1} e^{x_{t,t+1} \ln(1-\Delta) + \mu + \varepsilon_{z,t+1}} \right)$$

$$\Leftrightarrow \quad E_t \left(Q_{t,t+1} \right) = \frac{E_t \left(\tilde{Q}_{t,t+1} \right)}{e^{\mu + \varepsilon_{z,t+1}} (1 - \theta_t \Delta)} \tag{37}$$

such that, by (23), we have the real return on capital as

$$R_{t+1}^{k,real} = (1 - \theta_t \Delta) \frac{1 + r_t}{1 + \pi_{t+1}} \tag{38}$$

which further defines the numerator of our risk premium. Note that we can equivalently write it as

$$R^{k,real}_{t+1} = (1-\theta_t\Delta)\frac{\delta_0\eta u_t^{\eta-1}}{P_t^{k,real}}P^{k,real}_{t+1}\left\{u_{t+1} + \frac{1}{\delta_0\eta u_{t+1}^{\eta-1}}\left[1-\delta_0 u_{t+1}^{\eta}\right.\right.$$
$$\left.\left.+\tau\frac{i_{t+1}}{k_{t+1}}\left(\frac{i_{t+1}}{k_{t+1}}-\frac{\bar{i}}{\bar{k}}\right)-\frac{\tau}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\frac{\bar{i}}{\bar{k}}\right)^2\right]\right\} \tag{39}$$

or again, using the Tobin's q, as

$$R^{k,real}_{t+1} = (1-\theta_t\Delta)\left\{\frac{P^{k,real}_{t+1}u_{t+1}}{q_t} + \frac{q_{t+1}}{q_t}\left[1-\delta_0 u_{t+1}^{\eta}\right.\right.$$
$$\left.\left.+\tau\frac{i_{t+1}}{k_{t+1}}\left(\frac{i_{t+1}}{k_{t+1}}-\frac{\bar{i}}{\bar{k}}\right)-\frac{\tau}{2}\left(\frac{i_{t+1}}{k_{t+1}}-\frac{\bar{i}}{\bar{k}}\right)^2\right]\right\} \tag{40}$$

A.4 The role of the EIS on households' decisions

A.4.1 The response of the discount factor to the disaster risk

The EIS is given by the following combination of parameters in our model

$$EIS = \frac{1}{1-(1+\varpi)(1-\psi)}$$

so that the time-varying discount factor (6) can be rewritten as

$$\beta(\theta) = \beta_0\left[1-\theta_t\left(1-e^{(1-\gamma)\ln(1-\Delta)}\right)\right]^{\frac{1-1/EIS}{(1-\gamma)(1+\varpi)}}$$

Taking the derivate with respect to the probability of disaster gives

$$\frac{\partial\beta(\theta)}{\partial\theta} = \beta_0\underbrace{\frac{1-1/EIS}{(1-\gamma)(1+\varpi)}}_{A}\underbrace{\left[e^{(1-\gamma)\ln(1-\Delta)}-1\right]}_{B}\underbrace{\left[1-\theta_t\left(1-e^{(1-\gamma)\ln(1-\Delta)}\right)\right]^{\frac{1-1/EIS}{(1-\gamma)(1+\varpi)}-1}}_{C}$$

The sign of this expression crucially depends on the value of the EIS. Given $\varpi > 0$, $\Delta > 0$, $\theta > 0$, $\beta_0 > 0$, we have:

- With EIS < 1 and $\gamma > 1$, A>0, B>0, C>0, so $\frac{\partial\beta(\theta)}{\partial\theta} > 0$;
- With EIS < 1 and $0 \leq \gamma < 1$, A<0, B< 0, C>0, so $\frac{\partial\beta(\theta)}{\partial\theta} > 0$;
- With EIS > 1 and $\gamma > 1$, A<0, B>0, C>0, so $\frac{\partial\beta(\theta)}{\partial\theta} < 0$;

- With EIS > 1 and $0 \leq \gamma < 1$, A>0, B<0, C>0, so $\frac{\partial \beta(\theta)}{\partial \theta} < 0$;
- With $\lim_{EIS \to 1} \frac{\partial \beta(\theta)}{\partial \theta} \to 0$.

Overall, an increase in the probability of disaster makes agents more patient (higher $\beta(\theta)$) when the EIS is below unity, and inversely, more impatient (lower $\beta(\theta)$) when the EIS is above unity. This holds for all degrees of risk aversion (all values of γ), including risk neutrality.

A.4.2 The response of the riskfree rate to the disaster risk (along the balanced growth path)

Along the balanced growth path, the riskfree rate is given by

$$R^f = \frac{\left[1 - \theta\left(1 - e^{(1-\gamma)\ln(1-\Delta)}\right)\right]^{\frac{\varpi(1-\gamma)+1/EIS-\gamma}{(1+\varpi)(1-\gamma)}}}{\beta_0 e^{-\mu\frac{\varpi+1/EIS}{1+\varpi}}\left[1 - \theta\left(1 - e^{-\gamma\ln(1-\Delta)}\right)\right]}$$

The derivative $\partial R^f / \partial \theta$ is always negative, i.e the riskfree rate decreases in the disaster risk for all values of the EIS and risk aversion. However, the magnitude of the slump is sensitive to the value of the EIS: the riskfree rate decreases more with the disaster risk for an EIS below unity than for an EIS above unity, given the degree of risk aversion (including risk neutrality). For instance, with the baseline calibration we find

- With EIS $= 0.5$ and $\gamma = 3.8$, $\frac{\partial R^f}{\partial \theta} \approx -0.666$;
- With EIS $= 2$ and $\gamma = 3.8$, $\frac{\partial R^f}{\partial \theta} \approx -0.504$;
- With EIS $= 0.5$ and $\gamma = 0.5$, $\frac{\partial R^f}{\partial \theta} \approx -0.324$;
- With EIS $= 2$ and $\gamma = 0.5$, $\frac{\partial R^f}{\partial \theta} \approx -0.217$;
- With EIS $= 0.5$ and $\gamma = 0$, $\frac{\partial R^f}{\partial \theta} \approx -0.291$;
- With EIS $= 2$ and $\gamma = 0$, $\frac{\partial R^f}{\partial \theta} \approx -0.190$

Note again that this is not a general equilibrium effect.

A.4.3 The response of the return on capital to the disaster risk (along the balanced growth path)

Along the balanced growth path, the return on capital is given by

$$R^k = \frac{1-\theta\left(1-e^{\ln(1-\Delta)}\right)}{\beta_0 e^{-\mu\frac{\varpi+1/EIS}{1+\varpi}}\left[1-\theta\left(1-e^{(1-\gamma)\ln(1-\Delta)}\right)\right]^{\frac{1-1/EIS}{(1-\gamma)(1+\varpi)}}}$$

The derivative $\partial R^k/\partial\theta$ is also always negative, i.e the rate of return on capital decreases in the disaster risk. However, just as for the riskfree rate, the decrease is larger when the EIS is below unity (rather than above), for all values of risk aversion (including risk neutrality). For instance, we have

- With EIS = 0.5 and $\gamma = 3.8$, $\frac{\partial R^k}{\partial\theta} \approx -0.332$;
- With EIS = 2 and $\gamma = 3.8$, $\frac{\partial R^k}{\partial\theta} \approx -0.169$;
- With EIS = 0.5 and $\gamma = 0.5$, $\frac{\partial R^k}{\partial\theta} \approx -0.295$;
- With EIS = 2 and $\gamma = 0.5$, $\frac{\partial R^k}{\partial\theta} \approx -0.188$;
- With EIS = 0.5 and $\gamma = 0$, $\frac{\partial R^k}{\partial\theta} \approx -0.291$;
- With EIS = 2 and $\gamma = 0$, $\frac{\partial R^k}{\partial\theta} \approx -0.190$

A.4.4 The response of the risk premium to the disaster risk (along the balanced growth path)

Finally, along the balanced growth path, the risk premium is given by

$$Premium = \frac{\left[1-\theta\left(1-e^{\ln(1-\Delta)}\right)\right]\left[1-\theta\left(1-e^{-\gamma\ln(1-\Delta)}\right)\right]}{1-\theta\left(1-e^{(1-\gamma)\ln(1-\Delta)}\right)}$$

The derivative, calculated under our calibration values, gives

- With $\gamma = 3.8$, $\frac{\partial E(R^k)/R^f}{\partial\theta} \approx 0.333$;
- With $\gamma = 0.5$, $\frac{\partial E(R^k)/R^f}{\partial\theta} \approx 0.029$;
- With $\gamma = 0$, $\frac{\partial E(R^k)/R^f}{\partial\theta} = 0$.

The risk premium reacts positively to the disaster risk, and the larger the risk aversion the larger its magnitude. It does not directly depend on the value of the EIS along the balanced growth path, in line with Gourio (2012). However, in general equilibrium, the EIS plays a qualitative role: the larger the EIS, the smaller the risk premium in response to the disaster risk shock (comparing the impulse response functions from Figures 1 to 4).

B Firms' problem

B.1 Production aggregation

The aggregate of intermediate goods is given by

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

so that the representative firm in the final sector maximizes profits as

$$\underset{Y_{t,j}}{max}\ p_t \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} - \int_0^1 p_{j,t} Y_{j,t} dj$$

The first-order condition with respect to $Y_{t,j}$ yields a downward sloping demand curve for each intermediate good j as

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

The nominal value of the final good is the sum of prices times quantities of intermediates

$$p_t Y_t = \int_0^1 p_{j,t} Y_{j,t} dj$$

in which Y_t is substituted to give the aggregate price index as

$$p_t = \left(\int_0^1 p_{j,t}^{1-\nu} dj \right)^{\frac{1}{1-\nu}}$$

B.2 Cost minimization

Firms are price-takers in the input markets, facing (non-detrended) nominal wage W_t^{nom} and capital rental rate P_t^k. They choose the optimal quantities of labor and capital given the input prices and subject to the restriction of producing at least as much as the intermediate good is demanded at the given price. The intra-temporal problem is thus

$$\min_{L_{j,t},\tilde{K}_{j,t}} \quad W_t L_{j,t} + P_t^k \tilde{K}_{j,t}$$

$$\text{s.t.} \quad \tilde{K}_{j,t}^{\alpha}(z_t L_{j,t})^{1-\alpha} \geq \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

The (detrended) first-order conditions are

$$(L_{j,t}:) \quad w_t = mc_{j,t}^{nom}\,(1-\alpha)\left(\frac{\tilde{k}_{j,t}}{L_{j,t}}\right)^{\alpha}$$

$$(\tilde{K}_{j,t}:) \quad P_t^k = mc_{j,t}^{nom}\alpha\left(\frac{\tilde{k}_{j,t}}{L_{j,t}}\right)^{\alpha-1}$$

in which the Lagrange multiplier denoted $mc_{j,t}^{nom}$ can be interpreted as the (nominal) marginal cost associated with an additional unit of capital or labor. Rearranging gives the optimal capital over labor ratio as

$$\left(\frac{\tilde{k}_{j,t}}{L_{j,t}}\right)^{*} = \frac{w_t}{P_t^k}\frac{\alpha}{(1-\alpha)}$$

in which none of the terms on the right hand side depends on j, and thus holds for all firms in equilibrium, *i.e.*, $\frac{\tilde{k}_t}{L_t} = \frac{\tilde{k}_{j,t}}{L_{j,t}}$. Replacing in one of the first-order conditions above gives

$$mc_t^{nom*} = \left(\frac{P_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}$$

Reexpressing in real terms $mc_t^* = mc_t^{nom*}/p_t$, we finally have

$$mc_t^* = \left(\frac{P_t^{k,real}}{\alpha}\right)^{\alpha}\left(\frac{w_t^{real}}{1-\alpha}\right)^{1-\alpha}$$

where $P^{k,real}$ and w^{real} are the real capital rental rate and (detrended) wage.

B.3 Profit maximization

Let us now consider the pricing problem of a firm that gets to update its price in period t and wants to maximize the present discounted value of future profits. The (nominal) profit flows read as

$$p_{j,t}Y_{j,t} - W_t L_{j,t} - P^k_{j,t}\tilde{K}_{j,t} = (p_{j,t} - mc^{nom}_t)Y_{j,t}$$

which can be reexpressed in real terms as $\frac{p_{j,t}}{p_t}Y_{j,t} - mc^*_t Y_{j,t}$. These profit flows are discounted by both the stochastic discount factor, $Q_{t,t+s}$, and by the probability ζ^s that a price chosen at time t is still in effect at time s. Finally, given $Y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$, the maximization problem is thus

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\left(\frac{p_{j,t}}{p_{t+s}}\right)^{1-\nu} Y_{t+s} - mc^*_{t+s} \left(\frac{p_{j,t}}{p_{t+s}}\right)^{-\nu} Y_{t+s} \right)$$

which can be further simplified, using $mc^*_t = \frac{mc^{nom}_t}{p_t}$ and factorizing, as

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p^{\nu-1}_{t+s} Y_{t+s} \left(p^{1-\nu}_{j,t} - mc^{nom}_t p^{-\nu}_{j,t} \right)$$

The first-order condition is then

$$E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p^{\nu-1}_{t+s} Y_{t+s} \left((1-\nu) p^{-\nu}_{j,t} + \nu\, mc^{nom}_t p^{-\nu-1}_{j,t} \right) = 0$$

which simplifies as

$$p^*_{j,t} = \frac{\nu}{\nu-1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p^{\nu}_{t+s} Y_{t+s} mc^*_{t+s}}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p^{\nu-1}_{t+s} Y_{t+s}}$$

Note that this optimal price depends on aggregate variables only, so that $p^*_t = p^*_{j,t}$. Expressed as a ratio over the current price, we thus have

$$\frac{p^*_t}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t}\right)^{\nu} Y_{t+s} mc^*_{t+s}}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t}\right)^{\nu-1} Y_{t+s}}$$

which can be written recursively as

$$\frac{p^*_t}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\Xi_{1_t}}{\Xi_{2_t}} \quad \text{with}$$

$$\Xi_{1_t} = Y_t mc^*_t + \zeta E_t Q_{t,t+1} \left(\frac{p_{t+1}}{p_t}\right)^{\nu} \Xi_{1_{t+1}}$$

$$\Xi_{2_t} = Y_t + \zeta E_t Q_{t,t+1} \left(\frac{p_{t+1}}{p_t}\right)^{\nu-1} \Xi_{2_{t+1}}$$

Replacing $Q_{t,t+1} \equiv \tilde{Q}_{t,t+1} \frac{z_t}{z_{t+1}}$, and detrending, these are simplified as

$$\tilde{\Xi}_{1_t} = y_t mc_t^* + \zeta E_t \tilde{Q}_{t,t+1} \left(\frac{p_{t+1}}{p_t}\right)^{\nu} \tilde{\Xi}_{1_{t+1}} \quad (41)$$

$$\tilde{\Xi}_{2_t} = y_t + \zeta E_t \tilde{Q}_{t,t+1} \left(\frac{p_{t+1}}{p_t}\right)^{\nu-1} \tilde{\Xi}_{2_{t+1}} \quad (42)$$

where $\tilde{\Xi}_t \equiv \Xi_t / z_t$, the detrended variable.

C Aggregation

C.1 Bonds market

Market-clearing requires the public debt to equate the quantity of bonds purchased by the households at each time, i.e $Debt_t = B_t$, therefore making the public debt symmetrically affected by disasters. Moreover, we assume that the public budget has to be balanced every period, i.e the sum of tax revenues and new debt issuance must be equal to the current debt issuance to be repaid with interest rates (as for the non-disaster part), i.e

$$T_t p_t + B_{t+1} = [B_t(1 + r_{t-1})] e^{x_{t+1} \ln(1-\Delta)}$$

or, in detrended terms,

$$\frac{T_t}{z_t} + \frac{b_{t+1}}{p_t} = \frac{b_t(1 + r_{t-1})}{p_t e^{\mu + \varepsilon_{z,t+1}}}$$

C.2 Aggregate demand

Replacing the tax level above into the household's budget constraint gives

$$c_t + i_t + \left(\frac{b_t(1 + r_{t-1})}{p_t e^{\mu + \varepsilon_{z,t+1}}} - \frac{b_{t+1}}{p_t}\right) = w_t^{real} L_t + \left(\frac{b_t(1 + r_{t-1})}{p_t e^{\mu + \varepsilon_{z,t+1}}} - \frac{b_{t+1}}{p_t}\right) + P_t^{k,real} u_t k_t + \frac{D_t}{z_t}$$

which just simplifies as

$$c_t + i_t = w_t^{real} L_t + P_t^{k,real} \tilde{k}_t + \frac{D_t}{z_t}$$

i.e, in non-detrended terms, as

$$C_t + I_t = W_t^{real} L_t + P_t^{k,real} \tilde{K}_t + D_t$$

where we now have to verify that the RHS is equal to Y_t. Total dividends (or profits) D_t must be equal to the sum of dividends (or profits) from intermediate good firms, i.e

$$D_t = \int_0^1 D_{j,t} dj$$

The (real) dividends (or profits) from intermediate good firms j are given by

$$D_{j,t} = \frac{p_{j,t}}{p_t} Y_{j,t} - W_t^{real} L_{j,t} - P_t^{k,real} \tilde{K}_{j,t}$$

Substituting $Y_{j,t}$, we have

$$D_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t - W_t^{real} L_{j,t} - P_t^{k,real} \tilde{K}_{j,t}$$

Therefore, knowing that $D_{t(real)} = \int_0^1 D_{j,t(real)} dj$, we get

$$D_t = \int_0^1 \left(\left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t - W_t^{real} L_{j,t} - P_t^{k,real} \tilde{K}_{j,t} \right) dj = \int_0^1 \left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t dj - \int_0^1 W_t^{real} L_{j,t} dj - \int_0^1 P_t^{k,real} \tilde{K}_{j,t} dj$$

$$D_t = \int_0^1 \left(\left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t - W_t^{real} L_{j,t} - P_t^{k,real} \tilde{K}_{j,t} \right) dj = Y_t \frac{1}{p_t^{1-\nu}} \int_0^1 (p_{j,t})^{1-\nu} dj - W_t^{real} \int_0^1 L_{j,t} dj - P_t^{k,real} \int_0^1 \tilde{K}_{j,t} dj$$

Given that (i) the aggregate price level is $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$, (ii) aggregate labor demand must equal supply, i.e $\int_0^1 L_{j,t} dj = L_t$, and (iii) aggregate supply of capital services must equal demand $\int_0^1 \tilde{K}_{j,t} dj = \tilde{K}_t$, the aggregate (real) dividend (or profit) is

$$D_t = Y_t - W_t^{real} L_t - P_t^{k,real} \tilde{K}_t$$

Replaced into the household's budget constraint, this finally gives the aggregate accounting identity as

$$Y_t = C_t + I_t$$

or in detrended terms

$$y_t = c_t + i_t$$

C.3 Inflation

Firms have a probability $1-\zeta$ of getting to update their price each period. Since there are an infinite number of firms, there is also the exact fraction $1-\zeta$ of total firms who adjust their prices and the fraction ζ who stay with the previous period price. Moreover, since there is a random sampling from the entire distribution of firm prices, the distribution of any subset of firm prices is similar to the entire distribution. Therefore, the aggregate price index, $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$, is rewritten as

$$p_t^{1-\nu} = \int_0^{1-\zeta} p_t^{*1-\nu} dj + \int_{1-\zeta}^{1} p_{j,t-1}^{1-\nu} dj$$

which simplifies as

$$p_t^{1-\nu} = (1-\zeta) p_t^{*1-\nu} + \zeta p_{t-1}^{1-\nu}$$

Let us divide both sides of the equation by $p_{t-1}^{1-\nu}$

$$\left(\frac{p_t}{p_{t-1}}\right)^{1-\nu} = (1-\zeta)\left(\frac{p_t^*}{p_{t-1}}\right)^{1-\nu} + \zeta\left(\frac{p_{t-1}}{p_{t-1}}\right)^{1-\nu}$$

and define the *gross inflation rate* as

$$1+\pi_t \equiv \frac{p_t}{p_{t-1}}$$

and the *gross reset inflation rate* as

$$1+\pi_t^* \equiv \frac{p_t^*}{p_{t-1}}$$

we get

$$(1+\pi_t)^{1-\nu} = (1-\zeta)(1+\pi_t^*)^{1-\nu} + \zeta$$

Finally, since we know that

$$\frac{p_t^*}{p_t} = \frac{\nu}{\nu - 1} E_t \frac{\Xi_{1_t}}{\Xi_{2_t}}$$

we have the *reset inflation rate* as

$$(1 + \pi_t^*) = (1 + \pi_t) \frac{\nu}{\nu - 1} E_t \frac{\Xi_{1_t}}{\Xi_{2_t}}$$

with the expressions given previously for Ξ_1 and Ξ_2 (see Appendix B.3), or equivalently with the detrended terms as

$$(1 + \pi_t^*) = (1 + \pi_t) \frac{\nu}{\nu - 1} E_t \frac{\tilde{\Xi}_{1_t}}{\tilde{\Xi}_{2_t}}$$

C.4 Aggregate supply

We know that the demand to individual firm j is given by

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

and that firm j hires labor and capital in the same proportion as the aggregate capital to labor ratio (common factor markets). Hence, substituting in the production function for the intermediate good j we get

$$\left(\frac{\tilde{K}_t}{z_t L_t} \right)^{\alpha} z_t L_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

which can be rewritten with detrended variables as

$$\left(\frac{\tilde{k}_t}{L_t} \right)^{\alpha} L_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} y_t$$

Then, summing up across the intermediate firms gives

$$\left(\frac{\tilde{k}_t}{L_t} \right)^{\alpha} \int_0^1 L_{j,t} dj = y_t \int_0^1 \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} dj$$

Given that aggregate labor demand and supply must equal, i.e $\int_0^1 L_{j,t} dj = L_t$, we have

$$\int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj y_t = \tilde{k}_t^{\alpha} L_t^{1-\alpha}$$

Thus, the aggregate production function can be written as

$$y_t = \frac{\tilde{k}_t^{\alpha} L_t^{1-\alpha}}{\Omega_t}$$

where $\Omega_t = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj$ measures a distortion introduced by the dispersion in relative prices.[35] In order to express Ω_t in aggregate terms, let us decompose it according to the Calvo pricing assumption again, so that

$$\Omega_t = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj = p_t^{\nu} \int_0^1 p_{j,t}^{-\nu}$$

$$p_t^{\nu} \int_0^1 p_{j,t}^{-\nu} = p_t^{\nu} \left(\int_0^{1-\zeta} p_t^{*-\nu} dj + \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj \right)$$

$$p_t^{\nu} \int_0^1 p_{j,t}^{-\nu} = p_t^{\nu} (1-\zeta) p_t^{*-\nu} + p_t^{\nu} \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj$$

$$p_t^{\nu} \int_0^1 p_{j,t}^{-\nu} = (1-\zeta) \left(\frac{p_t^*}{p_t}\right)^{-\nu} + p_t^{\nu} \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj$$

$$p_t^{\nu} \int_0^1 p_{j,t}^{-\nu} = (1-\zeta) \left(\frac{p_t^*}{p_{t-1}}\right)^{-\nu} \left(\frac{p_{t-1}}{p_t}\right)^{-\nu} + p_t^{\nu} \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj$$

$$p_t^{\nu} \int_0^1 p_{j,t}^{-\nu} = (1-\zeta)(1+\pi_t^*)^{-\nu}(1+\pi_t)^{\nu} + p_{t-1}^{-\nu} p_t^{\nu} \int_{1-\zeta}^1 \left(\frac{p_{j,t-1}}{p_{t-1}}\right)^{-\nu} dj$$

Given random sampling and the fact that there is a continuum of firms

$$\Omega_t = (1-\zeta)(1+\pi_t^*)^{-\nu}(1+\pi_t)^{\nu} + \zeta(1+\pi_t)^{\nu}\Omega_{t-1}$$

D Full set of equilibrium conditions

Processus for the disaster risk

$$\log \theta_t = (1-\rho_\theta) \log \bar{\theta} + \rho_\theta \log \theta_{t-1} + \sigma_\theta \varepsilon_{\theta_t} \tag{43}$$

[35] This distortion is not the one associated with the monopoly power of firms but an additional one that arises from the relative price fluctuations due to prie stickiness.

Time-varying discount factor

$$\beta(\theta) = \beta_0 \left[1 - \theta_t + \theta_t e^{(1-\gamma)\ln(1-\Delta)}\right]^{\frac{1}{1-\chi}} \tag{44}$$

Households

$$\tilde{Q}_{t,t+1} = \left(\frac{c_{t+1}}{c_t}\right)^{-\psi} \left(\frac{1-L_{t+1}}{1-L_t}\right)^{\varpi(1-\psi)} \beta(\theta_t) e^{(1-\psi)\mu} \frac{e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{-\chi}}{\left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi}\right]^{\frac{-\chi}{1-\chi}}} \tag{45}$$

$$\frac{1-L_t}{c_t} = \frac{\varpi}{w_t^{real}} \tag{46}$$

$$E_t\left(\tilde{Q}_{t,t+1} \frac{1+r_t}{1+\pi_{t+1}}\right) = e^{\mu+\varepsilon_{z,t+1}} \tag{47}$$

$$\frac{P_t^{k,real}}{\delta_0 \eta u_t^{\eta-1}} = \frac{1}{1-\tau\left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}}\right)} \tag{48}$$

$$\frac{1}{1-\tau\left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}}\right)} = q_t \tag{49}$$

$$\begin{aligned} E_t \Bigg\{ \tilde{Q}_{t,t+1} P_{t+1}^{k,real} \Bigg\{ u_{t+1} + \frac{1}{\delta_0 \eta u_{t+1}^{\eta-1}} \Bigg[1 - \delta_0 u_{t+1}^{\eta} + \tau \frac{i_{t+1}}{k_{t+1}} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}}\right) \\ - \frac{\tau}{2}\left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}}\right)^2 \Bigg] \Bigg\} \Bigg\} = \frac{P_t^{k,real}}{\delta_0 \eta u_t^{\eta-1}} e^{\mu+\varepsilon_{z,t+1}} \end{aligned} \tag{50}$$

$$v_t = \left[c_t(1-L_t)^{\varpi}\right]^{1-\psi} + \beta(\theta_t) e^{(1-\psi)\mu} E_t \left[e^{(1-\psi)(1-\chi)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{51}$$

$$k_{t+1} = \frac{(1-\delta_t)k_t + S\left(\frac{i_t}{k_t}\right) k_t}{e^{\mu+\varepsilon_{z,t+1}}} \tag{52}$$

Asset pricing

$$Q_{t,t+1} = \frac{\tilde{Q}_{t,t+1}}{e^{\mu+\varepsilon_{z,t+1}}(1-\theta_t \Delta)} \tag{53}$$

$$E_t\left[Q_{t,t+1}R^f_{t+1}\right] = 1 \tag{54}$$

$$R^{k,real}_{t+1} = (1-\theta_t\Delta)\,\frac{1+r_t}{1+\pi_{t+1}} \tag{55}$$

$$E_t(Premium_{t+1}) \equiv E_t\left(\frac{R^{k,real}_{t+1}}{R^f_{t+1}}\right) \tag{56}$$

Firms' constraints

$$y_t = \frac{\tilde{k}^\alpha L_t^{1-\alpha}}{\Omega_t} \tag{57}$$

$$\tilde{k}_t = u_t k_t \tag{58}$$

$$w^{real}_t = mc^*_t\,(1-\alpha)\left(\frac{\tilde{k}_t}{L_t}\right)^\alpha \tag{59}$$

$$P^{k,real}_t = mc^*_t\alpha\left(\frac{\tilde{k}_t}{L_t}\right)^{\alpha-1} \tag{60}$$

$$(1+\pi^*_t) = (1+\pi_t)\frac{\nu}{\nu-1}E_t\frac{\tilde{\Xi}_{1_t}}{\tilde{\Xi}_{2_t}} \tag{61}$$

$$\tilde{\Xi}_{1_t} = y_t mc^*_t + \zeta E_t[\tilde{Q}_{t,t+1}\,(1+\pi_{t+1})^\nu\,\tilde{\Xi}_{1_{t+1}}] \tag{62}$$

$$\tilde{\Xi}_{2_t} = y_t + \zeta E_t[\tilde{Q}_{t,t+1}\,(1+\pi_{t+1})^{\nu-1}\,\tilde{\Xi}_{2_{t+1}}] \tag{63}$$

Price distorsion:

$$\Omega_t = (1-\zeta)(1+\pi^*_t)^{-\nu}(1+\pi_t)^\nu + \zeta(1+\pi_t)^\nu\Omega_{t-1} \tag{64}$$

Price index:

$$(1+\pi_t)^{1-\nu} = (1-\zeta)(1+\pi^*_t)^{1-\nu} + \zeta \tag{65}$$

Taylor rule:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left[\Phi_\pi(\pi_t-\bar{\pi}) + \Phi_Y\,(y_t-y^*) + r^*\right] \tag{66}$$

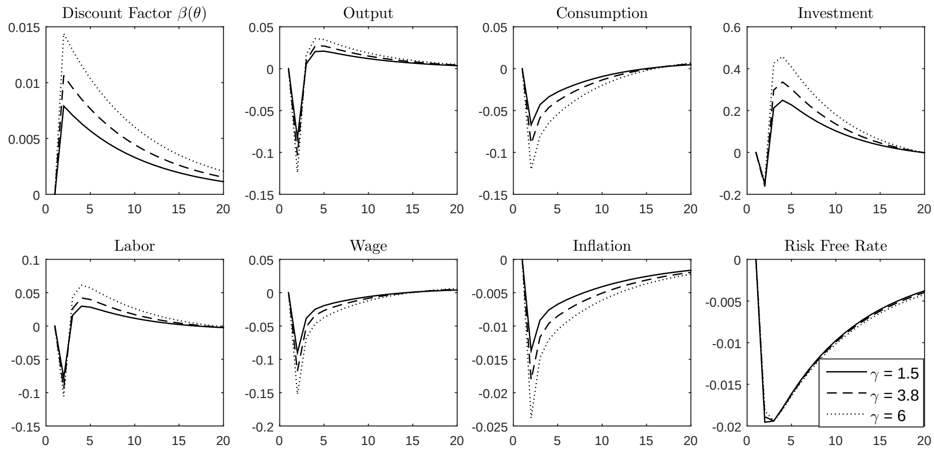
Aggregate resource constraint:

$$y_t = c_t + i_t \tag{67}$$

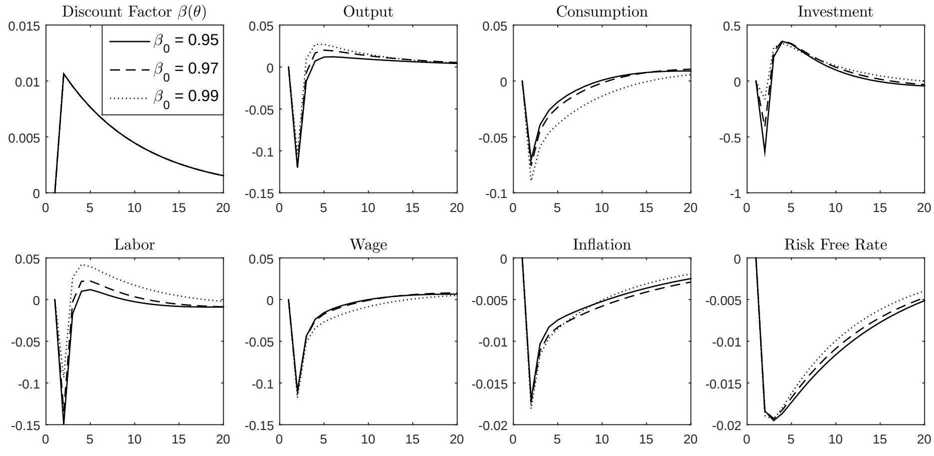
This is a system of 25 equations in 25 unknowns: $\Big\{\theta, \beta(\theta), y, c, i, L, k, \tilde{k}, u, \tilde{Q}, Q, R^{k,real}, R^f, Premium, P^{k,real}, w^{real}, q, \Omega, \pi, \pi^*, \tilde{\Xi}_1, \tilde{\Xi}_2, mc^*, v, r\Big\}$. The responses to a disaster risk shock are computed with a third-order approximation around its steady-state using Dynare.

Figure 5: Responses of main macroeconomic variables to a disaster risk shock; baseline scenario (EIS $= 0.5$ and $\zeta = 0.8$) and sensitivity analysis to alternative levels of other parameters. Third-order approximation.

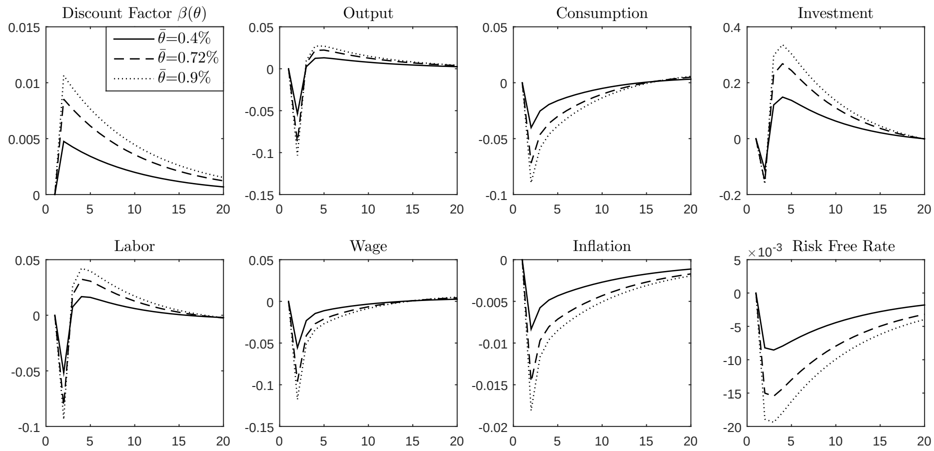
- Risk aversion coefficient, γ



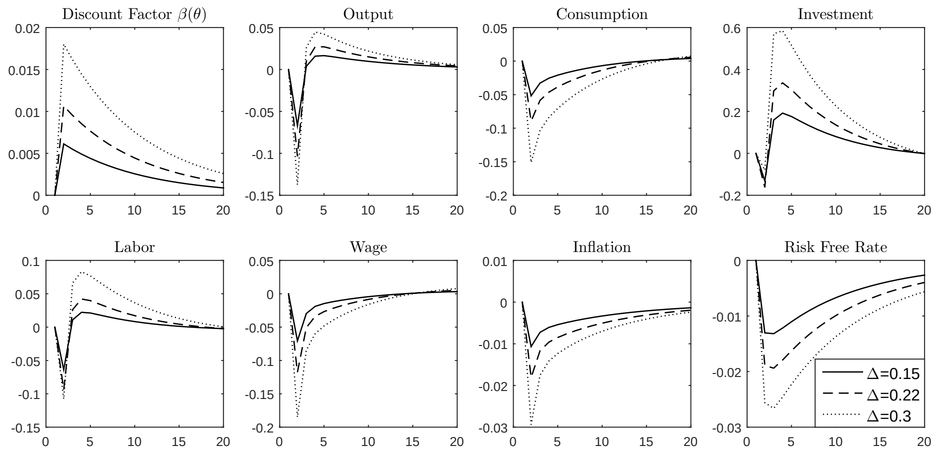
- Fixed discount factor parameter, β_0



- Steady-state probability of disaster, $\bar{\theta}$



- Size of disaster, Δ



- Persistence of disaster probability, ρ_θ

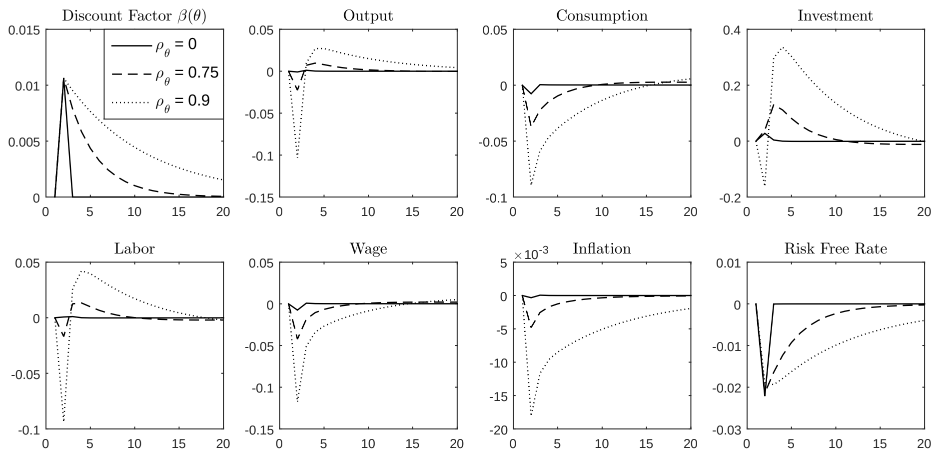
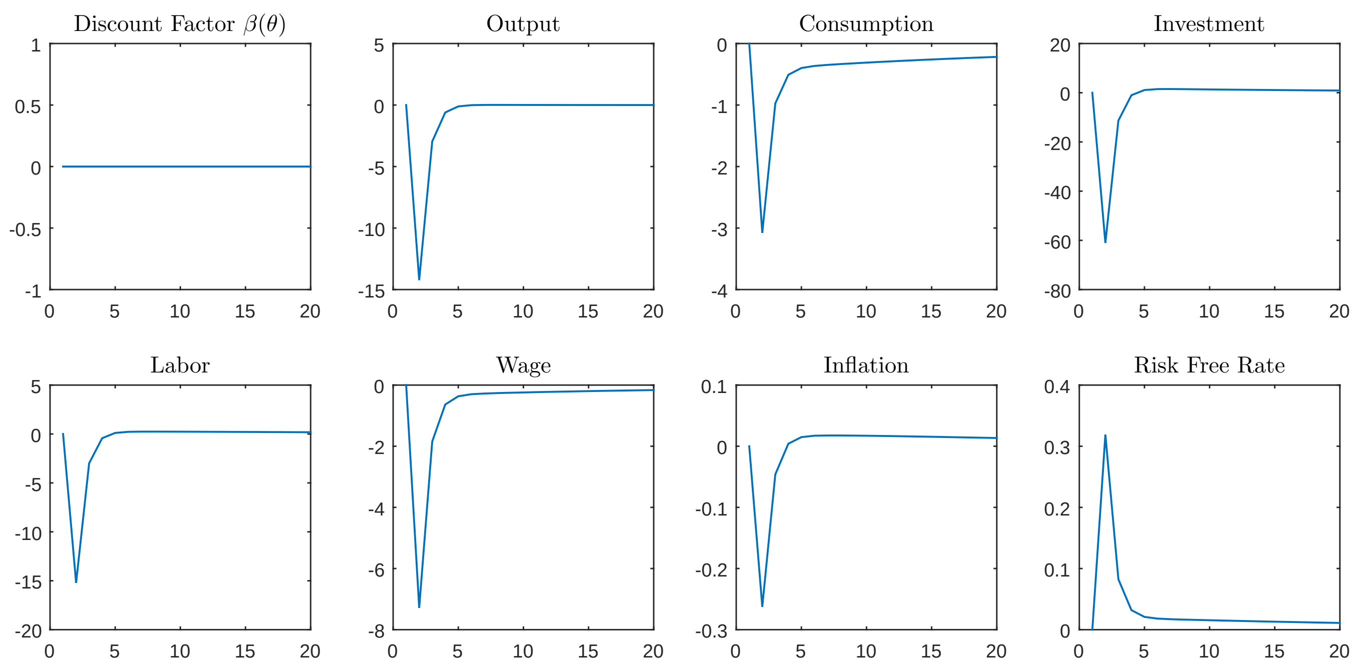


Figure 6: Responses (in percentage change) to a 1% change in the nominal interest rate on bonds; baseline scenario (EIS $= 0.5$ and $\zeta = 0.8$).



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