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Sequential Coordination, Higher-Order Belief Dynamics and the E-stability principle*

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†Banque de France, Monetary Policy Research Division [DGEI-DEMFI-POMONE], 31 rue Croix des Petits Champs 41-1391, 75049 Paris Cedex 01, France. Comments welcome at : *gaetano.gaballo@banque-france.fr*.

Résume. Cet article explore la convergence des croyances d'ordre supérieur - autrement dit la stabilité divinatoire - lorsque la coordination est séquentielle, c'est à dire, lorsque chaque agent d'un certain type décide de ses actions après avoir observé celles de types antérieurs, dans un ordre donné. La présence de types séquentiels améliore la coordination des anticipations en cas de substituabilité stratégique, mais pas dans le cas de complémentarité stratégique. La stabilité divinatoire peut être obtenue pour n'importe quel niveau de substituabilité stratégique, à condition que le nombre de types séquentiels soit sufficiement grand. Par conséquent, quand la coordination est séquentielle la convergence divinatoire se produit sous les mêmes conditions que la convergence adaptative, conformément au principe dit de E-stability.

Keywords: apprentissage eductive, équilibre aux anticipations rationnelles, ensemble rationalizable, apprentissage in macroéconomie, jeux de coordination.

JEL codes: D41, E30, B41.

Abstract. This paper explores convergence in higher-order beliefs - otherwise called eductive stability - when coordination is sequential, that is, when each agent of a given type fixes his own actions after observing the ones of earlier types in a given order. The presence of sequential types enhances expectational coordination in case of strategic substitutability, but not in case of strategic complementarity. In particular eductive stability can be obtained for any degree of substitutability, provided the number of sequential types is large enough. Therefore, sequential coordination opens up to the possibility that eductive convergence occurs at the same conditions of adaptive convergence, in accordance to the E-stability principle.

Keywords: eductive learning, rational expectation equilibria, rationalizable set, learning in macroeconomics, coordination games.

JEL code: D41, E30, B41.

Non-technical summary

One of the most delicate aspect of economic modelling is how agents form expectations about future outcomes. Most of the key macroeconomic dynamics are in fact highly sensitive to the specific hypothesis about expectation formation. The traditional view assumes that expectations are rational in the sense of Muth (1961), i.e. agents in the model share the same knowledge that the modeler has, and they can perfectly coordinate. This notion is the equivalent to the concept of Nash equilibrium in game theory, where agents have no uncertainty about which actions the others are taking. Therefore, a rational expectations equilibrium (REE) requires an highly demanding degree of coordination which is assumed but not derived from first principles.

The Eductive learning approach assesses whether or not such convergence in beliefs can be recovered from common knowledge of rationality. Eductive convergence occurs when the REE equilibrium is the only outcome logically implied by the structure of the model. Otherwise, the theory is somehow incomplete and requires to specify which coordination device ensures convergence of beliefs.

In alternative, one could think that the agents we want to model cannot be more sophisticated than econometricians in reality. Adaptive learning assumes that agents use past observables to form expectations about the future using standard statistical tools. According to this view, expectations are backward-looking and hinge on recent outcomes. The two approaches, eductive and adaptive, give quite different convergence results. In particular, eductive convergence implies adaptive convergence, whereas the opposite does not hold.

The contribution of this paper is to clarify that eductive convergence can be obtained at the same conditions of adaptive convergence when the coordination is sequential, that is, when agents act according to a given order and observe their predecessors' actions. This result establishes that the nature of the different convergence conditions is not due to the different degree of rationality assumed in the two approaches, but it is rather a consequence of the presence of time. In fact, the introduction of time by sequential coordination generates beneficial spillover effects in the eductive reasoning which I show can be also uncovered in the basic principle of the adaptive convergence.

1 Introduction

Eductive and adaptive learning provide different conditions under which out-of-the-equilibrium convergence on a particular rational expectation equilibrium (REE) occurs in macroeconomic models. Adaptive learning (Marcet and Sargent, (1989); Evans and Honkapohja, (2001)) assumes agents act like econometricians using standard statistical tools to calibrate a forecasting rule as new evidence becomes available over time. Eductive learning (Guesnerie (2005)) instead assumes more sophisticated agents who behave like game theorists reasoning on the higher-order implications of common knowledge (rationalizability). Typically the latter requires stricter convergence conditions than the former.

This paper demonstrates that sequential coordination enhances eductive stability opening up to the possibility that eductive and adaptive convergence occur at the same conditions. Sequential coordination introduces real time[1] into the eductive reasoning yielding epistemic implications in analogy with adaptive learning. When coordination is sequential, atomistic agents of a type form their own forecast after observing the average forecast, or equivalently the average action, of earlier types in a given order. Hence, a type cannot disagree on the beliefs of their predecessors, similarly with adaptive learning where agents cannot disagree on commonly observed past data. This paper shows that this particular form of epistemic restriction facilitates convergence in higher-order beliefs up to the point where eductive stability occurs at the same conditions of adaptive learning, that is, in accordance to the E-stability principle[2].

I investigate eductive learning in the context of self-referential liner economies populated by atomistic agents belonging to n sequential types with possibly disparate sizes. The sequential nature of coordination shapes a backward-induction rationalizability process. In the last subgame, agents type n observe the predetermined forecasts of other types, but need to anticipate the average expectation of their own type. Provided convergence in beliefs occurs, then it is common knowledge that agents type n will coordinate on the same response to the forecasts of earlier types from 1 to $n-1$. In this case, agents type $n-1$, who observe the average forecast of types from 1 to $n-2$, can also anticipate the indirect impact of their average forecast through the common response of type n. This spillover effect can stabilize or destabilize the higher-order reasoning of type $n-1$. Specifically, with strategic substitutability (resp. complementarity) the spillover spurs convergence: the average action type n counteracts to (resp. co-moves with) the common action type $n-1$ stabilizing (resp. destabilizing) the belief dynamics. If convergence is achieved, then it is common knowledge that agents type $n-1$ will coordinate on the same response to the actions of earlier types from 1 to $n-2$. Iterating we have that, as long as convergence conditions are met at later subgames, agents type τ, who directly observe the average forecast of types from 1 to $n-\tau$, can also anticipate the indirect impact of their average forecast through the common response of types from $\tau+1$ to n. Successful convergence at all subgames implies agents coordinate on the unique REE, which is therefore the only rationalizable outcome.[3]

[1] "Real" time is used in contrast with "notional" time which typically indexes the hierarchy of higher-order beliefs.

[2] George W. Evans and Seppo Honkapohja have introduced this terms in the literature.

[3] In other words, the REE is strongly rational (Guesnerie (1992)), that is, it is the only profile of

The introduction of sequential types enlarges the conditions for convergence of higher-order beliefs in case of strategic substitutability but *not* in case of strategic complementarity. In particular, there exists a correspondence between the degree of strategic substitutability and the minimal number of types needed to ensure coordination. I demonstrate that the impact of an additional type on the enlargement of the convergence region is increasing with the number of types. Put differently, very few sequential types make the difference at very high degree of strategic substitutability. With a sufficiently large number of types eductive convergence is achieved for any degree of strategic substitutability, as provided for by the E-stability principle. The final section depicts a parallel between the sequential eductive analysis and the convergence of adaptive learning rules. The presence of stabilizing sequential spillovers, introduced by a constant-gain correction term, also governs the emergence of adaptive convergence.

This paper is in line with an observation made in the seminal paper by Guesnerie (1992) which concerns the enlargement of convergence conditions in the case of strategic substitutability when coordination is sequential between two symmetrical types of agents.[4] Differently from the argument sketched in that paper, the present work considers a finite, arbitrarily large, number of types with disparate relative sizes and also looks at the case of strategic complementarity. Some efforts have been done before in the literature to bridge the eductive and the adaptive approach. Evans (1985), Evans and Guesnerie (1993) and Evans (2001) introduce the concept of iterative E-stability which concerns the discrete iteration of the map from the coefficients of a well-specified forecasting rule to the coefficients of the actual law of motion. The conditions for iterative E-stability are the same of eductive stability provided the Cobweb model as a sufficient homogeneous structure. Nevertheless, iterative E-stability substantially deviates from the idea of agents acting as econometricians and has no microfoundations.

2 Self-referential linear models

This section presents two benchmark linear models widely investigated by the rational expectation literature: the so-called Cobweb model, due to Muth (1961), and the island model proposed by Lucas (1973). The two economies have a common central feature - atomistic agents choose an action as a best reply to the average action - but they differ in the way agents react: in the latter the individual best action increases with the average one, a case of strategic complementarity, whereas in the former the individual best action decreases with the average one, a case of strategic substitutability.

Although simple, these models provide insights for the eductive dynamics in non-linear and/or stochastic models (see Guesnerie, 1992) and economies where expectations are not on the current but on the future state (Guesnerie, 1993). A transposition to more sophisticated settings can be recovered as explained in the aforementioned works.

actions contained in the rationalizable set as defined by Pearce (1984) and Bernheim (1984)).

[4] A pedagogical intuition, not a rigorous argument, is illustrated in Chamley (2004) pag. 246. That analysis leads to misleading conclusions in the case of strategic complementarity.

2.1 Strategic substitutability

Consider an infinite number of homogeneous firms, indexed in the unit interval. A firm $i \in (0,1)$ sets a positive quantity $q_i \geq 0$ to maximize expected profit $E^i(P)\, q_i - q_i^2/2c$ where $P > 0$ denotes price, $q_i^2/2c$ is a quadratic cost function parameterized by $c > 0$, and $E^i(\cdot)$ labels the expectation operator of supplier i. Optimal individual supply is given by $q_i = cE^i(P)$, that is, individual supply is proportional to the individual expected price.

We assume an exogenous aggregate demand curve given by $D = \mu - \phi P$ with $\mu, \phi, D > 0$. Clearing conditions imply total supply equals total demand. The relation between suppliers' expectations and the actual price is given by

$$p = \beta e, \tag{1}$$

where $p \equiv P - P^*$, $e_i \equiv E^i(p)$ and $e \equiv \int e_i \,\mathrm{di}$, with $\beta \equiv -c/\phi$ and $P^* \equiv \mu/(\phi + c)$ featuring the unique REE. Since both aggregate demand D and the market price P are required to be positive, then $p \in \aleph$ where $\aleph \equiv (-P^*, -\beta P^*)$.

In this model it is always $\beta < 0$. It entails strategic substitutability: the higher the average produced quantity (that is, the higher the average price expectation), the lower the actual price and hence the *lower* the optimal individual supply.

2.2 Strategic complementarity

Lucas (1973) provides a simple example of a macro-model featuring the same structural relation (1) but with a $\beta > 0$, which implies strategic complementarity. Suppose now that each supplier is placed in a island and produces a good which is traded at a price P_i. The "Lucas supply" function is $q_i = \bar{q} + \gamma(P_i - E^i(P))$ with $\gamma, \bar{q} > 0$, where $P \equiv \int P_i \mathrm{di}$. In words, a producer increases his output above $\bar{q}$ when his own price is greater than what he expects as the aggregate price in the economy. The aggregate demand D is the same as before with the restriction $\bar{q} < \mu$. Market clearing implies aggregate supply equals aggregate demand. The relation between suppliers' expectations and the actual price is the same as (1) where we redefine $\beta \equiv \gamma/(\gamma + \phi)$ and $P^* \equiv (\mu - \bar{q})/\phi$ being the unique REE. As before $p \in \aleph$.

In this model it is always $\beta > 0$. Unlike the Cobweb model, it entails strategic complementarity: the higher the average produced quantity (that is, the higher the average price expectation), the lower the actual average price and hence the *higher* the optimal individual supply.

3 Benchmark analysis of expectational coordination

This section will quickly recall the classical eductive analysis under simultaneous coordination firstly formalized by Guesnerie (1992) and reviewed in Guesnerie (2005). I will also briefly contrast eductive convergence conditions with the *E-stability principle* governing adaptive learning convergence. This establishes the benchmark over which we will later discuss sequential eductive convergence.

3.1 Simultaneous eductive coordination

The eductive approach concerns itself with the assessment of the iterative implications of common knowledge (CK) of the model and rationality. CK implies that agents commonly know that $p \in \aleph$ and so that necessarily $e \in \aleph$. Since the market clearing correspondence, which links an average expectation to an actual market price, is also CK, then agents will conclude that a possible outcome of the game will actually lie in $B(\aleph)$ where $B : \aleph \to \aleph$ according to (1). This is a second-order conjecture on the domain of possible aggregate outcomes. But given $B(\aleph)$ is also CK then, for the same argument, agents know that a rational outcome cannot lie out of $B^2(\aleph)$, and so on, along an infinite hierarchy of higher-order beliefs.

Finally, it is CK that the final outcome has to lie in $\lim_{\nu \to \infty} B^{\nu}(\aleph)$. If and only if B is a contracting map, that is $|\beta| < 1$, then iterative applications of rational responses pin down a single expectational profile as a limit fix point, which is the unique REE. In this case, we will say that the REE is strongly rational, or equivalently eductively stable, according to the definition provided by Guesnerie (1992). Otherwise, the REE is just one of the rationalizable outcome that agents can entertain given CK of rationality and market clearing conditions. In such a case the emergence of the unique REE is not guaranteed unless external coordination devices are introduced. In other words, eductive instability implies indeterminacy of rational beliefs.

The conditions for eductive simultaneous convergence among a finite number n of firms obtains as a generalization of the result whenever $n(2-n)^{-1} < \beta < 1$ (see Gaballo, 2013)[5]. In the limit of an infinite number of players ($n \to \infty$) the condition becomes $|\beta| < 1$ which corresponds to the classical result briefly discussed above.

3.2 Adaptive learning and the E-stability principle

The adaptive learning literature assesses whether or not agents, reasoning as econometricians, will coordinate on the equilibrium after enough observable outcomes are generated over time. According to this approach, agents are boundedly rational. They do not exploit implications from the model (which could be well unknown) but just reason as time-series econometricians who use past observations as predictors. The same B mapping governing eductive convergence is central to the so called *E-stability principle* establishing the conditions for adaptive convergence.

Consider the economy is taking place at time t so that $p_t = \beta e_t$. Suppose that e_t is the average expectation of a continuum of homogeneous adaptive learners who use OLS regression on past outcomes $\{p_h\}_0^{t-1}$ to calibrate their best forecast. The E-stability principle establishes that estimates would converge to the unique REE of the cobweb model, formally $\lim_{t \to \infty} e_t = 0$, if the ordinary differential equation

$$\frac{\partial e_t}{\partial t} = B(e_t) - e_t,$$

[5]With a finite number of suppliers β depends also on the degree of market power.

is stable at its fix point $e = 0$, namely the unique REE.[6] This condition is satisfied if and only if $\beta < 1$. Hence, on one hand eductive and adaptive convergence occur at the same conditions in case of strategic complementarity. On the other hand, adaptive convergence, in contrast to eductive convergence, can be obtained for any degree of strategic substitutability.

The main result of the following analysis is to demonstrate that, in the context of self-referential models with a reduced form (1), the adoption of sequentially enlarges the eductive convergence region with respect to the case of simultaneous coordination. In particular, provided number of types is sufficiently large, eductive convergence occurs in accordance with the E-stability principle, that is for any finite $\beta < 1$.

4 Sequential coordination and eductive convergence

This paper looks at the case where there is a finite number of types and coordination is sequential according to the following definition.

Definition 1 *Consider a linear economy whose reduced form encompasses (1) populated by a continuum of agents grouped into a finite number of types indexed by $\tau \in \{1, .., n\}$ with relative size λ_τ such that $\sum_{\tau=1}^{n} \lambda_\tau = 1$. Sequential coordination requires that agent type τ forms his expectation - or equivalently fixes his action - after observing the average forecast - or action - of earlier types from 1 to $\tau - 1$.*

Notice that, in both models presented in section 2, there is a one-to one relationship between an individual choice and an individual belief on the average action. In the following analysis we will refer mostly to expectations with the understanding that through the observation of actions agents elicit expectations. To better enlighten the main mechanism let us first focus on the simplest case of only two types, and then switch to the general case with a finite number.

4.1 Sequential coordination with two types

Consider the case in which the population is divided in two sequential types, namely type 1 and type 2. Agents type 2 of mass $\lambda \in (0, 1)$ form their expectations observing the average expectation held by agents type 1 of mass $1 - \lambda$. In the subgame played by type 2, the average price expectation of type 1 is fixed and directly observable; call it $\bar{e}_1$ where, from here onward, we will use an upper bar to denote a predetermined variable. Since agents type 2 commonly know (1), her individual price expectation $e_{i,2}$ is formed according to

$$e_{i,2} = \beta (1 - \lambda) \bar{e}_1 + \beta \lambda e_2, \tag{2}$$

that is, a map $B_2 : \aleph^2 \to \aleph$ which assigns an optimal individual action $e_{i,1}$ to each individual belief about a pair of average actions $(e_1, e_2) \in \aleph^2$ about both his own and

[6] For an exhaustive formulation of the E-stability principle see Evans and Honakpohja (2001).

the other type. Agents type 2 they all observe the average expectation of the other type $\bar{e}_1 \in \aleph$, that is therefore predetermined to their eductive reasoning. In analogy with the classical case, given CK that $e_2 \in \aleph$ and that all agents type 2 rationally conform to (2), then it is CK that a possible rational outcome lies in $B_2(\bar{e}_1, \aleph)$. But given $B_2(\bar{e}_1, \aleph)$ is also CK then, for the same argument, agents know that a rational outcome cannot lie out of $B_2^2(\bar{e}_1, \aleph)$, and so on along an infinite hierarchy of higher-order beliefs. Therefore is CK that a final outcome has to be in $\lim_{\nu \to \infty} B_2^{\nu}(\bar{e}_1, \aleph)$. If and if B_2 is a contracting map with respect to the variable e_2, that is $|\beta\lambda| < 1$, then the iterative applications of rational responses pin down

$$e_{i,2} = \hat{e}_2(\bar{e}_1) = \frac{\beta(1-\lambda)}{1-\beta\lambda}\bar{e}_1, \tag{3}$$

where $\hat{e}_2(\bar{e}_1)$ entails the unique common rational expectation of type 2 *conditional on* $\bar{e}_1$, the observed average price expectation of type 1.

Since the rationalizable outcome of the subgame played by type 2 is CK, this knowledge can be used to anticipate the rational actions by type 1 as follows. Consider the whole game where agents type 1 form their expectations according to

$$e_{i,1} = \beta(1-\lambda)e_1 + \beta\lambda e_2 \tag{4}$$

that is, the same map $B_2 : \aleph^2 \to \aleph$. Nevertheless, there is an important difference in the eductive reasoning of agents type 1: it is CK that *as long as* $|\beta\lambda| < 1$ holds then the action of agents type 2 will be $\hat{e}_2(e_1)$. In such a case, (4) can be written as an individual response to a conjecture on the average expectation of his own type only

$$e_{i,1} = \frac{\beta(1-\lambda)}{1-\beta\lambda}e_1 \tag{5}$$

after substituting (3) into (4) with due changes in upper bar subscripts. Notice that (5) is the same as (3) apart that type 1 does not observe their own average action which has to be anticipated (so no upper bar). Following again the standard higher-order beliefs argument, given CK that $e_1 \in \aleph$ and that all agents type 1 rationally conform to (5), then it is CK that a possible rational outcome lies in $B_{1,2}(\aleph) \equiv B_2(\aleph, \hat{e}_2(\aleph))$. But given $B_{1,2}(\aleph)$ is also CK then, for the same argument, agents know that a rational outcome cannot lie out of $B_{1,2}^2(\aleph)$, and so on along an infinite hierarchy of higher-order beliefs. Therefore it is CK that a final outcome has to lie in $\lim_{\nu \to \infty} B_{1,2}^{\nu}(\aleph)$. We have therefore the following definition.

Definition 2 *The unique REE is sequentially eductively stable if and only if both* B_2 *and* $B_{1,2}$ *are contracting maps in, respectively,* e_2 *and* e_1*, that is,*

$$|\beta\lambda| < 1 \tag{6}$$

and

$$\left|\frac{\beta(1-\lambda)}{1-\beta\lambda}\right| < 1, \tag{7}$$

for a given $\lambda \in (0,1)$.

The following proposition states when this condition is met.

Proposition 3 *In case of strategic complementarity ($\beta > 0$) the unique REE is sequentially eductively stable when*

$$\beta < 1,$$

independently of λ, the relative size of type 2. In case of strategic substitutability ($\beta < 0$) instead, the unique REE is sequentially eductively stable whenever

$$\beta > -\frac{1}{1-2\lambda},$$

with $\lambda \leq 1/3$, or

$$\beta > -\frac{1}{\lambda},$$

with $\lambda > 1/3$.

Proof. In case of strategic complementarity, $\beta > 0$, (6) implies $\beta\lambda < 1$, whereas (7) yields $\beta < 1$, which is a stricter condition.

In case of strategic substitutability instead, $\beta < 0$, (6) implies $-\beta\lambda < 1$ that is $\beta > -\lambda^{-1}$. The second inequality (7) is satisfied when $\beta > (2\lambda - 1)^{-1}$ provided $\lambda \in [0, 1/2]$, and always otherwise. The latter is binding whenever $-\lambda^{-1} < (2\lambda - 1)^{-1}$ from which the proposition. ■

The condition for eductive convergence when coordination is sequential is larger than the one obtained under simultaneous coordination. In particular, sequential coordination weakens the conditions for convergence in case of strategic substitutability, whereas does not affect the case of strategic complementarity. Figure 1 plots the increase of the parametric region of eductive convergence. Obviously, at the limits of $\lambda \to 0$ and $\lambda \to 1$ we get the same result obtained in the simultaneous setting.

The difference between the case of strategic complementarity and substitutability can be simply discussed looking at (6)-(7). The first condition (6) tell us that the smaller is the weight of agents type 2 the more likely is eductive convergence on their own common rational response. The same logic applies to (7) for type 1. Nevertheless the two differ for a factor $1/(1 - \beta\lambda)$, which measures the impact of the *sequential spillover* of agents type 2 on agents type 1.

With strategic complementarity, $\beta > 0$, the first condition (6) implies that $1/(1 - \beta\lambda)$ is strictly greater than one in the relevant case. This means that the common action of type 2 has a *destabilizing* feedback effect on the eductive reasoning of agents type 1: when agents type 1 conjecture how the actual price will react to their average expectation, they take into account the common reaction of type 2 which moves type 1's higher-order conjectures in the same direction *further away* from the unique REE. In particular, the additional feedback is such that there are no gains, in terms of the magnitude of the convergence region, in introducing sequential types.

With strategic substitutability, $\beta < 0$, the picture changes dramatically. The sequential factor $1/(1 - \beta\lambda)$ is positive but always smaller than one. This means that

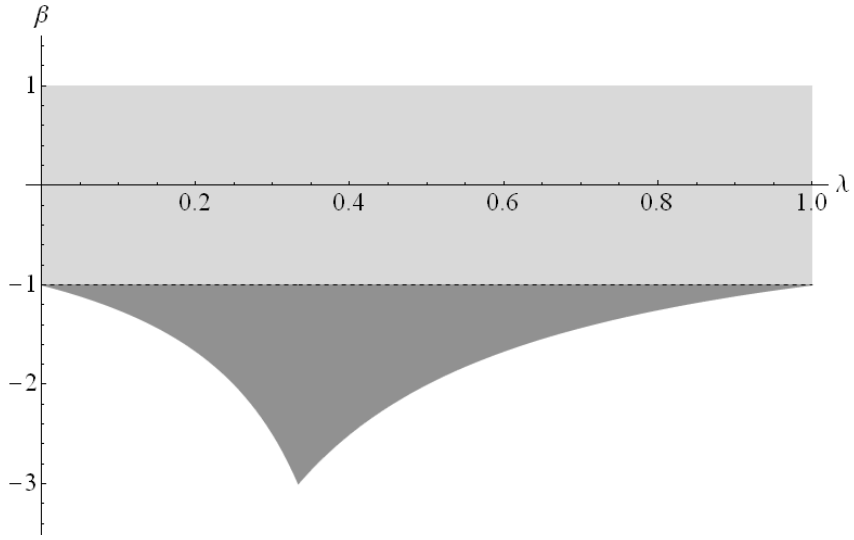

Figure 1: The parametric region of eductive convergence with two sequential types. Darker grey denotes the additional region obtained because of sequentiality.

the common action of agents type 2 has a *stabilizing* feedback effect on the eductive reasoning of agents type 1: when agents type 1 conjecture how the actual price will react to their average expectation they take into account the common reaction of type 2 which moves types 1's higher-order conjectures in the opposite direction *closer* to the unique REE. In fact, due to strategic substitutability the common action of type 2 will counteract to the average action of type 1. This effect stabilizes the eductive reasoning widening the eductive convergence region.

Notice that the size of population type 2 affects the stability conditions (6) and (7) in opposite ways. In the case of substitutability, the stabilizing sequential spillover from type 2 to type 1 gets stronger with λ. The intuition is simple. Let us take β slightly greater than -3 and $\lambda = 1/3$. Suppose, as a starting point, that type 1 considers an unit price increase away from the REE ($e_{i,1} = 1$). If actions of type 2 were fixed, type 1 should conclude that, in a second order guess, the price will drop of slightly less than two units ($\beta\,(1-\lambda)$). Nevertheless, type 1 knows that eventually type 2 will counteract to such (observed) drop making the price decreases only slightly less than one unity ($\beta\,(1-\lambda)\,/(1-\beta\lambda)$). This way a contraction in beliefs occurs for type 1. The contraction is stronger as λ gets larger. On the other hand, as λ increases, eductive convergence among agents type 2 becomes more demanding; hence $\beta = -3$ makes $\lambda = 1/3$ binding. This trade-off results in the asymmetric shape of the frontier of the eductive convergence region plotted in figure 2. The contracting properties of sequential coordination between two types are maximized exactly when type 2 has a relative size of 1/3. In such a case convergence occurs for any β-value belonging to the interval $(-3, 1)$.

4.2 Sequential coordination with a finite number of types

This section generalizes the previous result to a finite number n of types. The argument uses backward induction, so I firstly describe the rationalizable outcome in the last subgame played by agent type n and then I present the iterative step to recover convergence in the generic subgame.

The last subgame. Agents type n choose their own actions once all the other types already did. Agents type n only need to care about expectational coordination among themselves. The best reply function of agents type n is

$$e_{i,n} = \beta \bar{e}_{1|n-1} + \beta \lambda_n e_n, \tag{8}$$

where $e_{1|n-1} \equiv \sum_{h=1}^{n-1} \lambda_h e_h$ denotes a weighted sum of average expectations of the earlier types. Such expectations are observable by agents type n so they appear with an upper bar. (8) entails a map $B_n : \aleph^n \to \aleph$ which assigns an optimal individual action $e_{i,n} \in \aleph$ to an individual belief about the average action of all types, his own included. Nevertheless, agents type n observe the average actions of the other types $\bar{e}_{-n} \equiv \{\bar{e}_1, .., \bar{e}_{n-1}\} \in \aleph^{n-1}$ which are therefore predetermined to their eductive reasoning. Given CK that $e_n \in \aleph$ and that all agents type n rationally conform to the rule (2), then it is common knowledge that a possible outcome will lie in $B_n(\bar{e}_{-n}, \aleph)$. But given $B_n(\bar{e}_{-n}, \aleph)$ is also CK, then, for the same argument, agents know that a rational outcome cannot lie out of $B_n^2(\bar{e}_{-n}, \aleph)$, and so on along an infinite hierarchy of higher-order beliefs. Therefore is CK that a final outcome has to belong to $\lim_{\nu \to \infty} B_n^{\nu}(\bar{e}_{-n}, \aleph)$. If B_n is a contracting map in e_n, that is $|\beta \lambda_n| < 1$, then the iterative applications of rational responses pin down

$$e_{i,n} = \hat{e}_n(\bar{e}_{-n}) = \frac{\beta}{1 - \beta \lambda_n} \bar{e}_{1|n-1}, \tag{9}$$

that is, a common rational response $\hat{e}_n$ to a profile $\bar{e}_{-n}$ of average expectations held by types from 1 to $n-1$. Since the rationalizable outcome of this subgame is CK, then this knowledge can be used to anticipate the rational action of type n in earlier subgames as explained below.

The generic subgame. We initially guess and then verify that if eductive convergence occurs in the subgames from n to $\tau+1$, then it is CK that the common rational response of agents of types from n to $\tau+1$ is given by

$$e_{i,n} = ... = e_{i,\tau+1} = \hat{e}_{\tau+1}\left(\bar{e}_{-(\tau+1)}\right) = \frac{\beta}{1 - \beta \lambda_{\tau+1|n}} \bar{e}_{1|\tau}, \tag{10}$$

where $\lambda_{\tau+1|n} \equiv \sum_{h=\tau+1}^{n} \lambda_h$ and $\bar{e}_{-\tau} \equiv \{\bar{e}_1, .., \bar{e}_{\tau-1}\} \in \aleph^{\tau-1}$. By convention $\lambda_{n+1|n} = 0$ and $\bar{e}_{-1} = 0$. Consider now the subgame where agents type τ form their expectations according to

$$e_{i,\tau} = \beta \bar{e}_{1|\tau-1} + \beta \lambda_\tau e_\tau + \beta e_{\tau+1|n} \tag{11}$$

with $e_{\tau+1|n} \equiv \sum_{h=\tau+1}^{n} \lambda_h e_h$, that is the same map $B_n : \aleph^n \to \aleph$. However, as long as eductive convergence occurs for the following types, (11) can be written as an individual response to a conjecture on the average expectation of his own type only

$$e_{i,\tau} = \frac{\beta}{1 - \beta \lambda_{\tau+1|n}} \bar{e}_{1|\tau-1} + \frac{\beta \lambda_\tau}{1 - \beta \lambda_{\tau+1|n}} e_\tau \tag{12}$$

after substituting (10) with appropriate bar upscripts into (11).[7] It is useful to remark that the only difference between (12) and (10) is an epistemic one: the average action of type τ is unobserved to agents type τ (so it shows up without an upper bar in (12)) whereas it is a predetermined variable for agents type $\tau+1$. For each τ going from 1 to n let us denote

$$B_{\tau,n}\left(\bar{e}_{-\tau},\aleph\right)\equiv B_{\tau+1,n}\left(\bar{e}_{-\tau},\aleph,\hat{e}_{\tau+1}\left(\aleph\right)\right)$$

where $B_{\tau,n}\left(\bar{e}_{-\tau},\aleph\right):\aleph^{\tau}\rightarrow\aleph$ being $B_{n,n}\left(\bar{e}_{-n},\aleph\right)=B_{n}\left(\bar{e}_{-n},\aleph\right)$. The generic $B_{\tau,n}\left(\bar{e}_{-\tau},\aleph\right)$ is a map which assigns an optimal individual action $e_{i,\tau}\in\aleph$ to an individual belief about the average action of his own type $e_{\tau}\in\aleph$, conditionally on a given profile of observable expectations of the earlier types $\bar{e}_{-\tau}$.

The eductive argument repeats as usually. Given CK that $e_{\tau}\in\aleph$ and that all agents rationally conform to (2), then it is common knowledge that a possible outcome will lie in $B_{\tau,n}\left(\bar{e}_{-\tau},\aleph\right)$. But given $B_{\tau,n}\left(\bar{e}_{-\tau},\aleph\right)$ is also CK then, for the same argument, agents know that the rational outcome cannot lie out of $B^{2}_{\tau,n}\left(\bar{e}_{-\tau},\aleph\right)$, and so on along an infinite hierarchy of higher-order beliefs. Therefore is CK that the final outcome has to be in $\lim_{\nu\rightarrow\infty}B^{\nu}_{\tau,n}\left(\bar{e}_{-\tau},\aleph\right)$. The following extends definition 2 to the case with a finite number of types.

Definition 4 *The unique REE is sequentially eductively stable if and only if $B_{\tau,n}\left(\bar{e}_{-\tau},\aleph\right)$ are contracting maps for every type, that is when*

$$\left|\frac{\beta\lambda_{\tau}}{1-\beta\lambda_{\tau+1|n}}\right|<1 \tag{13}$$

for each $\tau\in\{1,..,n\}$.

The following proposition states when this condition is met.

Proposition 5 *In case of strategic complementarity ($\beta>0$) the unique REE is sequentially eductively stable when*

$$\beta<1,$$

independently on the number n and relative sizes $\{\lambda_{\tau}\}_{0}^{n}$ of sequential types. In case of strategic substitutability ($\beta<0$) instead, the unique REE is sequentially eductively stable whenever

$$\beta>\ell\left(\{\lambda_{\tau}\}_{0}^{n}\right)\equiv\max_{\tau\,:\,\lambda_{\tau}\geq\lambda_{\tau+1|n}}\left\{\frac{1}{\lambda_{\tau+1|n}-\lambda_{\tau}}\right\} \tag{14}$$

where $\ell\left(\{\lambda_{\tau}\}_{0}^{n}\right)<-1$, unless there exists a type $\lambda_{\tau}=1$ in which case $\ell\left(\{\lambda_{\tau}\}_{0}^{n}\right)=-1$.

[7] In particular, given the coordination of agents from type n to $\tau+1$, we have

$$e_{\tau+1|n}=\lambda_{\tau+1|n}\hat{e}_{\tau+1}=\frac{\lambda_{\tau+1|n}\beta}{1-\beta\lambda_{\tau+1|n}}e_{1|\tau}=\frac{\lambda_{\tau+1|n}\beta}{1-\beta\lambda_{\tau+1|n}}\left(\lambda_{\tau}e_{\tau}+e_{1|\tau-1}\right)$$

using the relation $e_{1|\tau}=\lambda_{\tau}e_{\tau}+e_{1|\tau-1}$.

Proof. The proof is about working out (13). Let investigate first the case of strategic complementarity $\beta > 0$. Notice that $\beta\lambda_{n+1|n} = 0$ so that (13) requires $\beta\lambda_{n|n} = \beta\lambda_n < 1$. Given that $\beta\lambda_{n|n} < 1$ then (13) requires $\beta\lambda_{n-1|n} < 1$. Iterating we have that, provided $\beta\lambda_{\tau+1|n} < 1$ then (13) requires $\beta\lambda_{\tau|n} < 1$, for each $\tau \in \{1, .., n\}$ where $\lambda_{\tau|n} = \lambda_\tau + \lambda_{\tau+1|n}$. Finally, given that $\beta\lambda_{2|n} < 1$ then (13) requires $\beta\lambda_{1|n} = \beta < 1$. Therefore, we can conclude that, in the case of complementarity, $\beta < 1$ is the relevant condition for sequential eductive convergence.

In case of strategic substitutability $\beta < 0$ we have to distinguish two cases. First, for a type τ such that $\lambda_\tau \leq \lambda_{\tau+1|n}$ then (13) is satisfied without further restrictions. Second, for the type τ for which instead $\lambda_\tau > \lambda_{\tau+1|n}$ then (13) requires

$$\beta > \frac{1}{\lambda_{\tau+1|n} - \lambda_\tau}.$$

Therefore, the most restrictive condition obtains for the τ^* such that $\lambda_{\tau^*} > \lambda_{\tau^*+1|n}$ which makes $\left(\lambda_{\tau^*+1|n} - \lambda_{\tau^*}\right)^{-1}$ maximal. Notice that by construction a τ^* always exists since $\lambda_n > \lambda_{n+1|n} = 0$ so that $-\lambda_n^{-1}$ is always a lower bound to β. ■

Like in the case of two types, the introduction of sequential types does not affect the conditions for eductive coordination in the case of strategic complementarity. With strategic substitutability instead the region of eductive convergence enlarges depending on the distribution of relative sizes of the types. In particular a negative β must be larger of any negative $\left(\lambda_{\tau+1|n} - \lambda_\tau\right)^{-1}$. Notice that, by construction, type $\tau = n$ has a size greater than the cumulative size of its predecessors, which is zero, so $-\lambda_n^{-1} < -1$ is always a lower bound to β. Also note that $\lambda_\tau - \lambda_{\tau+1|n}$ cannot be larger than one. We therefore conclude that sequential coordination strictly enlarges the parametric region of eductive convergence. This effect is due to the presence of a sequential spillover $1-\beta\lambda_{\tau+1|n}$ in (13) which (de)stabilizes the eductive reasoning of agent type τ in case of strategic substitutability (complementarity).

Sequential coordination enlarges the conditions for eductive convergence in the case of strategic substitutability but *not* in the case of strategic complementarity. The following proposition establishes when an equivalence between the E-stability principle and sequential eductive convergence conditions obtains.

Corollary 6 *The unique REE is sequentially eductively stable whenever $\beta < 1$, in accordance with the E-stability principle, provided: (i) the size of the last type n is smaller than $\left|\beta^{-1}\right|$, and (ii) the size of each type is at most as big as the cumulative weight of the subsequent ones.*

With a sufficiently large number of sequential types sequential eductive convergence can occur at the same conditions of E-stability. In particular, the size of type 1 can be large up to 1/2 without preventing eductive convergence at any negative β value. But this is only a necessary condition. With $\lambda_1 = 2^{-1}$ the weight of type 2 can be at most 2^{-2}. Along this line of reasoning, the distribution of types' sizes must follow the rule $\lambda_\tau = 2^{-\tau}$ for $\tau \in \{1, .., n\}$. In practice, the distribution of sizes that maximizes the region of eductive stability in case of strategic substitutability provides for later types having smaller relative sizes. The simple intuition beyond this feature is that later types have less of the following types from which benefit. The last type does not take advantage of any spillover, as a consequence the convergence conditions relative to it are the most stringent. The type before the last instead can have a greater relative size (meaning a greater impact on the average expectation) without affecting stability: its action, in fact, will be partly offset by the following type who counteracts because of strategic substitutability. Iterating the argument we easily conclude that the distribution of types' sizes that induces maximal stability has to be decreasing in the sequential order of types.

Another property of sequential coordination is that the eductive convergence region is widened soon as the number of types increases to few. The following proposition states the result.

Proposition 7 *Given a negative β, then the minimal number $n^*(\beta)$ of sequential types that can achieve coordination is the minimal $n \in \mathbb{N}$ such that $2^n > 1 - \beta$.*

Proof. With a $|\beta| < 1$ a single type is sufficient to achieve coordination. Consider now a particular $\beta < -1$. To obtain sequential eductive convergence for such value, the size of the last type n must be strictly smaller than $-1/\beta$. Hence, the maximal size of of the last type n obtains at the left-sided limit $\lambda_n \nearrow -1/\beta$.[8] In such a case, to comply with (14), the size of type $n-1$ must be strictly smaller than $-2/\beta$. When $\lambda_{n-1|n}$ is at its maximal size, that is with $\lambda_n \nearrow -1/\beta$ and $\lambda_{n-1} \nearrow -2/\beta$, then (14) requires λ_{n-2} must be strictly smaller than $-4/\beta$. In sum, to bind the stability conditions, the size of the generic type $\tau \in \{1, .., n\}$ must be

$$\lambda_\tau \nearrow -\frac{2^{n-\tau}}{\beta}. \tag{15}$$

In fact, since $2^{n-\tau} - \sum_{h=0}^{n-\tau-1} 2^h = 1$ for each $n - \tau \in \mathbb{N}$ then

$$\lim_{\lambda_\tau \nearrow -\frac{2^{n-\tau}}{\beta}} \frac{1}{\lambda_{\tau+1|n} - \lambda_\tau} < -\frac{\beta}{\sum_{j=0}^{n-\tau-1} 2^j - 2^{n-\tau}} = \beta$$

for each $\tau \in \{1, .., n\}$ for whatever n.

We now have to determine $n^*(\beta)$ that is the smaller n such that the sum of types with maximal size (with (15) binding) is at least one. The sum of the maximal weights of types is equal to

$$\sum_{\tau=1}^{n} \lambda_\tau = -\frac{\sum_{j=0}^{n-1} 2^j}{\beta} = -\frac{2^n - 1}{\beta}$$

so that $n^*(\beta)$ is the minimal n such that

$$-\frac{2^n - 1}{\beta} > 1$$

with $\beta < 0$, which finally gives the condition $2^n > 1 - \beta$. This means that for $n^*(\beta) - 1$ types is not possible to achieve sequential eductive convergence because necessarily at

[8] I use the notation $x \nearrow M$ to denote the left-sided limit otherwise denoted by $x \to M_-$.

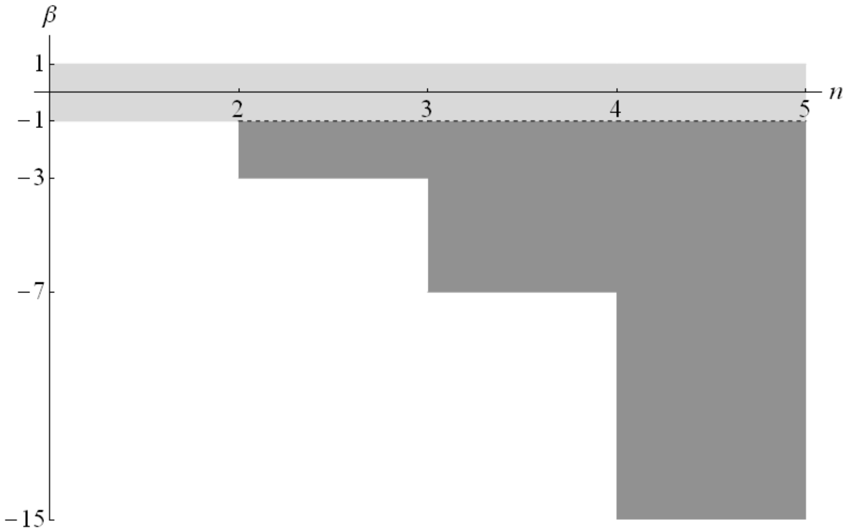

Figure 2: The parametric region of eductive convergence with a finite number of sequential types. Darker grey denotes the largest additional region obtained with the introduction of n sequential types.

least one type's size must violate (15). As a consequence at least $n^*(\beta)$ types are needed to achieve convergence. Of course, the sum of sizes of types must be exactly one. To construct a distribution of $n^*(\beta)$ types in order to bind stability conditions (14), the size of the first type must be chosen equal to

$$\lambda_1 = 1 - \sum_{\tau=2}^{n^*(\beta)} \lambda_\tau$$

with other types' sizes binding on (15). ■

The correspondence $n^*(\beta)$ is illustrated in figure 2. The implication is that relatively few sequential types are needed to ensure eductive coordination in linear self-referential economies with high degrees of strategic substitutability ($\beta < -1$). Put differently, the larger the number of types the larger the additional region of eductive convergence (denoted with a darkest gray) that one more sequential type ensures. The finding suggests that sequential coordination is a relatively cheap policy tool that can be used to implement a market mechanism granting expectational coordination according to the principle of eductive stability.

5 Sequential spillovers and adaptive learning

In this section I outline the relation between sequential eductive learning and adaptive learning as introduced in section 3.2. First of all, let me notice that under adaptive learning the possibility of observing others' expectations is irrelevant. Suppose agents are adaptive learners and they act sequentially *within the same period* according to their type, as assumed above. The first type will use only the series of past outcomes as regressors. The second one can use the same information set plus the observation of the average forecast of the first type, which in fact does not add new information. The argument can be easily iterated to establish that all players use the same information set at any period.

Nevertheless, the importance of sequential spillovers *between periods* also shows up in the adaptive setting. Let assume now that the economy repeats every period $t \in \mathbb{N}$. At each period the economy is populated by one single type t of agents which form their own expectations on the current market price using past data. First, let us consider the simplest form of adaptive (naive) learning

$$e_{i,t} = e_t = p_{t-1} \tag{16}$$

where agents just expect the current market price equal to the past one. This generates the well known Cobweb tatonnement. Convergence occurs with $|\beta| < 1$ as in the case of eductive learning with a single type. Consider now agents a bit more sophisticated that also include in their forecast rule a constant-gain correction term derived from the *observation* of the past price expectation e_{t-1} (which is evidently predetermined). That is

$$e_{i,t} = e_t = e_{t-1} + \gamma \left(p_{t-1} - e_{t-1}\right) \tag{17}$$

where γ is a gain measuring the impact of the forecast error last period. With $\gamma = 1$, (17) is the same as (16). With a time-varying decreasing gain $\gamma_t = t^{-1}$, (17) becomes a recursive expression for the average market price from time 1 to $t-1$: nothing else than an univariate OLS regression on a constant. Bray and Savin (1986) is the first paper establishing that the REE of the Cobweb model is stable under OLS learning whenever the E-stability condition $\beta < 1$ holds. With a constant gain $\gamma \in (0, 1)$ instead, (17) is a recursive expression for a weighted average with more recent updates having a higher impact. A classical result of adaptive learning literature maintains that the E-stability principle still applies, provided the gain γ is small enough (Evans and Honkapohja, 2001).

To see how sequential spillovers intervene in enlarging the convergence region into the strategic substitutability territory, let us rewrite (16) using (1) as

$$e_t = \left(1 + \gamma \left(\beta - 1\right)\right) e_{t-1}.$$

This is a map $A : \aleph \to \aleph$ linking an observed expectation $e_{t-1} \in \aleph$ into a subsequent expectation $e_t \in \aleph$. Convergence in the long-run to the unique REE occurs if and only if $\lim_{\nu \to \infty} A^{\nu}(\aleph) = 0$. This requires $|1 + \gamma(\beta - 1)| < 1$. In case of high strategic complementarity, $\beta > 1$, the sequential spillover $\gamma(\beta - 1)$ from expectation at time $t-1$ to expectations at time t will prevent a contraction no matter how small γ is. This is why, in case of strategic complementarity, the region of adaptive convergence does not enlarge using (17) instead of (16). In case of substitutability instead, for a given negative β, long-run adaptive convergence obtains as long as

$$\gamma < \frac{2}{1 - \beta},$$

that is, provided the gain γ is small enough. In the case of a type-varying decreasing gain this condition is satisfied from a certain time onwards no matter which negative value of β is considered, that is, the E-stability principle holds. With $\gamma = 1$ the convergence condition collapses to the case of naive expectations.

Notice how close is the condition above to the one recovered in proposition 7. We can rewrite that conditions as

$$2^{1-n^*} < \frac{2}{1-\beta}$$

which says sequential eductive convergence is ensured with a large-enough number of sequential types, similarly to the case of an adaptive constant-gain rule. Provided the number of types is large enough, this condition is satisfied independently for any negative value of β, in accordance with the E-stability principle. With $n^* = 1$ the convergence condition collapses to the classical case. In sum, the key force at play is the same in both settings: strategic substitutability introduces the possibility of favorable sequential spillovers which dampen the elasticity of agents' expectations and support expectational convergence.

6 Conclusions

This paper has explored eductive convergence when coordination is sequential, that is, when each agent of a given type fixes his own forecast after observing the ones of earlier types in a given order. When coordination is sequential, the unique REE of the benchmark Cobweb model is stable under sequential eductive learning for any level of strategic substitutability provided the number of types is large enough. That is, sequential eductive convergence can potentially occur in accordance with the E-stability principle. On the contrary, sequential coordination does not enlarge the region of eductive convergence in the case of strategic complementarity.

The finding sheds light on the subtle role played by the introduction of real time in eductive convergence. It implies that agents cannot disagree on the forecasts of their predecessors in analogy with adaptive learning where agents cannot disagree on commonly observed past data. This epistemic restriction originates sequential spillovers that stabilize the higher-order belief dynamics in case of strategic substitutability.

References

[1] Aumann R. (1976) "Agreeing to Disagree". *The annuals of Statistics*, vol. 4, 1236-1239.

[2] Bray M.M. and Savin N.E (1986) "Rational expectations equilibria learning and model specification". *Econometrica*, 54 , pages 1129–1160.

[3] Bernheim B.D. (1984) "Rationalizable Strategic Behavior". *Econometrica*, vol. 52, pages. 1007-1028.

[4] Borges T. and Janssen M.C.W. (1995). "On the Dominance Solvability of Large Cournot Games". *Games and Economic Behavior*, vol. 8, pages 297-321.

[5] Chamley P. C. (2004) "Rational Herds: Economic Models of Social Learning". Cambridge University Press, Cambridge.

[6] Evans G.W. (1985) "Expectational Stability and the Multiple Equilibria Problem in Linear Rational Expectations Models". *Quarterly Journal of Economics*, vol. 100, pages 1217-1233.

[7] Evans G.W. (2001) "Expectations in Macroeconomics Adaptive versus Eductive Learning" *Revue économique*, vol. 52, pages 573-582.

[8] Evans G.W. and Guesnerie R. (1993). "Rationalizability, Strong Rationalizability and Expectational Stability". *Games and Economic Behavior*, vol. 5, pages 632-646.

[9] Gaballo G. (2013) "Eductive Learning and the Rationalizability of Oligopoly Games". Economics Letters, vol. 120, pages 401–404.

[10] Guesnerie R. (1992) "An Exploration of Eductive Justifications of the Rational Expectations Hypothesis". *American Economic Review*, vol. 82, pages 1254-78.

[11] Guesnerie R. (1993) "Theoretical Tests of the Rational Expectations Hypothesis in Economic Dynamical Models". *Journal of Economic Dynamics and Control,* vol. 17, pages 847-864.

[12] Guesnerie R. (2005) " Assessing Rational Expectations 2: 'Eductive' Stability in Economics". Massachusetts Institute of Technology.

[13] Muth J.F. (1961) "Rational Expectations and the Theory of Price Movements". *Econometrica*, vol. 29 pages 315-335.

[14] Lucas R. (1973) "Some International Evidence on Output-Inflation Trade-offs", *American Economic Review*, vol. 63, pages 326-334.

[15] Pearce D.G. (1984) "Rationalizable Strategic Behavior and the Problem of Perfection". *Econometrica*, vol.52, pages 1029-1050.

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